Isostasy

- Refers to gravitational equilibrium
- Provides a physical rationale for the existence of mountains
- Based on force balance and buoyancy concepts

\[ P = \int_0^h \rho(h) g(h) dh \]

\( P \) = pressure (convention: compression is positive)
\( \rho \) = density
\( g \) = gravitational acceleration

For constant \( \rho \) and constant \( g \),
\( P = \rho gh \)

Isostasy

• Assumes a “compensation depth” at which pressures beneath two prisms are equal and the material beneath behaves like a static fluid, where $P_1 = P_2$
• Flexural strength of crust not considered
• Gravity measurements yield crustal thickness and density variations
• Complemented by seismic techniques

Isostasy
History

• Roots go back to da Vinci
• Term coined by Clarence Edward Dutton (USGS)
• Post-1800 interest triggered by surveying errors in India
• Two main models: Pratt, Airy
John Henry Pratt
(6/4/1809-12/28/1871)

- Pratt, J.H., 1855, On the attraction of the Himalaya Mountains, and of the elevated regions beyond them, upon the Plumb-line in India. Philosophical Transactions of the Royal Society of London, v. 145, p. 53-100.
- British clergyman and mathematician
- Archdeacon of India

Sir George Biddell Airy
(7/27/1801 - 1/2/1892)

- British Royal Astronomer from 1835-1881
- Determined the mean density of the Earth from pendulum experiments in mines
- Contributor to elasticity theory (telescope deformation)
- Opponent of Charles Babbage from 1842 to ??
Isostasy and Gravity Measurements

Expected deflection due to mtn mass

Mountain Mass

Region of inferred mass deficit

Vertical plumb bob

Observed deflection

Comparison of Isostatic Models

At isostatic equilibrium, $P = \text{constant}$ at depth of compensation, so no flow

Pratt Model

Airy Model (e.g., iceberg)

Depth of compensation = $h_2$

Depth of compensation = $h_2 = h_3$
Thermal Isostasy
(e.g., Turcotte and Schubert, 2002)

- Oceanic crust thickens and increases in density as it cools with time
- Oceanic crust thickens and increases in density with distance from ridge

Isostatic Rebound:
Lake Bonneville
Shorelines of Lake Bonneville
Tilt Away from Lake

20. Rheology & Linear Elasticity

I Main Topics
   A Rheology: Macroscopic deformation behavior
   B Linear elasticity for homogeneous isotropic materials
20. Rheology & Linear Elasticity

Viscous (fluid) Behavior

http://manoa.hawaii.edu/graduate/content/slide-lava

Ductile (plastic) Behavior

http://www.hilo.hawaii.edu/~csav/gallery/scientists/LavaHammerL.jpg
20. Rheology & Linear Elasticity

Elastic Behavior

http://www.earth.ox.ac.uk/__data/assets/image/0006/3021/seismic.hammer.jpg

https://thegeosphere.pbworks.com/w/page/24663884/Sumatra

Brittle Behavior (fracture)
20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
A Elasticity
1. Deformation is reversible when load is removed
2. Stress ($\sigma$) is related to strain ($\varepsilon$)
3. Deformation is not time dependent if load is constant
4. Examples: Seismic (acoustic) waves, rubber ball

http://www.fordogtrainers.com

Elastic

Shear stress

Linear

Slope = Young's Modulus

Shear strain

http://www.fordogtrainers.com
20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
B Viscosity
   1 Deformation is irreversible when load is removed
   2 Stress ($\sigma$) is related to strain rate ($\varepsilon$)
   3 Deformation is time dependent if load is constant
   4 Examples: Lava flows, corn syrup

http://wholefoodrecipes.net

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20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
C Plasticity
   1 No deformation until yield strength is locally exceeded; then irreversible deformation occurs under a constant load
   2 Deformation can increase with time under a constant load
   3 Examples: plastics, soils

http://www.therapputty.com/images/stretch6.jpg

II Rheology: Macroscopic deformation behavior
C Brittle Deformation
   1 Discontinuous deformation
   2 Failure surfaces separate

http://www.thefeeherytheory.com
20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
D Elasto-plastic rheology

E Visco-plastic rheology
20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
F Power-law creep
1 $\dot{\varepsilon} = (\sigma_1 - \sigma_3)^n e^{(-Q/RT)}$
2 Example: rock salt

Power-law creep $\sigma \sim (\dot{\varepsilon})^n$

Shear stress

Shear strain rate

20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior
G Linear vs. nonlinear behavior

Power-law creep $\sigma \sim (\dot{\varepsilon})^n$

Shear stress

Shear strain rate
20. Rheology & Linear Elasticity

II Rheology: Macroscopic deformation behavior

H Rheology = f(\(\sigma_{ij}\), fluid pressure, strain rate, chemistry, temperature)

I Rheologic equation of real rocks = ?

III Linear elasticity

A Force and displacement of a spring (from Hooke, 1676): F = kx

1. F = force
2. k = spring constant
   Dimensions: F/L
3. x = displacement
   Dimensions: length L)
20. Rheology & Linear Elasticity

III Linear elasticity (cont.)

B Hooke’s Law for uniaxial stress: \( \sigma = E \varepsilon \)

1. \( \sigma \) = uniaxial stress
2. \( E \) = Young’s modulus
   Dimensions: stress
3. \( \varepsilon \) = strain
   Dimensionless

\[ \varepsilon = \frac{\Delta L}{L_0} \]

20. Rheology & Linear Elasticity

III Linear elasticity (cont.)

B Hooke’s Law for uniaxial stress (cont.): \( \varepsilon = \frac{\sigma_1}{E} \)

1. \( \sigma_2 = \sigma_3 = 0 \)
2. \( \varepsilon_2 = \varepsilon_3 = -\nu \varepsilon_1 \)
   - \( \nu \) = Poisson’s ratio
   - \( \nu \) is dimensionless
   - Strain in one direction tends to induce strain in another direction

\[ \varepsilon = \frac{\Delta L}{L_0} \]
20. Rheology & Linear Elasticity

III Linear elasticity (cont.)

C Linear elasticity in 3D for homogeneous isotropic materials

By superposition:

1. \( \varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \left( \frac{\sigma_{yy} + \sigma_{zz}}{E} \right)(v/E) \)
2. \( \varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \left( \frac{\sigma_{zz} + \sigma_{xx}}{E} \right)(v/E) \)
3. \( \varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \left( \frac{\sigma_{xx} + \sigma_{yy}}{E} \right)(v/E) \)
20. Rheology & Linear Elasticity

III Linear elasticity

E Special cases

1 Isotropic (hydrostatic) stress
   a $\sigma_1 = \sigma_2 = \sigma_3$
   b No shear stress

2 Uniaxial strain
   a $\varepsilon_{xx} = \varepsilon_1 \neq 0$
   b $\varepsilon_{yy} = \varepsilon_{zz} = 0$

D Relationships among different elastic moduli

1 $G = \mu = \text{shear modulus}$
   $G = E/(2[1+v])$
   $\varepsilon_{xy} = \sigma_{xy}/2G$

2 $\lambda = \text{Lame' constant}$
   $\lambda = E(1 + v)/(1 + 2v)$

3 $K = \text{bulk modulus}$
   $K = E/(3[1 - 2v])$

4 $\beta = \text{compressibility}$
   $\beta = 1/K$
   $\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = -p/K$
   $p = \text{pressure}$

5 P-wave speed: $V_p$
   $V_p = \sqrt{\left( \frac{K + \frac{4}{3} \mu}{\rho} \right)}$

6 S-wave speed: $V_s$
   $V_s = \sqrt{\frac{\mu}{\rho}}$
Strike-view Cross Sections

• Prepared by projecting features along strike onto a cross section plane, where the cross section plane is perpendicular to strike
• Shows the true inclination and thickness of features
• Lines of strike lie in geologic planes and connect points of equal elevation
Fault Mechanics: Vertical Strike-slip Faults

Assume one principal stress is vertical, two principal stresses are horizontal.

The x-axis is parallel to fault strike.

The y-axis is normal to fault strike.

The z-axis points down.

If $\sigma_y > 0$, left-lateral faulting.

If $\sigma_y = 0$, no faulting.

If $\sigma_y < 0$, right-lateral faulting.

Draw the arrows showing how the faults would slip and determine whether the slip is right- or left-lateral.

Fault Mechanics: Dip-slip Faults

Assume one principal stress is vertical, two principal stresses are horizontal, and the horizontal principal stresses are parallel and normal to fault strike.

The x-axis is parallel to fault strike (at you).

The y-axis is normal to the fault.

The z-axis points down-dip.

If $\sigma_y > 0$, normal faulting.

If $\sigma_y < 0$, reverse faulting.

Draw the arrows showing how the faults would slip and determine whether the slip is normal or reverse.