NOTES FOR LECTURE 3: RADIATION

Planck's Law

In physics, Planck's law describes the amount of electromagnetic energy, or radiance, with a certain wavelength radiated by a black body in thermal equilibrium (i.e. the spectral radiance of a black body). The law is named after Max Planck, who originally proposed it in 1900. The law was the first to accurately describe black body radiation.

$$E_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{k\lambda T}} - 1}$$

- k is the Boltzmann constant, and is 1.38x10-23 J/K
- h is Planck's constant and is 6.626x10-34 Js
- c is the speed of light in a vacuum and is 2.9979x108 m s-1.
- Blackbody radiation is isotropic, homogeneous, unpolarized and incoherent.
- Planck's Law means that the sun isn't special, all objects radiate.

Stefan Boltzmann Equation:

1) All objects above Absolute Zero emit radiation (energy)
2) How much can be determined by the Stefan-Boltzman Equation

Stefan-Boltzman law is obtained by integrating Planck's Law. As the temperature of an object increases, the radiant energy (E) emitted by that object increases by a power of 4.

($E = \sigma T^4$) (E is in Watts/m^2)

E is referred to as irradiance

$P = A\sigma T^4$ P = Power (Watts, J/s) This is energy per time

A = Surface Area of the object, (m^2)

σ = sigma, the Stefan-Boltzman *constant*

= $5.67*10^{-8}$ W/(m^2*K^4)

T = Temperature in Kelvin

a) Provisions: Holds true only for objects that are "black bodies"
b) A "Black Body" is an object that follows Planck's Law.
c) Some objects emit less energy (lighter objects emit less energy than darker objects)
d) Nothing emits MORE than a black body

Example: **How much power does my head emit each second?**

Want to find Power, need Area and Temperature.

Power = Energy/Time = $A \sigma T^4$

Estimate: Body T is approx = 37˚C = 273 + 37 = 310K

Sphere =10 in diameter * (2.54 cm/1in) = 25 cm = 0.25 m

Area of circle = πR^2 where R = radius

Surface Area of Sphere = $4\pi R^2$ where R= radius

Diameter, D = 0.25 m so R = 0.125 m
Head Surface Area is approx = $4 * \pi * (0.125\ m)^2 = 0.2\ m^2$
Power = Energy/Time = $A\ \sigma\ T^4$
= $(0.2\ m^2)(5.67*10^{-8}\ W/(m^2*K^4))(310K)^4$
= 105 W $\approx$ 100 J/s

If we've been skiing for 4 hours: How much energy are we loosing?

$$\text{Energy} = 100\frac{J}{s} * \frac{60s}{1\min} * \frac{60\min}{1hr} * 4hr = 100 * (1.4 * 10^4)J$$

1 calorie = 4.2 J

Think about it: 1 big mac $\approx$ 1000 cal $\approx$ 4000 Joules

How many big macs = $1.4*10^6$ J/4000J = 250 big macs – oops, what went wrong here?
1 food calorie = 1000 heat calories. So need to divide by 1000. 250/1000 = .25 big mac
So why do I feel hungry enough to eat the equivalent of two big macs?

Wien's Law
λ = The wavelength at which the most amount of radiation is emitted by an object with Temp, T.

$$\boldsymbol{\lambda = b/T = 2897/T}$$

Where λ = wavelength in µm
b = Wien's displacement constant = 2897 has units of Kµm
T = is Temperature in Kelvin

People have a temperature of about 300K, we can calculate the peak wavelength of energy then:

$$\lambda_{peakE} = \frac{2900K\mu m}{300K} \approx 10\mu m$$ This is in the Infra Red (IR) range

4) What is the λ_{peak} for the sun?
Answer: The visible wavelength!! 0.4µm – 0.75µm
Let's say that it's 0.5µm just for the sake of the next question.

5) What is the (approx) temperature of the sun?

$$\frac{0.5\mu m}{2900K\mu m} = \frac{1}{T}$$

$$\frac{2900K\mu m}{0.5\mu m} = T = 5800K$$

Kirchhoff's law of thermal radiation

For an arbitrary body emitting and absorbing thermal radiation in thermodynamic equilibrium, the emissivity is equal to the absorptivity for each wavelength.

$a_\lambda = \epsilon_\lambda$

A good absorber is a good emitter, and a poor absorber is a poor emitter. Naturally, a good reflector must be a poor absorber. This is why, for example, lightweight emergency thermal blankets are based on reflective metallic coatings: they lose little heat by radiation.

Earth's Radiative Balance + Temperature

Box models
Characteristics:
1) Box: A system (usually draw an imaginary envelope around it, eg., the lake example)
2) Burden: The amount of material in the box, eg., the water measure in something (volume m^3; mass, kg; mole, number of molecules; energy, J)
3) Input Flux (Source Flux): The rate at which material (burden) is added to the box. The units are amount/time i.e. kg/s, J/s
4) Output Flux (Sink Flux): The rate at which material (burden) is removed from the box. The units are amount/time i.e. kg/s, J/s
5) Steady State: The state where the burden is constant over time
i) If the box is at steady state then: INPUT flux = OUTPUT flux
ii) If in = 1000 kg/day and out = 500 kg/day → Lake RISES
iii) If in = 500 kg/day and out = 1000 kg/day → Lake SINKS

Fact: Lake level is steady over time
Does this mean that no water flows into and out of the lake? NO
Inputs = Outputs, it must be in steady state
In any given period of time, the amount of water that entered the lake = amount that exited the lake
Dynamic Equilibrium = Steady State

Box Model of the Earth
Box: Earth
Burden: Amount of Energy falling on the Earth in W/m^2
How to calculate the input flux, given the temperature of the sun is 5800 K?
Use Stephan-Boltzmann's Law to calculate Solar Constant

Solar Constant = $S = E_s * (R_s / R_{sE})^2$

At the sun's surface:

$E_s = \sigma T^4 = 5.67*10^{-8}\ W/(m^2*K^4)*(5800K)^4 = 6.416X10^6\ W/m^2$

The radius of the sun $R_s = 6.955 \times 10^8$ m
Distance of sun to Earth $R_{sE} = 1.5 \times 10^{11}$ m

$S = E_s(R_s /R_{sE})^2 = 1379 \text{ W/m}^2$

Radiative Equillibrium – incoming = outgoing

Geometry –
sunlight falls on disk of the Earth = πr^2
Earthlight radiates in all directions = 4 πr^2
The radius of the Earth $r = 6.3781 \times 10^6$ m

$S(1\text{-}a)\, \pi r^2 = \sigma\, T_E^4\, (4\, \pi r^2)$

Available Incoming = (1379 W/m^2)*0.7/4 = 241 W/m^2

$T_E = [S\,(1\text{-}a)/4\sigma]^{1/4} = [241 \text{ W/m}^2 / 5.67 \cdot 10^{-8} \text{ W/(m}^2 \cdot \text{K}^4)]^{1/4} = 255\text{K} = -18^\circ\text{C} = 0^\circ\text{F}$*

This is the temperature of the earth as seen from space. We are actually seeing the temperature of one square meter of somewhere above the surface, not the surface of the earth itself.
For the earth to be at steady state we must be emitting at 255K, but that doesn't mean that's what our surface temperature has to be.
Temperature of the earth without green house gases (GHG) would be 255K
GHG layer is more or less transparent to visible light
GHG layer absorbs IR radiation
Additional energy from GHG layer (downward = 240W) is the reason why the surface is warmer than 255K.

The Real temperature of the earth's surface:
Input fluxes = Output fluxes for steady state
(Sunlight 343 W) + (IR_{ghg} 240W) = Albedo 103W + IR_{out} + $A\sigma T^4$ surface
343 + 240 = 103 W + IR_{out} + $A\sigma T^4$ surface
480 W = $A\sigma T^4$ surface

$$\left(\frac{480 Wm^{-2}}{5.67 \times 10^{-8} Wm^{-2} K^{-4}}\right)^{1/4} \quad T = 303\text{ K}, 30^\circ\text{C}, 85^\circ\text{F}$$

Average T_{earth} = 288K why? The GHG layer isn't a perfect absorber. So you don't have as much energy to warm the earth's surface, e.g., it is not a perfect blanket.

What is the surface T of Mars?
Distance of sun to Earth $R_{sE} = 1.5 \times 10^{11}$ m
Distance of sun to Mars $R_{sM} = 2.25 \times 10^{11}$ m
Solar constant for mars = $(R_{sE} / R_{sM})^2$ *1379 W/m^2 = .44*1379 W/m^2 = 612 W/m^2
Geometry requires we divide this by 4, so the output of mars must = 153 W/m^2
Albedo = 0.15

Output = 153 W * 0.15 = 130.05

$$\left(\frac{130.5Wm^{-2}}{5.67\times 10^{-8}Wm^{-2}K^{-4}}\right)^{1/4} = T = 218K = -55^{\circ}C$$

The measured answer from Mars is -50˚C

Q: What would Mars' T be if we terraformed it to add a perfect GHG layer? T= 260K, -13C

MARS	EARTH	VENUS
Little GHG	Some GHG	Run away GHG
T=-50°C	T = 15°C	T = 420°C
	CO_2 = 0.03%	CO_2 = 96%