Measures of Predictability with Applications to the Southern Oscillation*

PAO-SHIN CHU

Department of Meteorology, University of Hawaii, Honolulu, HI 96822

RICHARD W. KATZ

*Environmental and Societal Impacts Group, National Center for Atmospheric Research,** Boulder, CO 80307*

(Manuscript received 27 May 1986, in final form 29 December 1986)

ABSTRACT

A relative measure of actual, rather than potential, predictability of a meteorological variable on the basis of its past history alone is proposed. This measure is predicated on the existence of a parametric time series model to represent the meteorological variable. Among other things, it provides an explicit representation of forecasting capability in terms of the individual parameters of such time series models.

As an application, the extent to which the Southern Oscillation (SO), a major component of climate, can be predicted on a monthly as well as a seasonal time scale on the basis of its past history alone is determined. In particular, on a monthly time scale up to about 44% of the variation in SO can be predicted one month ahead (zero months lead time) and about 35% two months ahead (one month lead time), or on a seasonal time scale about 53% one season ahead (zero seasons lead time) and about 31% two seasons ahead (one season lead time). In general, the degree of predictability naturally decays as the lead time increases, with essentially no predictability on a monthly time scale beyond ten months (nine months lead time) or on a seasonal time scale beyond three seasons (two seasons lead time).

1. Introduction

Prediction of the weather and climate has always been a challenge to atmospheric scientists, especially when attempts have been made to forecast atmospheric behavior (e.g., the position and intensity of troughs at 500 mb level) beyond a few days. Because certain approximations are made in numerical model initialization schemes, the error associated with predictions based on such models inevitably increases as the lead time increases. As an alternative approach for monthly, or seasonal, or longer-range climate prediction, statistical techniques often offer promising results (e.g., Hastenrath, 1986; Namias, 1985). In particular, they provide a standard of comparison for dynamical methods.

One way to assess predictability, especially on time scales of a year or more, is by means of an indirect approach that produces estimates of "potential" predictability. Actual climatic variability (e.g., interannual variance derived from monthly or seasonal means), consisting of natural variability possibly confounded with a signal, is compared to an estimate of natural variability (or climatic "noise") obtained from the level of short-term weather fluctuations, regarded as unpredictable on a climatic time scale (e.g., Madden, 1976; Shukla and Gutzler, 1983; Trenberth, 1985). Although the potential predictability approach is able to detect the presence of low frequency signals, it does not explicitly identify the source of these signals (e.g., climatic trends). In particular, this approach does not provide a means of predicting future values.

In view of the disadvantages inherent in the potential predictability approach, the primary goal of this paper is to consider an alternative method to assess what could be termed "actual" predictability. Our approach is predicated on the availability of a statistical model to represent the temporal behavior of the meteorological variable under study. It involves an explicit estimate of how well, in a relative sense, such a time series model is able to forecast the future behavior of the meteorological variable.

It should be noted that Davis (1978) proposed a somewhat related measure of "intrinsic" predictability, and applied it to the forecasting of monthly pressure anomalies over the North Pacific Ocean from previous monthly anomalies for both pressure and sea surface temperatures. His approach was based on regression analysis and expressions were obtained for intrinsic predictability in terms of autocorrelations and cross correlations. Our approach involves the derivation of expressions for predictability in terms of the formal parameters of time series models and could be viewed

* Contribution 85-09, University of Hawaii, Dept. of Meteorology.
** The National Center for Atmospheric Research is sponsored by the National Science Foundation.

as a theoretical extension of certain aspects of Davis' work.

As an example of the application of this approach to actual data, a simple Southern Oscillation Index (SOI) will be used. This index is defined as the Tahiti (17.5°S, 150°W) minus Darwin (12.4°S, 130.9°E) normalized monthly (or seasonal) mean sea level pressure series. The SOI is regarded as a prime indicator for the strength and variation of the large-scale Walker-type circulation involving atmospheric mass exchanges between the Pacific and Indian Oceans. Normalization is achieved by taking the difference between raw monthly (seasonal) mean data and the long-term monthly (seasonal) average, and then by dividing this departure by the standard deviation for each month (season) at these two stations (see Chu and Katz, 1985; hereafter referred to as CK). This method of normalization implies that monthly and seasonal SOI time series both have zero mean.

Figure 1 displays the time series of seasonal SOI from spring 1935 to fall 1984, with the large negative anomalies between summer 1982 and spring 1983 being most evident. Note that this diagram is an updated version of that shown in CK. Since the 1982–83 anomalies were associated with profound socioeconomic consequences such as severe drought in some areas and unusually heavy rainfall in other regions (Glantz, 1984; Glantz et al., 1987), it is important to determine the extent to which skillful predictions of the Southern Oscillation (SO) can be made. In view of the readily available pressure observations at Tahiti and Darwin through the global telecommunication systems, it becomes increasingly imperative to know a priori the degree of predictability of such a large-scale circulation phenomenon.

Chu and Katz (1985) were primarily concerned with the identification of the best fitting autoregressive-moving average (ARMA) processes for the monthly and seasonal SOI time series. Some attention was also devoted to the degree of predictability of the 1982–83 extreme SOI anomalies (in conjunction with an intense El Niño event). The present paper extends this work by considering the overall degree of predictability of the SO associated with the ARMA models selected in CK. First, measures of predictability for general ARMA processes are discussed in section 2, and then their application to the SOI is treated in section 3. A summary and conclusion are found in section 4.

2. Measures of predictability

a. Definition

Attention is restricted to stationary stochastic processes, although there is some evidence that even time series derived from normalized data, such as the SOI, are phase-locked to the annual cycle (e.g., Wright, 1985). For simplicity, only a particular class of stationary models known as ARMA processes is actually considered. A stationary stochastic process, X_t, with mean zero (without loss in generality) and variance σ^2 is an ARMA(p, q) process if it can be expressed as

$$X_t = \sum_{i=1}^{p} \phi_i X_{t-i} + a_t - \sum_{j=1}^{q} \theta_j a_{t-j}. \tag{1}$$

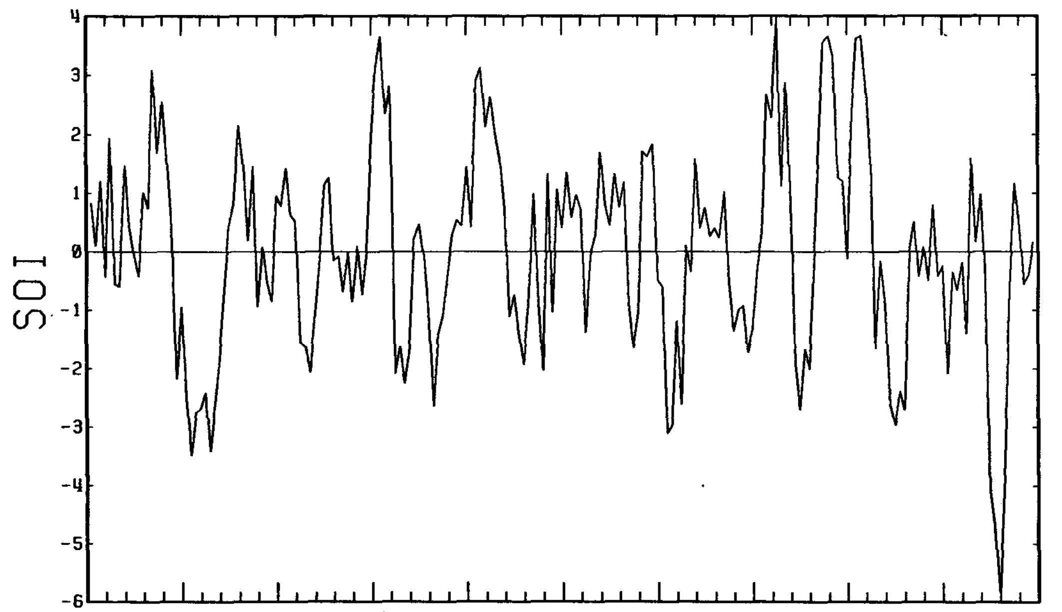

FIG. 1. Time series of seasonal SOI from spring 1935 through fall 1984.

Here the ϕ and θ are called autoregressive (AR) and moving average (MA) parameters, respectively. In (1), it is assumed that the a_t are uncorrelated random variables, each of which has a Gaussian distribution with mean zero and variance σ_a^2. It should be noted that certain constraints must be placed on the ϕ and θ in order for the X_t-process to actually be stationary. See Katz and Skaggs (1981) or CK for a more detailed discussion of the application of ARMA processes to meteorological data.

To define our measures of predictability, properties of the forecasts at time origin t for l time steps ahead ($l = 1, 2, \cdots$), $\hat{X}_t(l)$, produced by ARMA processes are examined. From (1), this forecast can be written as

$$\hat{X}_t(l) = \sum_{i=1}^{p} \phi_i X_{t+l-i} + a_{t+l} - \sum_{j=1}^{q} \theta_j a_{t+l-j}. \tag{2}$$

In (2), forecasted values (zeroes) are substituted for X's (a's) that have not yet been observed. To simplify the discussion, we assume for now that the parameters of the ARMA process (ϕ, θ, σ_a^2 and σ^2) are known. This assumption implies that the present and past error terms (a_t, a_{t-1}, a_{t-2}, . . .) are also known. In section 2c, this assumption will be relaxed.

We let

$$e_t(l) = X_{t+l} - \hat{X}_t(l) \tag{3}$$

denote the l-step ahead forecast error and $V(l) = \mathrm{var}[e_t(l)]$ denote its variance. An ARMA(p, q) process has an equivalent representation as an infinite-order MA process; namely,

$$X_t = \sum_{j=0}^{\infty} \psi_j a_{t-j}, \tag{4}$$

with $\psi_0 \equiv 1$ (Box and Jenkins, 1976, p. 47). Using (4), it is straightforward to show that the forecast error variance can be expressed as

$$V(1) = \sigma_a^2,$$

$$V(l) = (1 + \sum_{j=1}^{l-1} \psi_j^2)\sigma_a^2, \quad l = 2, 3, \cdots. \tag{5}$$

One relative measure, λ_l, of how well the ARMA process forecasts l time steps ahead would involve comparing the variance of the l-step ahead forecast errors to the variance of the X_t-process (namely, σ^2). Specifically, the proportion of variance "explained" by the l-step ahead forecasts is

$$\lambda_l = 1 - \frac{V(l)}{\sigma^2}, \quad l = 1, 2, \cdots. \tag{6}$$

It is convenient to define $\lambda_0 \equiv 1$. Using the relationship between the process and error variances, the l-step ahead predictability for any ARMA process can be expressed as solely a function of the ψ-weights. If we define

$$\Lambda(l) = \sum_{j=l}^{\infty} \psi_j^2, \quad l = 0, 1, \cdots, \tag{7}$$

then (6) becomes

$$\lambda_l = \Lambda(l)/\Lambda(0). \tag{8}$$

The stationarity constraint on the ARMA parameters (ϕ and θ) implies that (7) is a finite sum.

From this representation of the l-step ahead predictability in terms of the ψ-weights (7)–(8), it is clear that $0 \leq \lambda_l \leq 1$, with $\lambda_l = 0$ for the case of no predictability l-steps ahead and $\lambda_l = 1$ for the case of perfect predictability l-steps ahead. Moreover, λ_l necessarily tends to zero as l tends to infinity. For meteorological variables, one should naturally expect a value of λ_l intermediate between zero and one for relatively short lead times l.

This measure of predictability (for the case of $l = 1$) has been proposed by Box and Tiao (1977) and is the time series analogue of the so-called R^2 (or coefficient of multiple determination) in regression analysis. It has also been proposed as the appropriate standard of comparison when dealing with multiple time series; that is, when forecasts of the future behavior of the X_t-process are made on the basis of not only its own past history, but on the past history of other time series as well (Pierce, 1979). One by-product of this approach is that it provides an explicit representation of forecasting capability in terms of the parameters (ϕ and θ) of an ARMA process.

b. Expressions

In this section, the general definition of predictability (6) is applied to special cases of ARMA processes. We first note that λ_l can always be expressed as a function of the ϕ's and θ's alone. This result follows because the ψ-weights in (4) can be calculated by means of a recursion that involves only the ϕ and θ (Box and Jenkins, 1976, p. 134).

1) λ_1 FOR AR(p) PROCESS

For an AR(p) process, the one-step ahead predictability (6) reduces to

$$\lambda_1 = \sum_{i=1}^{p} \rho_i \phi_i. \tag{9}$$

Here ρ_i denotes the theoretical ith-order autocorrelation coefficient for an AR(p) process. As such, the ρ are related to the ϕ by the so-called Yule–Walker equations, implying that λ_1 can be determined from the ϕ alone. In fact, for very low-order AR processes, analytical expressions for λ_1 that involve only the ϕ can be obtained.

For an AR(1) process, (9) reduces to

$$\lambda_1 = \phi_1^2. \tag{10}$$

Hence the single parameter ϕ_1 plays precisely the same role as the correlation coefficient in simple linear regression. For an AR(2) process, (9) reduces to

$$\lambda_1 = \phi_1^2[(1+\phi_2)/(1-\phi_2)] + \phi_2^2. \quad (11)$$

An analytical expression for λ_1 in the case of an AR(3) process, which, incidentally, was found by CK to provide a good fit to the seasonal SOI time series, can also be derived [see (A3) in the Appendix].

2) $\lambda_l\ (l > 1)$ FOR AR(p) PROCESS

As an example, consider the case of predictability two steps ahead. Using (9) and the fact that $\psi_1 = \phi_1$ for an AR(p) process (Box and Jenkins, 1976, p. 134), (8) reduces to

$$\lambda_2 = (1+\phi_2^2)\sum_{i=1}^{p} \rho_i\phi_i - \phi_1^2. \quad (12)$$

For an AR(1) process, the predictability l steps ahead can be expressed in closed form as

$$\lambda_l = \phi_1^{2l}, \quad l = 1, 2, \cdots, \quad (13)$$

indicating that in this case predictability decreases at a geometric rate towards zero as the lead time increases.

3) λ_1 FOR ARMA(1, 1) PROCESS

Analytical expressions for the predictability of general ARMA processes, involving the simultaneous use of both autoregressive and moving average terms, are somewhat more difficult to derive. We present the results of only one special case, namely an ARMA(1, 1) process, which, incidentally, was found by CK to be the simplest model that provided a relatively good fit to the monthly SOI time series. For an ARMA(1, 1) process, the one-step ahead predictability (6) reduces to

$$\lambda_1 = (\phi_1 - \theta_1)^2/(1 + \theta_1^2 - 2\phi_1\theta_1). \quad (14)$$

4) $\lambda_l\ (l > 1)$ FOR ARMA(1, 1) PROCESS

Using the closed form expression for the ψ-weights of an ARMA(1, 1) process (Box and Jenkins, 1976, p. 154), the predictability at higher lead times is given by

$$\lambda_l = \phi_1^{2l-2}\lambda_1, \quad l > 1, \quad (15)$$

where λ_1 is specified by (14). (15) indicates that the predictability for an ARMA(1, 1) process decays at a geometric rate toward zero for higher lead times, the only difference from the result (13) for an AR(1) process being that the starting value involves both of the parameters ϕ_1 and θ_1.

c. Estimation

Strictly speaking, all of the expressions for the measures of predictability (6) that have been presented so far are actually only valid when the parameters of the ARMA process are known. In practice, it is necessary to estimate these parameters on the basis of a finite sample of observations for the meteorological variable. Standard techniques, many based on least squares or approximate least-squares criteria, are available to produce such parameter estimates (e.g., Box and Jenkins, 1976, pp. 495–516). The simplest approach to estimating predictability would be to substitute these estimated values in place of the corresponding ϕ and θ that appear in the expressions for predictability. We term such estimates as *theoretical predictability.* One potential drawback to this approach is the possibility of introducing a bias into the predictability estimates. Namely, because the uncertainty inherent in these parameter estimates is ignored in the expression (5) for the forecast error variance, the predictability estimates are positively biased (Akaike, 1969).

On the other hand, if unbiased estimates of the actual level of predictability that has been achieved on the basis of the finite samples currently available are desired, then adjustments could be made to remove this positive bias. One approximate procedure is based on the concept of Finite Prediction Error introduced by Akaike (1969). The approximate effect of estimating the $s = p + q$ parameters of an ARMA(p, q) process is to inflate the variance $V(l)$ of prediction errors l steps ahead (5) by the factor $1 + s/n$, where n denotes the total number of observations on which the parameter estimates are based (Jones, 1985). If we let λ_l^* denote the adjusted estimator of predictability l steps ahead that takes into account this inflation factor, then (6) becomes

$$\lambda_l^* = \lambda_l - \frac{s}{n}(1 - \lambda_l). \quad (16)$$

Davis (1978) also attempted to correct for this so-called "artificial skill" in a similar manner.

An alternative technique for estimating predictability would simply be to compute λ_l on the basis of actual l-step ahead prediction errors observed for an independent sample. Specifically, the definition (6) for λ_l suggests that observed predictability $\hat{\lambda}_l$ be calculated by

$$\hat{\lambda}_l = 1 - \frac{\frac{1}{T}\sum_{t=1}^{T}[X_t - \hat{X}_{t-l}(l)]^2}{\frac{1}{T-1}[\sum_{t=1}^{T}(X_t - \bar{X})^2]}, \quad (17)$$

where

$$\bar{X} = \frac{1}{T}\sum_{t=1}^{T} X_t. \quad (18)$$

Here X_t denotes an independent observation not employed in estimating the parameters of the time series model, and $\hat{X}_{t-l}(l)$ denotes the corresponding forecast based on the fitted model and (2), $t = 1, 2, \cdots, T$. We term such estimates as *observed predictability*. Such an exercise is useful for verifying whether the appropriate form of stochastic model has been fit to the data. However, from the perspective of assessing predictability, this approach has the disadvantage that it is no longer possible to specify the manner in which the individual model parameters contribute to the predictability estimates that are obtained.

3. Application to SOI

a. Observed monthly and seasonal predictability

We will first consider the degree of observed predictability for the monthly and seasonal SOI. For the sake of comparison, both AR(1) and ARMA(1, 1) processes are used to forecast the monthly SOI and both AR(1) and AR(3) processes to forecast the seasonal SOI. Keep in mind, nevertheless, that it was found in CK that the fit of an AR(1) process to the SOI was inferior to that of an ARMA(1, 1) process in the monthly case and to that of an AR(3) process in the seasonal case.

To illustrate the application of (17), we restrict consideration to the observed predictability one-step and two-steps ahead. In a way similar to the one-step ahead forecasts produced in CK, two-step ahead forecasts are computed at various time origins. For instance, a forecast for June 1982 is produced using ARMA parameter estimates based on monthly SOI observations from January 1935 through April 1982, and a forecast for July 1982 is made using parameter estimates based on observations through May 1982, and so on.

For an AR(p) process, the expression (2) for l-step ahead forecasts at time origin t is equivalent to

$$\hat{X}_t(l) = \sum_{i=1}^{p} \phi_i(l) X_{t-i+1}. \tag{19}$$

For one-step ahead forecasts, the weights are simply $\phi_i(1) = \phi_i$, $i = 1, 2, \cdots, p$. For two-step ahead forecasts, the weights are given by

$$\begin{aligned} \phi_i(2) &= \phi_1\phi_i + \phi_{i+1}, \quad i = 1, 2, \cdots, p-1, \\ \phi_p(2) &= \phi_1\phi_p. \end{aligned} \tag{20}$$

This result demonstrates how any l-step ahead forecast based on an AR(p) process can be expressed as a weighted sum of a finite number of current and past observations.

For a general ARMA(p, q) process, the expression for l-step ahead forecasts is more complex because noise terms are also involved. Nevertheless, a forecast still can be expressed as a weighted sum of current and past observations, with the sum now being infinite. From the representation of an ARMA(p, q) process as an infinite-order AR process, the expression (2) for l-step ahead forecasts at origin t is equivalent to

$$\hat{X}_t(l) = \sum_{i=1}^{\infty} \pi_i(l) X_{t-i+1}. \tag{21}$$

Because the π-weights constitute a convergent series, this sum can be treated as finite for operational purposes. For the case of an ARMA(1, 1) process, the π-weights used to obtain the one-step ahead forecast are given by

$$\pi_i(1) = (\phi_1 - \theta_1)\theta_1^{\,i-1}, \quad i = 1, 2, \cdots. \tag{22}$$

Similarly, the π-weights for the two-step ahead forecasts are given by

$$\pi_i(2) = \phi_1(\phi_1 - \theta_1)\theta_1^{\,i-1}, \quad i = 1, 2, \cdots. \tag{23}$$

Table 1 contains the estimates of one- and two-month ahead observed predictability of the monthly SOI time series (space does not permit individual monthly forecasts to be listed). Note that, since SOI is a time-averaged quantity, "one-month ahead" forecasts actually have zero months lead time and "two-month ahead" forecasts actually have one month lead time. The time period for which monthly SOI is forecast has been chosen in order to mask the onset of the large, negative SOI values that persisted from June 1982 until April 1983. The estimate of one-month ahead observed predictability is slightly greater for an ARMA(1, 1) process than for an AR(1) process, whereas the estimate of two-month ahead observed predictability is substantially greater for an ARMA(1, 1) process.

Table 2 gives seasonal SOI forecasts, based on AR(1) and AR(3) processes, as well as the actual SOI seasonal observations from summer 1982 through fall 1984. For both models, the one-season ahead individual forecast errors are usually smaller than the corresponding two-season ahead errors. In general, except for their onset in summer 1982, the major 1982–83 SOI anomalies, together with the more recent minor anomalies in 1984, could have been predicted with a considerable degree of skill at least one season in advance (i.e., zero seasons lead time) using an AR(3) process.

TABLE 1. Estimates of one- and two-month ahead observed predictability of monthly SOI time series.

	AR(1) process[b]		ARMA(1, 1) process[b]	
Time period[a]	λ_1	λ_2	λ_1	λ_2
Jun 1982–Nov 1984	51.1%	16.3%	52.1%	29.0%

[a] For which forecasts were made.

[b] Time period on which AR(1) and ARMA(1, 1) parameter estimates are based ranges from January 1935 through May 1982 for one-month ahead June 1982 forecast to January 1935 through October 1984 for one-month ahead November 1984 forecast; and from January 1935 through April 1982 for two-month ahead June 1982 forecast to January 1935 through September 1984 for two-month ahead November 1984 forecast.

TABLE 2. One- and two-season ahead forecasts and estimates of observed predictability of seasonal SOI time series.

	Forecast SOI				
	AR(1) process[a]		AR(3) process[a]		
Season	$l = 1$	$l = 2$	$l = 1$	$l = 2$	Actual SOI
Summer 1982	−0.311	0.435	−0.082	0.318	−4.08
Fall 1982	−2.628	−0.206	−3.059	−0.506	−4.73
Winter 1983	−3.093	−1.692	−3.996	−2.921	−5.88
Spring 1983	−3.898	−2.023	−3.732	−2.564	−3.57
Summer 1983	−2.445	−2.585	−2.362	−2.592	−0.62
Fall 1983	−0.427	−1.147	0.536	−0.528	1.17
Winter 1984	0.801	−0.293	1.741	1.613	0.53
Spring 1984	0.364	0.549	0.865	1.835	−0.55
Summer 1984	−0.377	0.249	−0.586	0.316	−0.38
Fall 1984	−0.260	−0.258	−0.568	−0.747	0.17
λ_l	46.8%	−15.1%	52.6%	1.7%	

[a] Time period on which AR(1) and AR(3) parameter estimates are based ranges from spring 1935 through spring 1982 for one-season ahead summer 1982 forecast to spring 1935 through summer 1984 for one-season ahead fall 1984 forecast; and from spring 1935 through winter 1982 for two-season ahead summer 1982 forecast to spring 1935 through spring 1984 for two-season ahead fall 1984 forecast.

Table 2 also contains the estimates of one- and two-season ahead observed predictability of the seasonal SOI time series. The estimate of one-season ahead observed predictability for an AR(3) process is somewhat greater than that for an AR(1) process. Neither model indicates any observed predictability two seasons ahead. Because the time period for which seasonal SOI is forecast has been intentionally chosen for its known anomalous characteristics, any predictability two seasons ahead might have been hidden.

b. *Theoretical monthly and seasonal predictability*

We now apply the expressions for the measure of *theoretical predictability* defined in section 2 to the monthly and seasonal SOI time series. For the monthly SOI data, the theoretical predictability is estimated using (13) when an AR(1) process is assumed and using (14) and (15) when an ARMA(1, 1) process is assumed. Table 3 gives estimates of one- and two-month ahead theoretical predictability when the corresponding ARMA parameter estimates are substituted into (13)–(15). These ARMA parameter estimates are obtained from fitting the monthly SOI data for the entire period of record (i.e., January 1935 through November 1984) and also for a somewhat shorter period (i.e., January 1935 through May 1982).

The estimates of one- and two-month ahead theoretical predictability with an AR(1) process using the entire SOI database are about 39% and 15%, respectively (see Table 3). In the case of an ARMA(1, 1) process, these estimates increase to 44% and 35%, respectively. They are also larger than those derived from the shorter database. A comparison of Tables 1 and 3

TABLE 3. Estimates of one- and two-month ahead theoretical predictability of monthly SOI time series.

	AR(1) process		ARMA(1, 1) process	
Time period[a]	λ_1	λ_2	λ_1	λ_2
Jan 1935–May 1982	34.9%	12.2%	40.9%	32.2%
Jan 1935–Nov 1984	39.0%	15.2%	44.1%	34.5%

[a] On which AR(1) and ARMA(1,1) parameter estimates are based.

indicates considerable discrepancies among observed and theoretical predictability estimates, although the patterns are relatively consistent. There are several possible explanations for these discrepancies. First, theoretical predictability is derived directly from ARMA parameter estimates based upon a relatively large sample size (e.g., 1935 to 1982 or 1984), whereas observed predictability is determined using forecasts for only a relatively short time period (i.e., 1982 to 1984). Further, these independent SOI observations for which forecasts were made constitute an extremely anomalous period, not particularly representative of the entire time series.

Figure 2 shows the estimates of theoretical predictability as a function of l for both AR(1) and ARMA(1, 1) processes based on parameter estimates obtained from the entire period of record. It is evident that these theoretical predictability estimates are systematically larger, especially at higher lead times, when an ARMA(1, 1) process is assumed. Although the predictability is negligible beyond three months (i.e., two months lead time) for an AR(1) process, it only becomes negligible beyond about 10 months (i.e., nine months lead time) for an ARMA(1, 1) process.

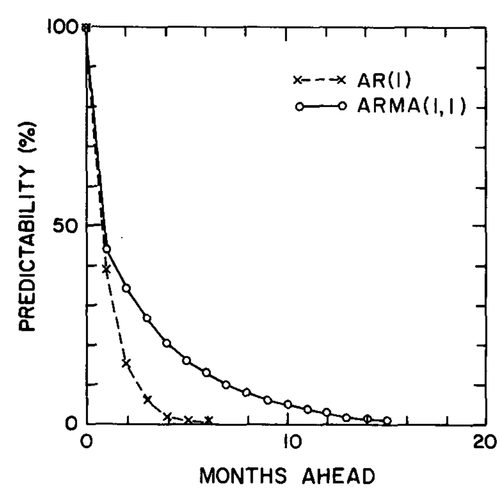

FIG. 2. Estimated theoretical predictability of monthly SOI based on AR(1) and ARMA(1, 1) parameter estimates obtained using data from January 1935 through November 1984.

For the seasonal SOI data, the theoretical predictability of an AR(3) process is estimated by substituting its parameter estimates into (A3) and (12). Table 4 gives estimates of one- and two-season ahead theoretical predictability for AR(1) and AR(3) processes. The estimates of one- and two-season ahead theoretical predictability with an AR(3) process based on parameter estimates obtained from the entire database (i.e., spring 1935 through fall 1984) are about 53% and 31%, respectively. These theoretical predictability estimates are correspondingly larger than those obtained using an AR(1) process. For an AR(3) process, a comparison of Tables 2 and 4 suggests that observed predictability is about the same as theoretical predictability one season ahead, but considerably smaller two seasons ahead. The same explanations provided for such discrepancies in the monthly case are relevant here. Figure 3 shows the estimates of theoretical predictability as a function of l, indicating that predictability becomes negligible beyond three seasons (i.e., two seasons lead time) whether an AR(1) or an AR(3) process is assumed.

In order to account for the fact that the time series models were fitted on the basis of finite sample sizes, it might be appropriate to apply a correction factor to the estimated levels of predictability just cited. Using (16) and parameter estimates obtained from the entire SOI database, a more nearly unbiased estimate of predictability for one-season ahead forecasts based on the AR(3) process is about 52.1%. In this case, because the total number of observations is relatively large, the adjustment is quite small.

4. Summary and conclusion

In this paper, we have introduced a measure for quantifying atmospheric predictability based on expressions for the relative prediction error of a parametric time series model. As a simple illustration, estimates of predictability of the monthly and seasonal SOI have been obtained. Based on an ARMA(1, 1) process for the monthly data, it is found that the theoretical predictability for the next month (i.e., zero months lead time) is about 44% and about 35% for two months ahead (i.e., one month lead time), with essentially no predictability beyond ten months (i.e., nine months lead time). Based on an AR(3) process

TABLE 4. Estimates of one- and two-season ahead theoretical predictability of seasonal SOI time series.

	AR(1) process		AR(3) process	
Time period[a]	λ_1	λ_2	λ_1	λ_2
Spring 1935–spring 1982	43.8%	19.2%	50.3%	30.0%
Spring 1935–fall 1984	46.9%	22.0%	52.8%	30.7%

[a] On which AR(1) and AR(3) parameter estimates are based.

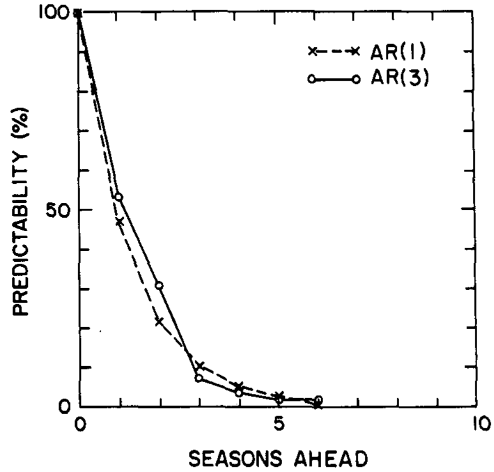

FIG. 3. Estimated theoretical predictability of seasonal SOI based on AR(1) and AR(3) parameter estimates obtained using data from spring 1935 through fall 1984.

for the seasonal data, it is found that the theoretical predictability for the next season (i.e., zero seasons lead time) is about 53% and about 31% for two seasons ahead (i.e., one season lead time), with essentially no predictability beyond three seasons (i.e., two seasons lead time). If AR(1) processes are assumed as models instead, then the predictability is underestimated at all lead times for the monthly SOI and at small lead times for the seasonal SOI. For comparative purposes, the degree of observed predictability is also assessed on the basis of a small but independent dataset.

Our method attempts to determine actual realized, rather than potential, predictability. It is predicated on the availability of a parametric time series model to represent adequately the given meteorological variable. In practice, the identification of such a model is not necessarily straightforward. For simplicity, we have restricted consideration to a class of stationary models known as ARMA processes. However, nonstationary models that allow for intra-annual differences in predictability might be more realistic for variables such as the SOI.

The type of SOI forecasts considered here could be made operationally, since the pressure observations at Tahiti and Darwin can be obtained on a timely basis. Note that the Climate Analysis Center of the National Oceanic and Atmospheric Administration uses a similar index for operationally monitoring the SO fluctuations. Due to the fact that pressure data are readily accessible, forecasting models such as those described in CK deserve further consideration as a practical tool.

However, if the goal is to achieve better predictability than that currently demonstrated, other variables (e.g., surface winds over the tropical Pacific Ocean) certainly need to be employed as additional predictors. In particular, certain precursors of changes in the SO have been identified by van Loon and Shea (1985) and

Wright (1986). To obtain this higher degree of predictability, the more sophisticated approach of multiple time series analysis is necessary.

Acknowledgments. We thank Barbara G. Brown for comments on a draft of this manuscript, and an associate editor and the reviewers for suggestions that led to improvements in the manuscript. This research was supported in part by the National Oceanic and Atmospheric Administration under Contract NA-80RAH00002 to JIMAR at the University of Hawaii.

APPENDIX

Predictability of an AR(3) Process

We demonstrate how the predictability λ_l of an AR(3) process can be expressed as a function of the autoregression coefficients (ϕs) alone. The Yule-Walker equations for this process (Box and Jenkins, 1976, p. 55) are

$$\begin{aligned} \rho_1 &= \phi_1 + \phi_2\rho_1 + \phi_3\rho_2, \\ \rho_2 &= \phi_1\rho_1 + \phi_2 + \phi_3\rho_1, \\ \rho_3 &= \phi_1\rho_2 + \phi_2\rho_1 + \phi_3. \end{aligned} \tag{A1}$$

Solving the system of equations (A1) to express ρ_1, ρ_2 and ρ_3 in terms of ϕ_1, ϕ_2 and ϕ_3 gives

$$\begin{aligned} \rho_1 &= (\phi_1 + \phi_2\phi_3)/\delta, \\ \rho_2 &= \phi_2 + (\phi_1 + \phi_3)(\phi_1 + \phi_2\phi_3)/\delta, \\ \rho_3 &= \phi_1\phi_2 + \phi_3 + [\phi_2 + \phi_1(\phi_1 + \phi_3)](\phi_1 + \phi_2\phi_3)/\delta, \end{aligned} \tag{A2}$$

where

$$\delta = 1 - (\phi_2 + \phi_1\phi_3 + \phi_3^2).$$

Inserting (A2) into (9), a value of λ_1 is obtained in terms of only ϕ_1, ϕ_2 and ϕ_3:

$$\begin{aligned} \lambda_1 = {} & \phi_2^2 + \phi_3^2 + \phi_1\phi_2\phi_3 + (\phi_1 + \phi_2\phi_3) \\ & \times [\phi_1 + (\phi_1 + \phi_3)(\phi_2 + \phi_1\phi_3) + \phi_2\phi_3]/\delta. \end{aligned} \tag{A3}$$

REFERENCES

Akaike, H., 1969: Fitting autoregressions for predictions. *Ann. Inst. Statist. Math.,* **21,** 243–247.

Box, G. E. P., and G. M. Jenkins, 1976: *Time-Series Analysis: Forecasting and Control* (rev.), Holden-Day, 575 pp.

——, and G. C. Tiao, 1977: A canonical analysis of multiple time series. *Biometrika,* **64,** 355–365.

Chu, P.-S., and R. W. Katz, 1985: Modeling and forecasting the Southern Oscillation: A time-domain approach. *Mon. Wea. Rev.,* **113,** 1876–1888.

Davis, R. E., 1978: Predictability of sea level pressure anomalies over the North Pacific Ocean. *J. Phys. Oceanogr.,* **8,** 233–246.

Glantz, M. H., 1984: El Niño—Should it take the blame for disasters? *Mazingira,* **8,** 21–26.

——, R. W. Katz and M. E. Krenz, Eds., 1987: *Economic and Societal Impacts of the 1982–83 Worldwide Climate Anomalies.* Report for the United Nations Environment Programme. National Center for Atmospheric Research, 105 pp.

Hastenrath, S., 1986: On climate prediction in the tropics. *Bull. Amer. Meteor. Soc.,* **67,** 696–702.

Jones, R. H., 1985: Time series analysis—time domain. *Probability, Statistics, and Decision Making in the Atmospheric Sciences,* A. H. Murphy and R. W. Katz, Eds., Westview Press, 223–259.

Katz, R. W., and R. H. Skaggs, 1981: On the use of autoregressive-moving average processes to model meteorological time series. *Mon. Wea. Rev.,* **109,** 479–484.

Madden, R. A., 1976: Estimates of the natural variability of time-averaged sea level pressure. *Mon. Wea. Rev.,* **104,** 942–952.

Namias, J., 1985: Remarks on the potential for long-range forecasting. *Bull. Amer. Meteor. Soc.,* **66,** 165–173.

Pierce, D. A., 1979: R^2 measures for time series. *J. Amer. Statist. Assoc.,* **74,** 901–910.

Shukla, J., and D. Gutzler, 1983: Interannual variability and predictability of 500 mb geopotential heights over the Northern Hemisphere. *Mon. Wea. Rev.,* **111,** 1273–1279.

Trenberth, K. E., 1985: Potential predictability of geopotential heights over the Southern Hemisphere. *Mon. Wea. Rev.,* **113,** 54–64.

van Loon, H., and D. J. Shea, 1985: The Southern Oscillation. Part IV: The precursors south of 15°S to the extremes of the Southern Oscillation. *Mon. Wea. Rev.,* **113,** 2063–2074.

Wright, P. B., 1985: The Southern Oscillation: An ocean-atmospheric feedback system? *Bull. Amer. Meteor. Soc.,* **66,** 398–412.

——, 1986: Precursors of the Southern Oscillation. *J. Climatol.,* **6,** 17–30.