

GG605: Lava Flow Rheology and Morphology (Fall 2017)

Project IV: Thermo-rheological flow model

Set: November 21st

Due: December 12th

General instructions: The aim of this project is to build on the notions and quantitative analyses of flow rheology and velocity (Project 1), effusion rates and viscosity (Project 2), heat loss and cooling (Project 3) to construct your own thermo-rheological model of lava flow propagation.

The basis of this project is the FLOWGO model initially distributed by Harris and Rowland (2001) and recently revised (Harris and Rowland 2015). This model simulates downflow changes in rheology. The foundation of the model is a series of heat loss equations (identical to Project 3). The model moves a ‘control volume’ of lava in one-meter increments down a pre-existing channel of known at-vent dimensions. At each increment step, the model calculates core cooling and crystallization, and hence the temperature and crystallinity-dependent rheological parameters: viscosity and yield strength. After setting up the initial inputs, the model is self-adaptive, allowing the control volume to move down-channel until cooling and crystallization cause its velocity to fall to zero.

We will simulate the flow conditions of the Mauna Ulu 1974 channel that we visited. Harris (unpublished) made measurements of the channel width from when the tube-fed part of the flow progressively gave way to the channelized part began (~1650 m from the vent), down to the portion of the flow that became dispersed (i.e. non-channelized) at ~ 6000 m from the vent. The flow continued for another ~2600 m after becoming dispersed.

Hint: it helps to try and draw simplified diagrams of your system/lava to visualize where different heat/gains losses and parameters come into play.

Step 1: Setting up the modified Jeffrey’s equation

The foundation of the FLOWGO model is Jeffrey’s **velocity equation** modified to account for yield strength by Moore (1987) (see Project 1 for details):

$$V = \frac{h\tau_b}{B\eta_{bulk}} \left[1 - \frac{4}{3} \left(\frac{\tau_0}{\tau_b} \right) + \frac{1}{3} \left(\frac{\tau_0}{\tau_b} \right)^4 \right] \quad \text{(Semi-circular channel)} \quad \text{(Eq. 1a)}$$

$$V = \frac{h\tau_b}{B\eta_{bulk}} \left[1 - \frac{3}{2} \left(\frac{\tau_0}{\tau_b} \right) + \frac{1}{2} \left(\frac{\tau_0}{\tau_b} \right)^3 \right] \quad \text{(Wide Channel)} \quad \text{(Eq. 1b)}$$

Where h is the channel/flow depth, τ_0 the yield strength, τ_b the shear stress at the base of the flow, and η_{bulk} is bulk viscosity (melt+bubbles+crystals). B is a shape constant (3 for wide, 8 for semi-circular).

The basal shear stress τ_b is defined as:

$$\tau_b = h \rho_{bulk} g \sin(\alpha) \quad (\text{Eq. 2})$$

Where g is acceleration due to gravity, ρ_{bulk} is bulk lava flow density (vesicles+crystals+melt), and α is slope.

To solve equation 1a or b, we will first need to set a ‘typical depth’ h . Let’s begin with the minimum depth over the most proximal section of the channel (C8), i.e. 1.9 m. For simplicity, we will first assume that channel width (w) is equal to height (h).

Step 2: Setting up initial Rheological parameters

The following other inputs are also needed to solve Eq. 1a or b:

The lava flow density ρ_{bulk} is related to the dense rock equivalent density (ρ_{DRE} , crystals+melt) and the lava flow vesicle fraction ϕ_B by:

$$\rho_{bulk} = (1 - \phi_B) \rho_{DRE} \quad (\text{Eq. 3})$$

The yield strength τ_0 can be expressed as a function of temperature and crystal content ϕ_C () following:

$$\tau_0(T, \phi_C) = \tau_0^{liq} \left[\exp^{b(T_{erupt} - T_{core})} - 1 \right] + 6500 \phi_C^{2.85} \quad (\text{Eq. 4})$$

Where τ_0^{liq} is the yield strength at liquidus conditions (0.01 Pa for basalt) and b is a constant (here equal to 0.08 K^{-1} for basalt) that also depends on lava composition. T_{erupt} is the at vent temperature, and T_{core} is the temperature at the core of the flow.

Note: this form combines the Ryerson et al. (1988) expression for crystal-content-dependence

$$\tau_0(\phi_C) = 6500 \phi_C^{2.85} \text{ with an expression for } T\text{-dependence } \tau_0(T, \phi_C) = \tau_0^{liq} \left[\exp^{b(T_{erupt} - T_{core})} - 1 \right] \text{ from}$$

Dragoni (1989), similar to the form of Chester et al. (1985 we saw in class).

The bulk viscosity η_{bulk} is a function of composition (+volatiles), temperature, crystal, and bubble content. Here, we will take the approach of Harris and Allen (2008) to calculate the three-phase viscosity using expressions that incorporate relative sizes of crystals and bubbles:

Case 1: Size of crystals < Size of bubbles

$$\eta_{bulk} = \eta_{melt} \left(\frac{1-\phi_c}{1-\phi_b} \right)^{-\frac{5}{2}} (1-\phi_b)^{-1} \quad (\text{Eq. 5a})$$

Case 2: Size of crystals = Size of bubbles

$$\eta_{bulk} = \eta_{melt} (1-\phi_c - \phi_b)^{-\left[\frac{5\phi_c + 2\phi_b}{2(\phi_c + \phi_b)} \right]} \quad (\text{Eq. 5b})$$

Case 3: Size of crystals > Size of bubbles

$$\eta_{bulk} = \eta_{melt} \left(\frac{1-\phi_b}{1-\phi_c} \right)^{-1} (1-\phi_c)^{-\frac{5}{2}} \quad (\text{Eq. 5c})$$

Where η_{melt} is the melt viscosity. For the 1974 lavas, we will assume that the crystals and bubbles had similar sizes. As you have done for project 2, the melt viscosity can be obtained using the all-encompassing general model of Giordano et al. (2008), or, more simply, using a melt viscosity model applicable to Hawaiian basalt only (e.g. Dragoni 1989):

$$\eta_{melt}(T) = \eta_0^{erupt} \exp^{a(T_{erupt} - T_{core})} \quad (\text{Eq. 6})$$

Where η_0^{erupt} is the bulk viscosity at eruption temperature T_{erupt} and a is a constant (here equal to 0.04 K^{-1} for basalt) that also depends on lava composition. To calculate η_0^{liq} - the viscosity of the 1974 Mauna Ulu lava at eruption temperature - use your Giordano et al. (2008) spreadsheet or matlab code using $T_{erupt} = 1170^\circ\text{C}$, and a lava composition shown on the right.

Oxide	wt. %
SiO ₂	50.47
TiO ₂	2.64
Al ₂ O ₃	13.11
FeO	11.31
MnO	0.17
MgO	8.40
CaO	11.02
Na ₂ O	2.39
K ₂ O	0.53
P ₂ O ₅	0.27
F	0.056
Cl	0.014
H ₂ O	0.076
Total	100.45

Note: For a more precise liquid viscosity model, you are welcome to use Giordano et al. (2008) instead of Eq. 6 throughout (not well suited for excel calculations but can be more easily done in Matlab).

Step 3: Setting up the Heat Loss model

We now need to calculate the heat loss per meter of advance. This will enable us to calculate core cooling with distance. First, we need to define our thermal surface.

Thermal structure: We will use a two-component model (e.g. crust with temperature T_c and a core with temperature T_h), where the effective **radiation** temperature of the flow surface (T_{rad}) can be described by:

$$T_{rad} = \left[fT_{surf}^4 + (1-f)T_{hot}^4 \right]^{0.25} \quad (\text{Eq. 7})$$

And the effective **convection** temperature (T_{conv}) by:

$$T_{conv} = \left[fT_{surf}^{1.333} + (1-f)T_{hot}^{1.333} \right]^{0.75} \quad (\text{Eq. 8})$$

In which f is the surface fraction occupied by the cold component (crust) at temperature T_{crust} (K). We discussed in class that in many cases, the exposed ‘hot’ parts of lava flows may not actually be as hot as the core (rapid formation of visco-elastic layer etc...). To account for this in our model, we will assume that the maximum temperature encountered at the surface is somewhat lower than the true core temperature. We will call this a temperature offset or buffer T_{buff} , which one can express as:

$T_{core} = T_{hot} + T_{buff}$, or, since we are interested in T_{hot} for Eq. 7 and 8,

$$T_{hot} = T_{core} - T_{buff} \quad (\text{Eq. 9})$$

The crust temperature can be set to be constant through the model, or vary according to the Hon et al. (1994) model we discussed in class:

$$T_{surf} = (-140 \times \log(t) + 303) + 273.15 \quad (\text{Eq. 10a})$$

Note that this is the Hon et al. 1994 expression simply converted to K. To solve this equation, we will need to estimate time (t) since the beginning of the flow emplacement using:

$$t = \sum \frac{\Delta x}{V(x)} \quad (\text{Eq. 10b})$$

Where Δx is our distance step and $V(x)$ is the mean velocity at each distance increment.

Note that through these instructions I'll be using small deltas (δ) to express incremental changes (e.g. per meter, per degree), whereas big delta (Δ) refers to change integrated over a given value (e.g. over the 200 m distance step). For distance, ‘ x ’ refers to the position in x , and ‘ X ’ is the cumulative distance.

We now need to estimate the fraction of crust f . Harris et al. (2007) proposed that crust growth depended on flow/channel velocity V (Eq. 1a and b) and followed an exponential growth law of form:

$$f(V) = g_1 \exp(g_2 \times V) \quad (\text{Eq. 11})$$

Where g_1 and g_2 are constants that depend on lava composition and properties (here taken to be 0.9 and -0.16 respectively). Note that it is velocity variations here controlling the amount of crust cover. Alternatively, one can also set crust cover f to be constant downstream (0 to 1).

Heat losses: We will only consider heat losses associated with radiation, conduction, and forced convection for this exercise. These are written:

Forced convection:

$$Q_{force} = C_H \rho_{air} C_P^{air} U (T_{conv} - T_{amb}) w \quad (\text{Eq. 12})$$

in which C_H is the square of the ratio of wind speed to the slip speed of wind across the ground (taken to equal to 0.0036), C_P^{air} is the heat capacity of air ($\text{J kg}^{-1} \text{K}^{-1}$), ρ_{air} is the air density, and U is mean wind speed (m s^{-1}). Compared to the previous version you worked on for Project 3, the heat loss is integrated along the channel width (w is added) to get ‘sectional’ heat loss at different distances travelled (W m^{-1}) instead of heat loss per unit area (W m^{-2}). Also, the air density and specific heat capacities can be calculated using:

$$\rho_{air} = \frac{352.6}{T_{mean}} \quad (\text{Eq. 13a})$$

$$C_P^{air} = 947 + 0.191 T_{mean} \quad (\text{Eq. 13b})$$

Note: These forms save you from checking the look up tables like you did in Project 3.

Like in project 3, T_{mean} is the average temperature of the boundary layer:

$$T_{mean} = 0.5(T_{amb} + T_{surf}) \quad (\text{Eq. 14})$$

Radiation:

$$Q_{rad} = \sigma \epsilon T_{rad}^4 w \quad (\text{Eq. 15})$$

Remember that T is in K, and that is the Stefan Boltzmann constant ($5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$) and ϵ is emissivity (~ 0.95 for lava).

Conduction through base:

$$Q_{cond} = k \frac{T_{core} - T_{base}}{h_{base}} w \quad (\text{Eq. 16})$$

In which k is the lava thermal conductivity and $\frac{T_{core} - T_{base}}{h_{base}}$ is the temperature gradient across the basal crust, with h_{base} the distance between the core interior and the point at which T_{base} (here

assumed to be 500°C) is reached. For h_{base} , we can assume that this distance is 19% of the total thickness:

$$h_{base} = 0.19h \quad (\text{Eq. 17})$$

We previously assumed that thermal conductivity k was constant but it depends in fact on temperature and on porosity (or vesicularity ϕ_b in our case) of the lava. We will only account for the vesicularity-dependence, using (Peck 1978):

$$k = (1.929 - 1.554\phi_b)^2 \quad (\text{Eq. 18})$$

Heat budget and cooling rate

In contrast to the scenarios we talked about in class, where there often is advection of new lava included in the heat budget, here, we will assume that there are no further advection heat gains (e.g. no more input of lava once we release our ‘control volume’). Thus:

$$Q_{adv} = 0$$

As in Project 3, you can now calculate heat loss and core cooling per meter of advance $\frac{\delta T}{\delta x}$ using:

$$\frac{\delta T}{\delta x} = \frac{-Q_{force} - Q_{rad} - Q_{cond}}{V \rho_{bulk} h C_L \left(\frac{\delta \phi}{\delta T} \right) w}$$

Where V is the mean flow velocity (m s^{-1}), ρ_{bulk} bulk lava density, C_L is the latent heat of crystallization ($3.5 \times 10^5 \text{ J kg}^{-1}$), $\frac{\delta \phi}{\delta T}$ is the volume fraction of crystallization per degree cooling and h is flow depth (m).

Assuming that effusion rate can be expressed as the product of width, depth, and velocity:

$$E_r = whV \quad (\text{Eq. 19})$$

The degree of cooling is now:

$$\frac{\delta T}{\delta x} = \frac{-Q_{force} - Q_{rad} - Q_{cond}}{E_r \rho_{bulk} C_L \left(\frac{\delta \phi}{\delta T} \right)} \quad (\text{Eq. 20})$$

This gives us core cooling per meter. To obtain the core temperature change over the $\Delta x = 200$ m step that we are considering, we can simply calculate:

$$\Delta T(x) = -\frac{\delta T}{\delta x} \times \Delta x \quad (\text{Eq. 21})$$

Finally, we can use this change in temperature to update the core temperature for the next distance step:

$$T_{core}(x + \Delta x) = T_{core}(x) - \Delta T(x) \quad (\text{Eq. 22})$$

Crystallization rate

By now you know that as lava flow temperature decreases crystallization occurs. Several assumptions can be made: (1) crystal content increases linearly with decreasing temperature (e.g. Harris and Rowland 2001), (2) crystal content increases linearly but at different rates depending on which phase(s) are crystallizing (e.g. Riker and Cashman 2009), or (3) Crystal content increases non-linearly (more realistic but requires more complex model inputs, e.g. from the MELTS program). Here, we will use assumption (2), where:

From eruption temperature to 1160°C:

$$\frac{\delta \phi}{\delta T} = 0.00083 \text{ K}^{-1} \quad (\text{Eq. 23a})$$

Below 1160°C:

$$\frac{\delta \phi}{\delta T} = 0.002 \text{ K}^{-1} \quad (\text{Eq. 23b})$$

We can now calculate the mass fraction of crystallization per meter $\frac{\delta \phi}{\delta x}$ from:

$$\frac{\delta \phi}{\delta x} = -\frac{\delta T}{\delta x} \cdot \frac{\delta \phi}{\delta T} \quad (\text{Eq. 24})$$

To obtain total fraction of crystals grown for any given position we can use:

$$\phi_c(x + \Delta x) = \phi_c(x) + \frac{\delta \phi}{\delta x} \times \Delta x \quad (\text{Eq. 25})$$

Step 4: Moving and evolving the control volume down flow

We now need to move the control volume downslope, adapting the core temperature, crystallinity, yield strength, viscosity as we go (see flow chart for detailed ‘map’). We need to make the following changes:

- (1) We need to conserve mass. We will do this by assuming that depth is constant and width is variable. In short, the effusion rate at each point downflow (E_r) should match the at-vent effusion rate i.e., $E_r = whV$. Thus, if V is varying and h is constant, w must vary to keep E_r constant so that:

$$w = \frac{E_r}{hV} \quad (\text{Eq. 26})$$

- (2) Now we need to reduce the core temperature by ΔT .
- (3) Viscosity and yield strength will need to be recalculated using this new core temperature and the new crystallinity calculated.

- (4) Check that:

- Your T_{hot} calculation uses the new core temperature
- Your f_{crust} calculation uses the new velocity
- Your T_{rad} and T_{conv} calculations use the new T_{hot} and f_{crust} values
- Your Q_{rad} calculation uses the new T_{rad} and w values
- Your Q_{force} calculation uses the new T_{conv} and w values
- Your Q_{cond} calculation uses the new T_{core} and w values
- Your new $\frac{\delta T}{\delta x}$ calculation use the new Q_{rad} , Q_{force} , and Q_{cond} values.
- Your $\frac{\delta \phi}{\delta x}$ calculation uses the new $\frac{\delta T}{\delta x}$ value.

- (5) Finally, we need to add the new crystals grown to obtain a new total crystallinity by using:

$$\phi_c(x + \Delta x) = \phi_c(x) + \frac{\delta \phi}{\delta x} \times \Delta x$$

Step 5: Testing and applying the model

To facilitate setting up your model and completing your mission, you are given the following:

- An excel spreadsheet that contains labeled columns and rows as well as most input parameters.
- Also in the same document, you’ve been given a set of field measurements (distance, width) of the channel to compare with your model outputs.

- A stolen top-secret document from Tom's computer that shows three rows of the table filled with what you might assume are the correct values (hopefully...). The damn CIA...
- A summary of initial input parameters set (see Table)
- A flow chart "cheat sheet" to remember the different steps involved and roughly the order to follow

Model test 1:

With the initial parameters given to you in the table, and assuming that crystals and bubbles are of similar sizes, run a first model up to 6000 m.

Question 1a: Does your simulated flow reach 6000 m? Are the initial model conditions we used compatible with the observed distance of 6 km for the Mauna Ulu channel?

Question 1b: Plot the downflow variations in: (i) Flow core temperature, (ii) Total crystallinity, (iii) Yield strength, (iv) Viscosity, (v) Velocity, (vi) the three main heat losses on one graph, and (vii) channel width on top of the measured field width data. **For each plot, say in a couple sentences what you observe and how you might interpret the trend.**

Question 1c: Write down the effusion rate and velocity you obtain at the vent. Write down crystallinity that the maximum distance reached.

Model test 2:

For the channel to extend further downslope, we can change one or several parameters. From field estimates at a few locations, we initially assumed that channel depth was equal to width (1.9m). Let's now make the channel deeper.

Question 2a: Set $h=5$ m. Write down the effusion rate and velocity you obtain at the vent. Write down crystallinity that the maximum distance reached. **How do those values compare to the shallower channel from Model 1?**

Question 2b: The 1974 flow was active for 50 hr, and emplaced lava over an area of $\sim 1.38 \text{ km}^2$, with an average thickness 3-7 m (Note: Thickness is not equivalent to flowing channel depth but rather represents accumulated basal unit, overflows, levees). The time-averaged discharge rate is therefore between 23-54 m^3/s (Harris et al. 2009). **How do your model 1 and 2 values compare with the TADR estimated from volume measurements? If they are different, explain why that might be.**

Model test 3:

We know from our hike on the Mauna Ulu day that the 1974 flow was not fully channelized from the vent. A tube-fed portion of the flow extended from vent to $\sim 1600\text{m}$. Furthermore, we saw plenty of collapsed tube roofs along the way on the transitional part of the channel. But our model only applies to channelized flows right? Actually, we can simulate tube flow by artificially forcing the channel surface to be 100% crusted. You can do this by setting the

fraction of crust f to 1. We are going to extend the tube-transported part of our model all the way down to 3600 m (not too far from the Chain of Craters road where we left the car). Andy Harris also measured tube heights from collapsed skylights of ~ 1.6 m, we will use this value for flow depth h . We will also need to set a lower, fixed surface temperature for the tube roof $T_{surf} = 60^\circ\text{C}$, i.e. assuming a fairly thick roof.

Note: Remember to only assign these special 'tube' conditions (T_{surf} , f) to the first 3600 m of your model, the remaining distance should be calculated in 'channel' conditions as in Models 1 and 2. The height $h = 1.6\text{m}$ will be the same for tube and channel.

Question 3a: Is this tube-channel hybrid model still capable of feeding dispersed flows beyond 6km?

Question 3b: Like in Question 1, produce a set of plots of (i) Flow core temperature, (ii) Total crystallinity, (iii) Yield strength, (iv) Viscosity, (v) Velocity, (vi) the three main heat losses on one graph, and (vii) channel width on top of the measured field width data. **For each plot, say in a couple sentences what you observe and how you might interpret the trend.**

Question 3c: How does the effusion compare with the other models and the estimated TADR for the eruption?

Question 3d: Which of the three models do you prefer and why?

Bonus/extra credit points: Change any other two (or more) parameter values of your choice and explain how/why they modify flow behavior. Remember to write down E_r , V , distance reached, or any other parameter you deem important for me to understand your model.