

INTERNAL GRAVITY WAVES AND MIXING

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Oceanic internal gravity waves span a range between mesoscale eddies and small-scale turbulence, providing an important link in the overall energy cascade from the large scales of generation to the small scales of dissipation. To discuss progress at understanding internal-wave dynamics, the sixth 'Aha Huliko'a Hawaiian Winter Workshop brought together oceanographers and meteorologists. Conclusions of the workshop are

- Considerable progress is being made in predicting the diapycnal mixing rates associated with the shear instability of small-scale internal waves (Figure 1).
- Large-scale internal waves are forced by the atmosphere and (possibly) other oceanic flows. These waves are the most energetic and can propagate over distances of $O(1000 \text{ km})$.
- Intermediate-scale waves couple the large-scale waves to the scale waves by a spectral cascade. There exist a good kinematic description of these waves (Garrett-Munk model) and a reasonable account of their dynamics (nonlinear interactions).
- It is timely to proceed toward a global model to predict the internal wave field and diapycnal mixing.

This report discusses these conclusions in more detail.

Parameterization of Dissipation

Gregg (1989) suggested a parameterization of the rate of kinetic energy dissipation in terms of the internal-wave 10-m vertical shear and the buoyancy frequency, which is consistent with McComas and Müller's (1981) and Henyey et al.'s (1986) theoretical predictions of the energy cascade rate to high wavenumbers. This parameterization has considerable skill even in complex environments, as seen in Figure 2, which compares predicted with observed dissipation rates in the Florida Straits (M. Gregg). Also, the large

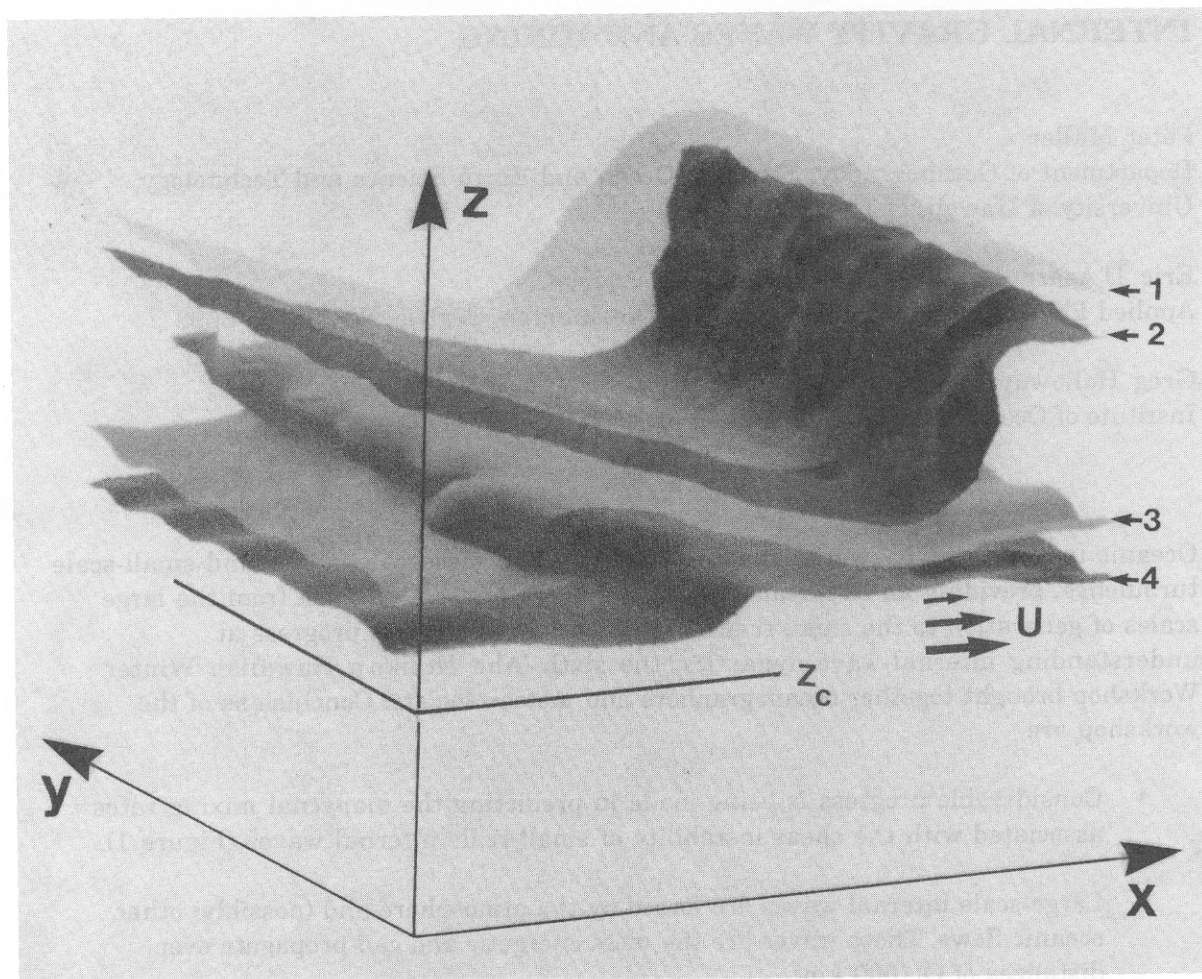


Figure 1. Visualization of an internal wave packet breaking at a critical level. Shown are four isopycnal surfaces from a numerical simulation in which a wave packet propagates downward through a horizontal shear flow $U(z)$ and encounters a critical level z_c . Near z_c , regions of high shear and overturned isopycnals develop. The wave eventually "breaks", owing to a three-dimensional instability. The results of this and other simulations are being used to investigate diapycnal mixing in the deep ocean. (Figure courtesy of Winters and D'Asaro, Applied Physics Laboratory, University of Washington.)

diapycnal diffusivity inferred from a tracer experiment in the Santa Monica Basin implies an elevated level of internal-wave shear that was indeed found (M. Gregg, E. Kunze). On the other hand, Gregg's scaling law has been questioned by Gargett (1990) and regions exist where the scaling is not observed, such as the Yermak Plateau in the Arctic Ocean where dissipation rates much higher than predicted have been found (L. Padman).

The observed scaling of the dissipation rate in terms of internal-wave parameters provides a stringent test of nonlinear wave-wave interaction theories.

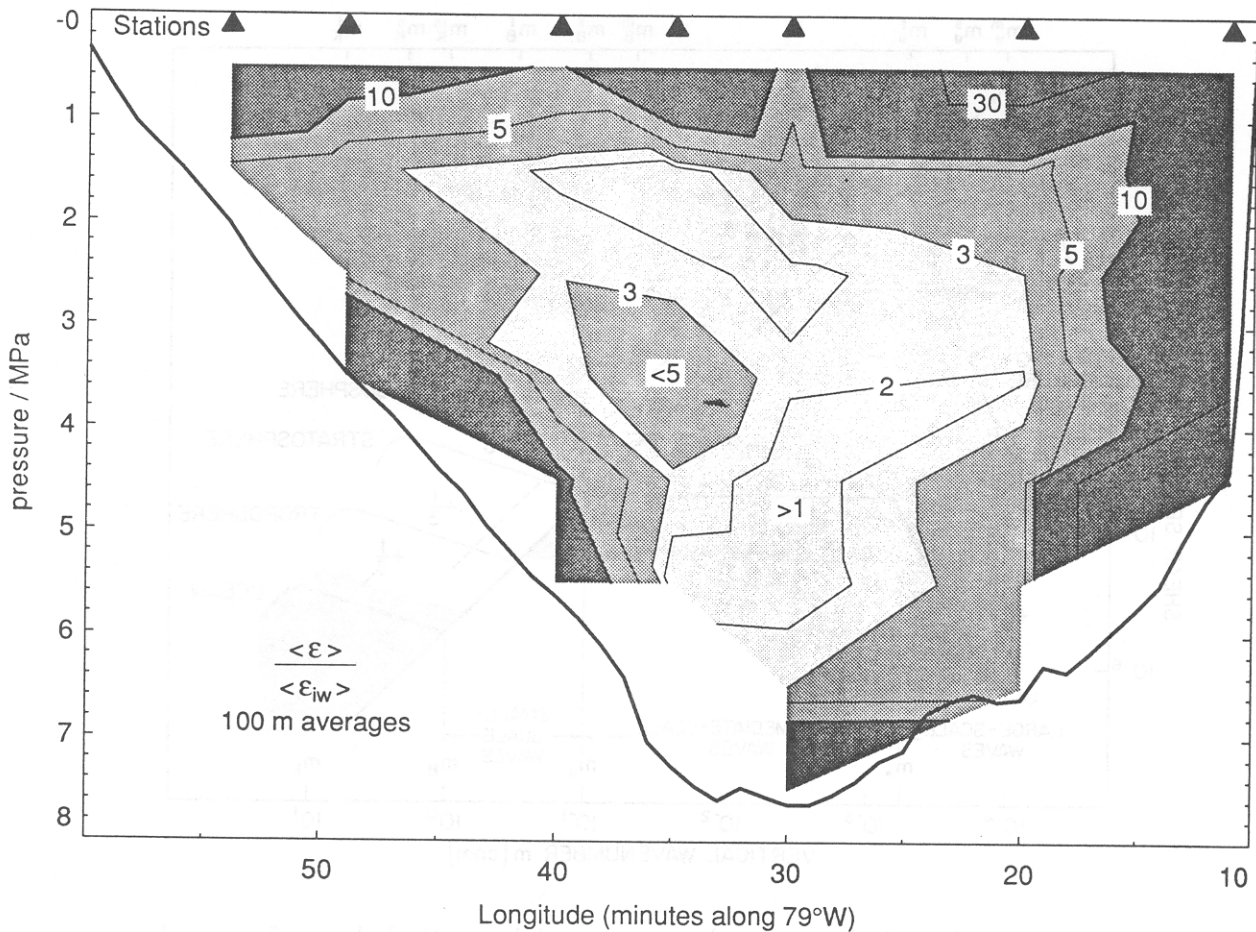


Figure 2. Contours of normalized kinetic energy dissipation rate along a section through the Florida straits (along 28° N from West Palm Beach to Little Bahama Bank). In the interior, the observed rates $\langle \epsilon \rangle$ are comparable to the rates $\langle \epsilon_{iw} \rangle$, which are inferred from internal wave parameters using Gregg's (1989) scaling law. The higher values around the periphery are likely due to interactions with topography. (Courtesy of M. Gregg, University of Washington.)

The Saturation Range

Existing parameterizations, such as that of Gregg (1989), bridge the gap from small-scale internal waves of $O(10\text{ m})$ wavelengths to the dissipation scales in the centimeter range (Figure 3). Considerable progress has been made in describing these small-scale internal waves, which are not represented by the Garrett and Munk spectral model. First, a universal "saturation" range with a -1 spectral slope in the shear or strain spectra is observed between the roll-off wavenumber m_u and the Ozmidov or buoyancy wavenumber m_B . The spectral level is proportional to the buoyancy frequency N squared. Above and below this saturation range, the spectrum is much more variable. Second, the low

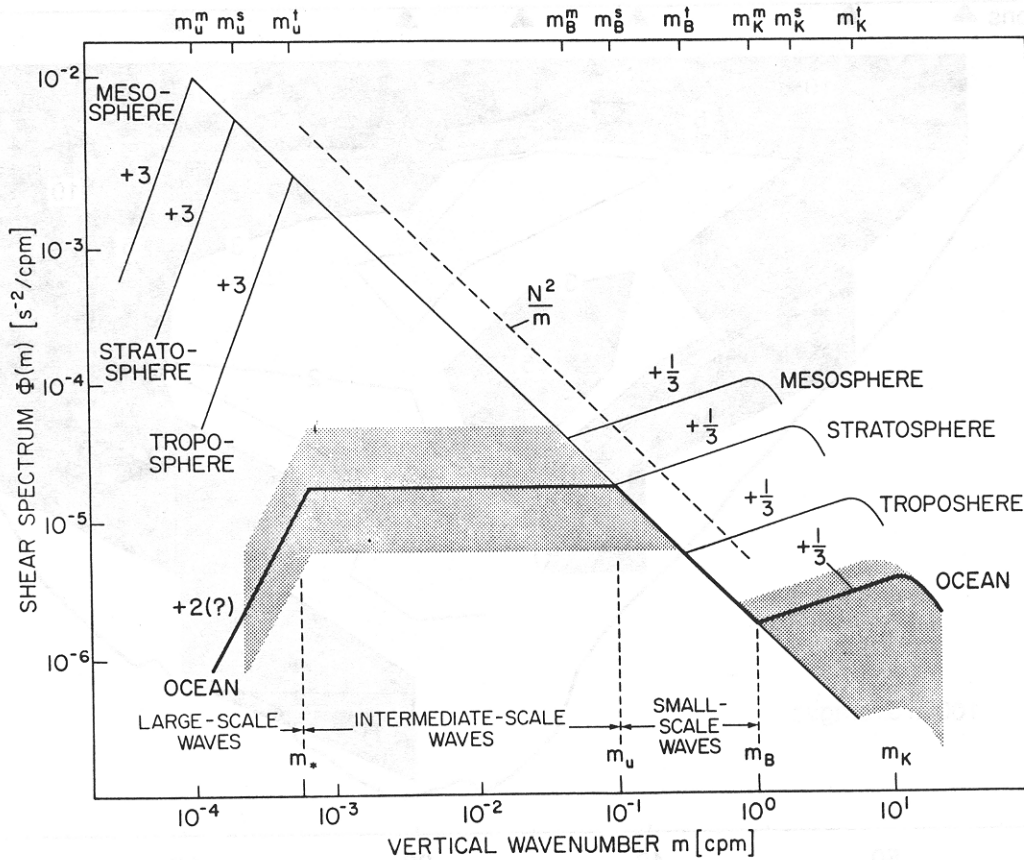


Figure 3. Schematized vertical wavenumber spectra of the vertical shear of internal gravity waves in the ocean, troposphere, stratosphere, and mesosphere, scaled to a common value of the buoyancy frequency N . The wavenumber power law ranges are indicated. The transitional wavenumbers for the ocean are the bandwidth m^* , the cut-off or roll-off wavenumbers m_u , the buoyancy or Ozmidov wavenumber $m_B = (N^3 \epsilon)^{1/2}$, and the Kolmogorov dissipation wavenumber $m_K = (\epsilon / \nu^3)^{1/4}$ with ϵ being the kinetic energy dissipation rate and the molecular viscosity ν . The analogous wavenumbers for the tropo-, strato-, and mesosphere are indicated by superscripts t , s and m , respectively. The oceanic spectrum consists of a large-scale part ($m < m^*$) which is not well established, an intermediate-scale part ($m^* < m < m_u$) which is well described by the Garrett and Munk spectral model, a small-scale or "saturation" range between m_u and m_B with a -1 slope, and an inertial and viscous dissipation range for $m > m_B$. The variability of the spectrum is indicated by the shading. The "saturation" range is much less variable than the other ranges. The atmospheric spectra also show a saturation range with the same spectral slope and level as the oceanic spectra. The line N^2/m represents theoretical predictions by Lumley (1964) and Holloway (1983) for buoyant turbulence or nonlinear wave interactions.

wavenumber ($m < m_u$), i.e., Garrett and Munk, spectral level and the roll-off wavenumber m_u vary in such a way as to maintain the universal saturation range (M. Gregg). Third, the probability distribution function of the observed strain field is well represented by a gamma distribution. Moments at different scales are thus related to a single (dimensional) parameter (R. Pinkel).

Atmospheric spectra have also been included in Figure 3 because they share the "saturation" slope and spectral level. Atmospheric gravity waves are generated at low altitudes by topography and/or convection. When these waves propagate upward their amplitudes increase due to the decrease in density, and the wave spectrum slides along the -1 slope and "saturates" at the roll-off wavenumber (D. Fritts). Figure 1 shows that atmospheric gravity waves are a narrow-band process with a peak at m_u , whereas oceanic internal waves represent a broad-band process with a roll-off at m_u . Also, atmospheric gravity waves have shorter lifetimes than oceanic waves, and dissipation rates are higher in the atmosphere than in the ocean.

Resonant or eikonal wave-wave interaction calculations predict only the energy transfer from low wavenumbers to the roll-off wavenumber m_u , not the energy transfer across the "saturation" range to the dissipation wavenumbers. The classical "buoyancy subrange" theory of Lumley (1964) predicts the observed N^2/m spectral form on the premise that the buoyancy flux term is dominant in the turbulence kinetic energy equation. The same spectrum is also predicted by Holloway (1983) who assumes that strong wave-wave nonlinearities transfer kinetic and potential energy nearly independently to high wavenumbers. For the atmosphere a wave theory has been advanced that ascribes the "saturation" range to random refractive broadening of an upward propagating band-limited wave field (C. Hines). This wave theory also reproduces the observed spectral slope and level. On the other hand, rapid changes of the atmospheric spectrum during vertical propagation suggest that wave breaking may occur at "saturation" scales (D. Fritts). In the ocean, structures at "saturation" scales often persist for many buoyancy periods; this is indicative of wave rather than turbulent dynamics (R. Pinkel). The distinction between wave and turbulent processes may be one of degree. Numerical simulations show a smooth transition from wave refraction to vigorous overturning with increasing Froude number (D. Ramsden).

Diapycnal Mixing

Internal wave theory predicts the energy flux to mixing scales; observations provide the kinetic energy dissipation rate. Additional assumptions are needed to infer the diapycnal mixing rate. Traditionally, diapycnal mixing is inferred from the buoyancy flux, which is assumed to be equal to the dissipation rate multiplied by a mixing efficiency factor. Therefore, to infer mixing rates from internal wave parameters one needs to quantify the mixing efficiency. An increasingly useful tool to address this problem is direct numerical simulation. Both visualization of the flow field (D. Siegel, K. Winters) and diagnosis of the dynamics are now possible. It appears that spatial separation of sources and sinks, which clearly occurs in the ocean, affects the dynamical balances and must be taken into account (D. Ramsden). Multiple scale interactions and separated source and sink regions require extreme numerical resolution. To infer mixing rates one must also carefully separate changes in available and base-state potential energy (K. Winters).

Overall, present observations and theory indicate a typical diapycnal diffusivity near $10^{-5} \text{ m}^2\text{s}^{-1}$ in the upper ocean and thermocline, with much larger values in some areas due to local wave generation (L. Padman), storm-induced mixing (C. Eriksen), or double diffusive processes (M. Gregg). This value is generally consistent with estimates from inverse models, although in some places these estimates are subject to great uncertainties due to incorrect formulation and neglect of the nonlinearities in the equation of state (T. McDougall).

Whereas the internal wave-induced diapycnal diffusivity that describes the fluxes of buoyancy and tracers across isopycnals is being quantified, there is little information about the internal wave-induced diapycnal viscosity that describes the momentum flux across isopycnals. Indeed, even the sign of the internal wave or "eddy" Prandtl number is subject to dispute. There is also little information about isopycnal transports induced by internal waves.

Boundary and Abyssal Mixing

Several distinct regions of mixing can be identified in the ocean (C. Garrett). In the ventilated part of the thermocline, vertical transport is primarily caused by advection along isopycnals, and diapycnal mixing of $O(10^{-5} \text{ m}^2\text{s}^{-1})$ might perhaps be secondary. Most of our measurements are from this region or from the even more variable upper ocean. In contrast, in the unventilated abyssal ocean mixing processes are likely to be more important. There are few internal-wave measurements in the deep ocean. The few that have been made indicate that the spectral distribution of internal-wave energy might be different there (T. Sanford). Characteristics of the internal-wave field in shallow seas have also not been established yet.

A special mixing regime may occur in the vicinity of benthic boundaries. Understanding such boundary mixing is important for the parameterization of mixing and the formulation of boundary conditions in large-scale numerical models. Internal waves might play a very important role in boundary mixing because both the reflection at critical angle slopes (Eriksen, 1985) and the scattering at rough bottom topography (N. Xu) result in a transfer of energy to high wavenumber waves that are more likely to break and mix. A careful search for signatures of critical angle reflection at two locations has yielded frustratingly ambiguous results (D. Gilbert).

The stratification of water mixed at the boundary is a key factor in determining how efficiently the surrounding ocean is mixed. Simple models have been developed for the restoration of stratification by buoyancy-driven secondary flows and for the net buoyancy flux (C. Garrett).

Potential Vorticity

Potential vorticity is a conserved quantity that is not carried by internal gravity waves. Theoretically we expect non-wave motions, often called "vortical" motions, to carry potential vorticity at internal-wave scales. Vortical motions can interact with internal waves (J. Riley) and may thus affect the internal-wave field. Potential vorticity

conservation expresses the invariance of fluid motion under fluid particle relabeling. A formulation of the equations of motion in terms of truncated Hamiltonian equations that exhibit this invariance has been achieved (H. Abarbanel).

Attempts to observe small-scale potential vorticity carrying motions have yielded ambiguous results. Analysis of IWEX data shows an excess of vertical relative vorticity at 25 m horizontal separation above that expected for internal waves but no such excess at 250 m (R. C. Lien). Analysis of data taken downstream of Ampere Seamount shows no contribution to potential vorticity fluctuations from relative vorticity but only from vortex stretching (E. Kunze).

Much of the velocity and density finestructure often attributed to vortical motion can be explained equally well as distortions of an internal-wave field that is vertically advected and strained. The use of vertically Lagrangian (isopycnal following) coordinates that eliminate the effects of vertical advection might simplify the kinematical description and dynamical evolution of internal waves (F. Henyey).

Forcing

Various sources have been suggested for the oceanic internal-wave field. Fluctuations in the windstress excite mixed-layer inertial oscillations and a large fraction of energy from these oscillations penetrates the ocean as near inertial, low-modenumber internal waves. Although the details of this process can be complicated by mesoscale eddies and storm-induced mixing (C. Eriksen) there is little reason to doubt that windstress fluctuations are a major source of large-scale internal waves (E. D'Asaro).

Internal waves of tidal frequency are commonly observed in the ocean and may also be a major energy source. Several clear examples of generation by the barotropic tide have been found. However, the amplitude and propagation direction of the internal tide are highly variable and this variability is not well understood (M. Levine).

Surface waves have also been suspected as being a source for internal waves. Recent calculations (K. Watson) show, however, that surface waves draw energy from the high-frequency, low-modenumber internal-wave field, except during periods of very strong surface waves. These energy transfers indicate that imbalances between the internal and surface wave fields may relax rapidly.

Despite the large energy contained in mesoscale low frequency motions there is little evidence that these motions are a major source of energy for the internal-wave field. Refraction and the resulting trapping of waves in frontal or eddy features can lead to strong localized wave dissipation and mixing (Kunze, 1985). Similar interactions in a weaker eddy field have been predicted to result in a net energy flux from the eddy field to the internal-wave field (Watson, 1985). Recently, radiation of internal waves from newly formed small coherent eddies has been observed (T. Sanford, R. Pinkel).

A Global Internal Wave Model

Sufficient progress in theory and observations has been made to outline the structure of a global model to predict the internal-wave field and diapycnal mixing. Such a model would follow the propagation of large-scale internal waves from their sources, with transfer of energy across the time-variable Garrett and Munk spectrum to its local dissipation. To achieve the understanding of the physics necessary for the construction of such a model the following issues need to be addressed:

- *Generation:* Wind and tides may be sufficient to drive the global internal-wave field. The wind source function is limited by the resolution of wind data and the accuracy of the drag coefficient. Nevertheless, the general spatial distribution and seasonal cycle can perhaps be estimated from current operational weather products. Presently, we have less confidence in the construction of the source function for internal tides.
- *Propagation:* Estimates of the mean free path from group velocity and nonlinear interaction rates indicate that wind-generated, large-scale waves propagate significant distances, $O(1000 \text{ km})$, from their sources (E. D'Asaro, E. Hirst). The generation and propagation of large-scale waves might thus be a basically linear problem (D. Rubenstein) and can perhaps be modeled by ray tracing. The interaction of these large-scale waves with topography and the mesoscale eddy field remain challenging research areas.
- *Spectral Transfer:* We anticipate that resonant interaction theory will predict the transfer of energy from the propagating large-scale waves to the Garrett and Munk spectrum with sufficient accuracy (E. Hirst). The necessary calculations have not yet been done for a highly directional wave field. Energy transfer by scattering at topography is probably important as well, but has also not been investigated yet. The subsequent energy transfer across the Garrett and Munk spectrum, its spectral shape, and the final dissipation and diapycnal mixing rate will be controlled by more highly nonlinear interactions. To test the accuracy of nonlinear interaction theory, comparisons with direct numerical simulations need to be carried out.
- *Dissipation:* We also anticipate that present research linking dissipation and its associated mixing to internal-wave parameters at the roll-off wavenumber m_u will converge on a parameterization with significant skill. This area remains a high priority for research since it will likely result in the ability to estimate diapycnal mixing from routine internal wave measurements. However, such measurements alone will be insufficient to prescribe mixing rates in large-scale models. A global internal wave model is needed.

To make progress on these issues requires coordinated theoretical and observational studies of specific processes. We suggest as priority issues the quantification of internal-wave sources, of the propagation of large-scale waves from their sources, and of the transfer of energy from these waves to the rest of the spectrum.

Internal Gravity Waves and Mixing

Direct numerical simulation will be a powerful tool in this effort. Although the dynamics are clearly three-dimensional (3D), some processes might be modeled in two dimensions (2D, vertical plane) affording higher resolution. By careful intercomparison of 2D and 3D models, the range of applicability of 2D can be addressed (D. Ramsden).

Observationally, the measurement of the directional spectrum of large-scale waves requires techniques that integrate vertically (or horizontally). Acoustics is a natural candidate for this (T. Ewart). Measuring the directionality of large-scale waves away from obvious sources may yield clues about the location of sources.

Finally, comparison of data and model prediction might require multivariate analysis to arrive at definite conclusions (C. Frankignoul). Actual predictive models will undoubtedly also need to be integrated with models of other oceanic processes that affect the internal-wave field (A. Warn-Varnas).

Conclusions

Internal waves and their effect on larger scales remain a basic and important issue. Recent progress suggests that we have the tools and conceptual framework to predict diapycnal diffusivities globally in the not too distant future. Such optimism is not warranted for the prediction of internal-wave induced momentum fluxes and isopycnal dispersion. In these problems the vortical mode may turn out to be important. Present understanding is focused on the upper ocean and thermocline. We are less clear about internal-wave behavior in shallow seas, the abyss and near boundaries.

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