The role of meteorological focusing in generating rogue wave conditions

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Abstract. Rogue waves are believed to be the consequence of focusing of wave energy. While there are several ways that energy may be focused, we concentrate here on the role of meteorological patterns in generating mixed sea conditions. We demonstrate the sharp increase in the probability of high wave crests for a given significant height when the sea is mixed, i.e., consists of wave trains arriving from very different directions. Then, using a full spectral wave prediction model forced by NCEP winds on the Atlantic Ocean, we illustrate the role of meteorological focusing in enhancing the probability of occurrence of rogue waves.

Introduction

The existence of "rogue waves" on the oceans is established through the recorded encounters with ships or manned offshore platforms. Seafarers and offshore workers quickly gain a healthy respect for the sea and, in particular, the energy containing surface gravity waves that, at times, present a hazard to ships and structures. The statistics of wave heights is well-known in a typical narrow banded (in frequency and direction) wind driven sea. A wave exceeding the significant height occurs roughly every 7.5 periods, while waves of crest-to-trough extent more than twice the significant height (defined as "freak", *Kjeldsen*, 1996) typically occur less frequently than one in three thousand waves. If the criterion for freakness is taken to be just 10% more, i.e., 2.2 H_s then Raleigh statistics appropriate to narrow banded (linear) gaussian seas would suggest that one in 17,000 is closer to the expectation for a very unusual ("freak") wave. For typical storm wave periods of 15 seconds, this means that such a wave would occur on average every 70 hours at a given location.

Thus, the concept of "freakness" is clearly bound up with the expectation of rarity, in the context of local conditions. But is a wave twice as big as the significant height likely to damage a well-designed structure or ship operated in a standard sea-keeping fashion, or are there characteristics of these rogue waves over and above mere height that render them particularly hazardous? Taking the fluid drag on a fixed piling as an example of the damaging force per unit area (P_c) that may be exerted by the crest of waves near the spectral peak, this is proportional to the steepness, $\pi H/\lambda_p$ and the phase propagation speed c_o:

$$P_c \propto \left(\frac{\pi H}{\lambda_p} c_o\right)^2 \tag{1}$$

where H is trough to crest height and λ_p wavelength of the spectral peak waves.

Thus a rogue wave of double the significant height would produce a crest horizontal pressure of four times that of the significant height waves. If the rogue wave is breaking, the water velocity at the crest travels at the phase speed, c_o producing a further increase of a factor of at least ten, based on the observation that the spectral peak waves do not break when $\pi H/\lambda_p < 0.3$. Thus a monster wave packing a four-fold wallop may be a hazard to ships and offshore structures, but, if it is also breaking, the forty fold increase above the design conditions will almost certainly produce significant damage.

Focusing of wave energy

The self-similarity of a wind-generated sea is well-established and the significant height H_s and peak period T_p and peak wavelength λ_p are given by:

$$H_s = 0.2\,\frac{U_{10}^2}{g}\left(\frac{C_p}{U_{10}}\right)^{1.65}$$
$$T_p = 2\pi\,\frac{U}{g}\times\frac{C_p}{U_{10}} \qquad (2)$$
$$\lambda_p = 2\pi\,\frac{U^2}{g}\times\left(\frac{C_p}{U_{10}}\right)^2$$

So that the steepness of the peak waves of significant height, SS_{H_s} , is given by:

$$SS_{H_s} = \pi\,\frac{H_s}{\lambda_p} = 0.1\left(\frac{C_p}{U_{10}}\right)^{-0.35} \qquad (3)$$

where $\frac{C_p}{U_{10}}$ is the wave age.

These relationships are taken from observations of fetch-limited waves (*Donelan et al.*, 1985). Breaking of the peak waves in a wind generated spectrum occurs for $SS > 0.3$. Consequently breaking of these peak waves will occur when their height exceeds the significant height by this ratio:

$$\frac{H_B}{H_S} > 3\left(\frac{C_p}{U_{10}}\right)^{0.35} \qquad (4)$$

For full development, corresponding to wave age of 1.2, the spectral peak waves break when H exceeds 3 H_S, while in strongly forced storm conditions (for wave age of 0.3, say) breaking of the spectral peak waves will occur at more modest heights of $2\,H_S$.

Wave energy may be focused in deep water by their interaction with current gradients or simply through the propagation of energy from different areas where the meteorological forcing yields waves propagating towards a common destination. This may be through different meteorological disturbances in the same ocean basin or even within a single storm where the curvature of the streamlines generates waves propagating from one sector of the storm to another producing a mixed sea of recent swell and local wind sea. This circumstance is common and can lead to hazardous conditions with enhanced probability of rogue waves and of breaking rogue waves.

The enhanced probability of rogue waves in mixed seas

The statistics of occurrence of wave crest heights (or, equivalently, the group envelope) in a random sea is quite well described by the Rayleigh distribution (*Longuet-Higgins*, 1952) based on the linear superposition of random wave components in a narrow-banded sea, although various higher order refinements (*e.g. Tayfun*, 1983) yield marginally better descriptions of the tail of the distribution. For illustrative purposes in considering the additive effect (superposition) of mixed seas on the statistics of high wave crests, the Rayleigh distribution is adequate.

It is important to note that the theoretical height distributions have been derived for the group envelope amplitude or equivalently, crest height above mean water level. The theoretical height distribution for linear seas is actually just that of twice the group envelope magnitude. The group envelope can only be observed at the crest or trough of the underlying wave making it impossible in a dispersive wave field to measure the actual height of a given wave from time series at a point. Furthermore in most cases rogue waves are dangerous not because their heights are extraordinary, but rather because their crest heights are. Nonlinear distortions to large waves tend to steepen the crests and flatten the troughs, with little change in the total height. Consequently, for both theoretical and practical reasons it is appropriate to discuss the probability of rogue waves in terms of crest heights above mean water level rather than crest to trough heights. Where we refer to height, H this is just double the crest envelope, 2A. In a wind-sea of mean variance of elevation about the mean sea level, σ^2, the probability of occurrence $p(A)$ of wave crests of height A is

$$p(A) = \frac{A}{\sigma^2}\exp\left\{\frac{-A^2}{2\sigma^2}\right\} \qquad (5)$$

so that the probability of exceedance of any crest height, A is

$$\int_A^\infty p(A)\,dA = \exp\left\{\frac{-A^2}{2\sigma^2}\right\} \qquad (6)$$

This is graphed in Figure 1 against the ratio of the exceedance of crest height A to σ. The vertical lines on the figure (eq. 4) indicate the heights that must be exceeded for breaking of the spectral peak waves to occur. At full development (wave-age = 1.2) this probability is essentially zero $\left(10^{-9}\right)$, whereas in the teeth of an intense storm with wave age of 0.3 the probability rises to 5×10^{-4} or about one breaker every eight hours for 15 second peak waves.

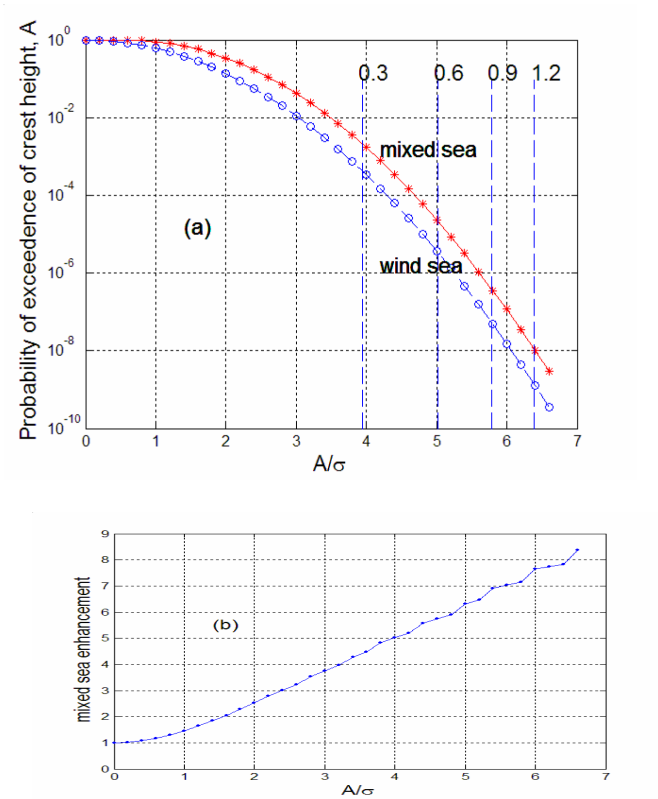


Figure 1. (a) the probability of exceedance of crest height in relation to the ratio of height to the standard deviation of surface elevation for both wind seas (lower curve) and mixed seas (upper curve). The vertical dashed lines indicate the heights above which breaking of the (spectral) peak waves will occur for the wave ages indicated at the top of the lines; (b) the enhancement (multiplier) to (a) for mixed seas.

Now consider a mixed sea consisting of two wave trains arising from different meteorological sources and therefore different directions and having variances of σ_1^2 and σ_2^2, where $\sigma_1^2 + \sigma_2^2 = \sigma^2$, i.e., the total wave energy in the mixed sea is the same as the reference wind sea. In a narrow banded spectrum of waves in frequency and direction eq. 5 applies to the envelope (or crest) heights along the propagation direction of the waves. It helps to consider an arbitrarily narrow spread of each wave train, so that the crests extend unattenuated across the propagation direction. We are seeking the probability of occurrence of crests of a given height in an area defined by the wavelengths of the two trains. Assuming statistical independence of these two trains in the mixed sea, the joint probability of occurrence of crest heights A_1 and A_2 in the mixed sea is therefore:

$$p(A_1, A_2) = \frac{A_1 A_2}{\sigma_1^2 \sigma_2^2} \exp\left\{ -\frac{A_1^2}{2\sigma_1^2} - \frac{A_2^2}{2\sigma_2^2} \right\} \qquad (7)$$

Under the constraint of linear superposition, this is also the probability of occurrence of crest height in the mixed sea, $A_m = A_1 + A_2$. Thus, the probability of mixed sea crest height, A_m exceeding $n\sigma$ may readily be computed from the integral of eq. 7: $\int_{A_1}^{\infty} \int_{n\sigma - A_1}^{\infty} p(A_1, A_2) dA_1 dA_2$ (*Donelan*, 2005b). The highest probabilities of extreme waves occur for the case of equal energy components $(\sigma_1 = \sigma_2)$ of the mixed sea and these are shown in Figure 1a. It is clear that the intersection of two trains produces an enhancement of the probability of extreme crests. The probability of exceedance of twice the standard deviation in a mixed sea is 2.5 times higher than for a pure wind sea and the enhancement increases with A/σ (Figure 1b). Furthermore these enhanced probabilities of higher crests raises the probability of high waves being also breakers and hence far more dangerous. In these mixed seas, the probability of the spectral peak waves breaking is still negligible for mature seas, but increases to 1/550 for wave age of 0.3. For 15 second waves this translates into a mean time between giant breakers of 2.5 hours. Thus, the maturity of the storms has a direct bearing of the danger of the seas, i.e., breaking or non-breaking giant waves, but the extreme crest heights are largely dependent on the mixture of focused seas. We further note that the intersection of the crests of the two wave trains travels in a direction between those of the two trains and at a speed, c_m, that is larger than the phase speed of either train:

$$c_m = \frac{c_1}{\cos\theta_1} = \frac{c_2}{\cos\theta_2} \qquad (8)$$

where c_1, c_2 are the phase speeds of the two trains and θ_1, θ_2 are their propagation directions with respect to the propagation direction of the intersecting crests. If the intersecting crest is breaking, then the water at the crest is traveling at this enhanced speed, c_m, greatly adding to the destructive force of the rogue.

The predictability of occurrence of mixed seas

Having identified the importance of mixed seas in increasing the probability of occurrence of rogue waves and the direct connection between age of sea and the probability that the (spectral) peak waves will break, the task of predicting the likelihood of dangerous rogue wave conditions boils down to basin scale wave forecasting of the size, age and directional spread of seas. Basin scale forecasting is emphasized because waves may arrive at a given location from storms in various parts of the basin. This "meteorological focusing" is one of several common ways of producing mixed seas. Other means of focusing wave energy at particular locations are due to interaction with currents, interaction with topography – both bathymetry and coastlines. For simplicity in the following discussion of the predictability of mixed seas, we limit ourselves to prediction of waves in deep water on the Atlantic Ocean without specified currents. In this case, the wave prediction model (*Donelan*, 2005a) will yield focused mixed seas due to meteorological patterns and to coastline topography. The latter focusing affect occurs in offshore winds when, even for a constant wind direction, mixed seas may result from fetch variations along the coast (*Donelan et al.*, 1985; *Walsh et al.*, 1989).

A convenient way of examining the directional properties of mixed seas is to generate "data" from the running of a full spectral model in realistic ocean conditions with the usual mix of storms in various parts of the ocean basin. In view of the focus of this paper we have selected a period in which there is a clear record of wave damage to a ship. A dredger broke in two and sank off the island of Madeira on March 25, 1996. The wind data used to drive the model is taken from the NCEP reanalysis archived data base (*Kalnay et al.*, 1996). Figure 2 shows the significant height map on the Atlantic, for UTC hour 00 on the previous day, computed from the model hindcast of the full wavenumber spectrum in 10 degree directional bins and Figure 3 is a blowup of the section of highest waves. The grid locations are the same as the NCEP wind data – approximately 2 degree resolution in latitude and longitude. The variations in color reflect the significant heights. On every 4th grid location, indicated by magenta dots, a "necklace" hangs. This is meant to reflect the directional distribution of wave energy integrated over all wavenumbers, so that the magnitude of the vectors from the magenta dot to each black dot in the necklace is proportional to the "partial significant height" in the direction of the vector, i.e., it is 4 times the square root of the sum of the energy integrated over all wavenumbers in that 10 degree directional bin. Consequently, the square root of the sum of the squares of the partial significant heights is the significant height at that grid point in the usual sense, i.e., 4 times the standard deviation of surface elevation regardless of the underlying directions of the component wave trains. Thus a circular necklace centered on the magenta dot corresponds to an isotropic height spectrum, while a long narrow ellipse hanging from the magenta dot corresponds to a highly directed height spectrum. In fact all cases, for which there is appreciable wave energy, lie between these extremes (Figure 2).

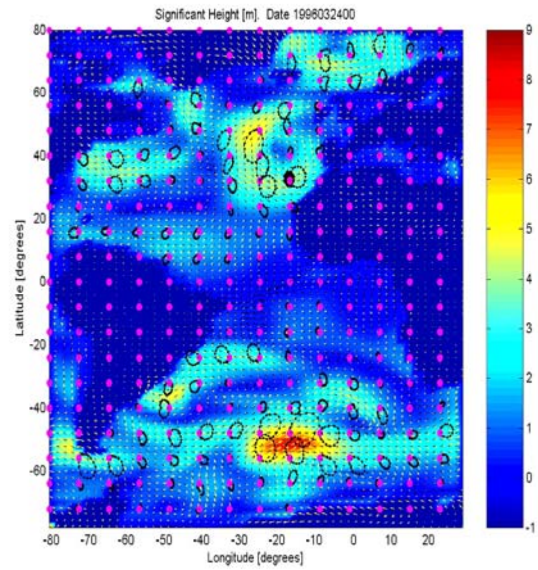


Figure 2. Wave height map of the Atlantic on 24 March, 1996 at 00 UTC. The color indicates significant heights in the range of 0 to 9 m. The 10 m wind vectors are represented by yellow arrows, while the black arrows are in the mean wave direction and have magnitude proportional to the square root of the mean wavelength (eq. 2). Every 4th grid point is marked with a magenta dot and includes a connected "necklace" of dots corresponding to the "partial significant height" in the direction of the dots. The nearest grid point to Madeira is indicated with a heavy black circle.

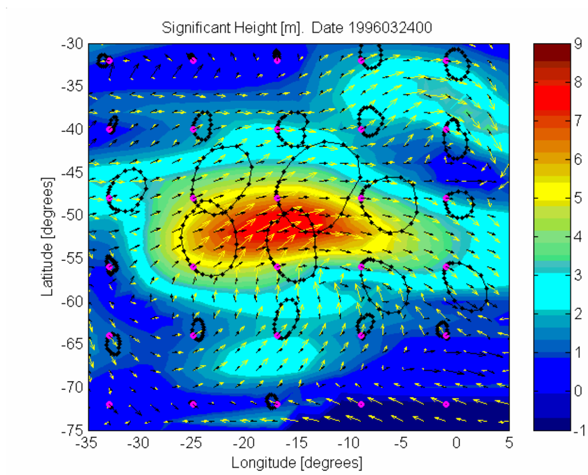


Figure 3. Blowup of a section of Figure 2 to illustrate the presence of mixed seas in and near the centre of storms.

This method of distinguishing between mixed seas and wind seas is qualitative and therefore not well suited to making a clear choice. In view of the very different statistics of high waves a definite indicator of wind sea or mixed sea is warranted. Our approach is to find the mean direction for the wavenumber spectrum at each grid point and then compute the mean directions and partial significant heights over 90 degrees to right (clockwise) and left. The difference in direction of the right and left parts is a clear indicator of the directional spread of the sea. Of course, this measure works best when there is sufficient wind to generate a reasonable sea and we are interested in high sea states, so we consider cases with significant heights over 2 m. Figure 4 is a histogram of all the significant heights from 12 hours of the hindcast (UTC 00 to 11 on March 24th 1996). The probability of occurrence of heights decays exponentially with height. The choice of 2 m for the lower limit to be considered helps focus our attention on moderate and strong storms.

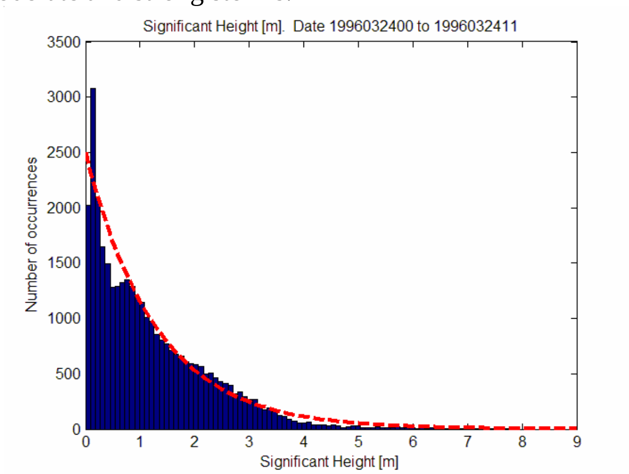


Figure 4. Histogram of all the significant heights from 12 hours of the hindcast (UTC 00 to 11 on 24 March 1996) of waves on the Atlantic Ocean. The red dashed line is the fitted exponential decay with increasing height.

The histogram of the angular difference between left and right components of the wave trains, exceeding 2 m significant height from 12 hours of the hindcast (UTC 00 to 11 on March 24th 1996) of waves on the Atlantic Ocean, is graphed in Figure 5. The (pronounced) mode of the distribution is at 35.5^{o} and the distribution is strongly skewed towards larger values. It is likely that the values around the mode represent the spreading of pure wind seas with more or less variability in the wind direction along the generating fetch of the sea. The red dashed line is the normal distribution with mean of 35.5^{o} and standard deviation of 3.5^{o} reflecting the variability of the wind along fetch. The much larger spreading angles probably arise from the confluence of wind seas and swell, i.e., mixed seas. We take the threshold spreading angle distinguishing wind sea from swell to be 42.5^{o}, i.e., two standard deviations above the mode.

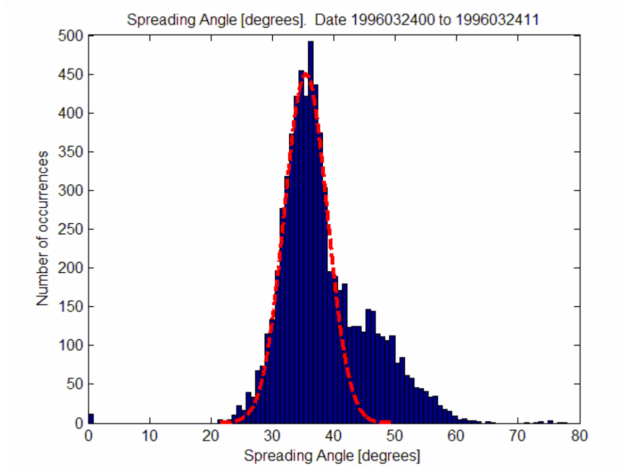


Figure 5. Histogram of the angular difference between left and right components of the wave trains exceeding 2 m significant height from 12 hours of the hindcast (UTC 00 to 11 on 24 March 1996) of waves on the Atlantic Ocean. The red dashed line is the normal distribution with mean of 35.5^{o} and standard deviation of 3.5^{o}.

Figure 6 is an example of these mixed sea indicators, and the angular differences of the left (blue arrows) and right (magenta arrows) partial significant heights is seen to be broad in regions where the wind direction (yellow arrows) changes rapidly along the fetch.

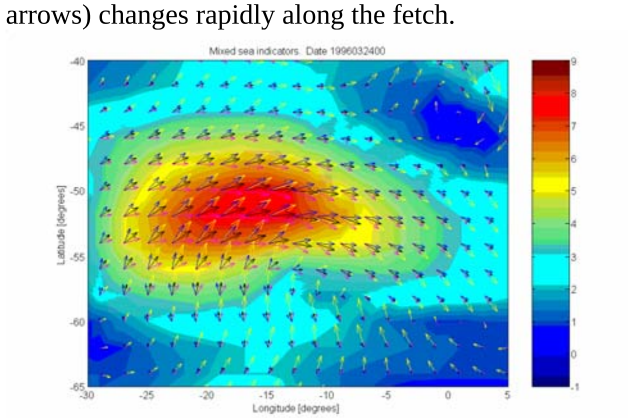
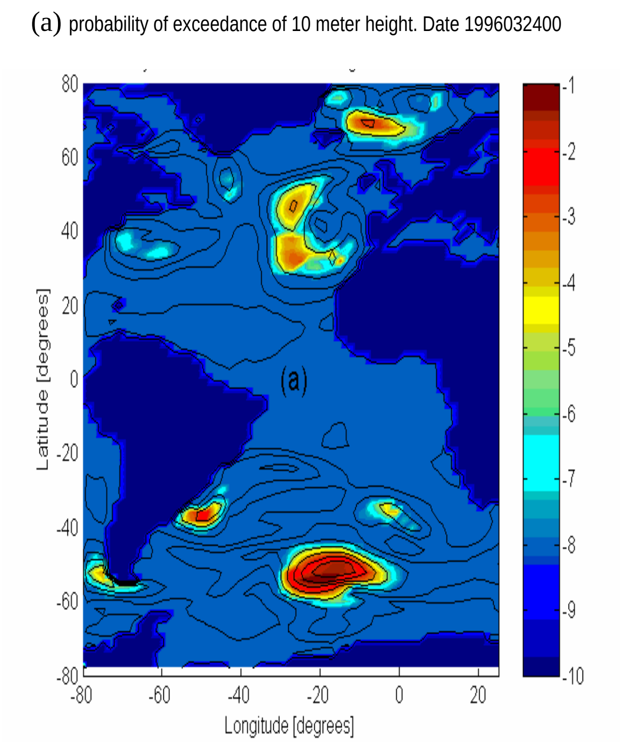
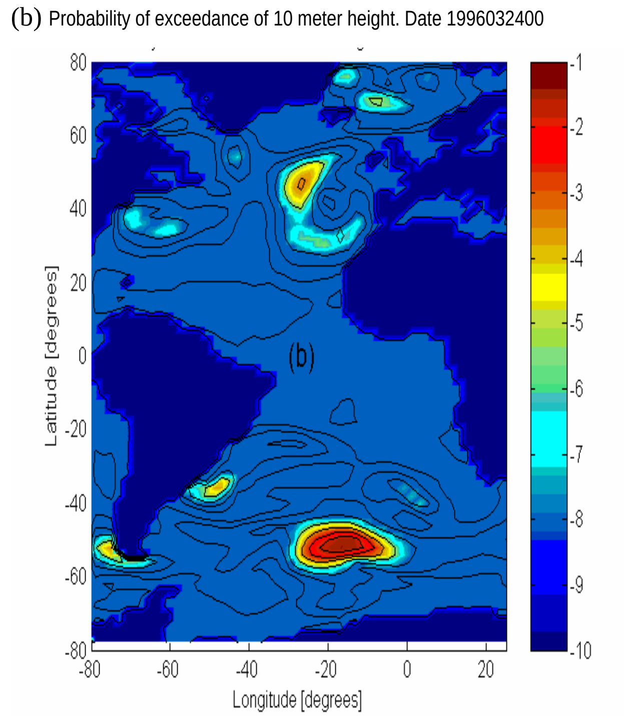


Figure 6. Map of wave heights and mixed sea indicators in the Southern Ocean on 24 March 1996 at 00 UTC. The color indicates significant heights in the range of 0 to 9 m. The 10-m wind vectors are represented by yellow arrows, while the black arrows are in the mean wave direction and have magnitude proportional to the significant height; the blue arrows have magnitude and direction proportional to the partial significant height in the 90^{o} sector to the left of the mean direction; the magenta arrows have magnitude and direction proportional to the partial significant height in the 90^{o} sector to the right of the mean direction.

The probability of exceedance of certain heights may be computed using the Rayleigh distribution (eq. 6) for wind seas (directional difference < 42.5^{o}), and the distribution appropriate to independent intersecting wave trains (eq. 7) otherwise. This is shown in Figure 7, panel (a) for exceedance of 4 m. Significant probabilities (say > 10^{-3}) occur in the Southern Ocean and the North Atlantic, in particular near Madeira. The effect of mixed seas on these probabilities is seen in comparison with panel (b), where the effect of intersecting wave trains is not considered. In view of the enormous enhancement of force a breaking wave can deliver, the probabilities of waves that are higher than 5 m and are also breaking is graphed in panel (c). There remain a few places – generally on the edges of storms where one would expect to encounter truly dangerous conditions.





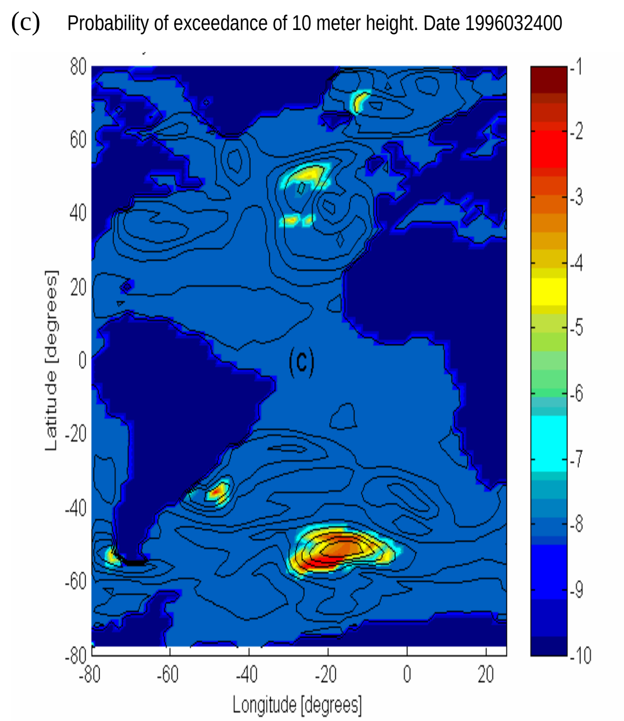


Figure 7. Probability of exceedance of 4 meter wave crest height on the Atlantic on 24 March, 1996 at 00 UTC. The color indicates the $\log_{10}$(probability of exceedance of 4 m). The contours are of significant height (see Figure 2). Panel (a): probabilities are based on eq. 7 when mixed seas are indicated, eq. 6 otherwise. Panel (b): probabilities are based on eq. 6 only, i.e., single component wind sea assumed everywhere. Panel (c): as in panel (a) except probabilities are for crest height exceedance of 5 m and also breaking.

Conclusions

The role of focusing in generating conditions that are likely to produce dangerous rogue waves has been explored for the case of meteorological focusing. Unlike the other common forms of focusing due to interaction with strong ocean currents or bathymetry, focusing arising from the wind patterns is not specific to particular locales and thus presents an interesting challenge to the marine wind and wave forecasting community. Both accurate wind vector forecasts and a directionally well-resolved wave model are prerequisites for the eventual goal of providing a reliable warning system for rogue waves.

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