CHARACTERIZING EARTHQUAKE SOURCES (13)

I Main Topics
   A Elastic rebound theory
   B Slip on a fault with a uniform stress drop
   C Seismic moment
   D Energy release during an earthquake

II Elastic rebound theory (H.F. Reid, 1908, v. 2 of 1906 Earthquake report)
   A Founded by comparing pre- and post-quake survey lines across SAF
   B Seismic energy source: elastic potential energy of rock around fault

GEODETIC OBSERVATIONS SUPPORTING REID'S ELASTIC REBOUND THEORY

1 6m slip on fault in 1906
2 Straight-line survey
   in 1851-1865
3 ~3m far-field relative displacement
   in ~50 years from 1851/1865 - 1906. This is half the 1906 slip.
4 ~2m far-field relative displacement
   from 1874/1892 - 1906.
5 3m of slip probably accumulated before 1851/1865.
6 Last quake 100 (= 50+50) years ago?
III Slip ($\Delta u$) on a fault with a uniform shear stress drop ($\Delta \tau = \tau_1 - \tau_2$)

A Rock is elastic, homogenous, isotropic, isothermal material
B Shear stress on fault prior to slip = $\tau_1$; post-slip shear stress = $\tau_2$
C Slip profile is related to the shape and size of the rupture
D Slip distribution is particularly sensitive to the short dimension
E For a "2-D" rupture (one dimension $>>$ other dimension = $2a$)
1. \[ \Delta u = 2(1-\nu) \left( \frac{\Delta \tau}{\mu} \right) (a^2 - x^2)^{1/2} \]
\[ \nu = \text{Poisson's ratio} \]

2. \[ \Delta u_{\text{max}} = \Delta u(x = 0) = 2(1-\nu) \left( \frac{\Delta \tau}{\mu} \right) a = 3 \times 10^{-4} a \]

3. \[ \Delta u_{\text{min}} = \Delta u(x = \pm a) = 0 \]

4. \[ \Delta u_{\text{ave}} = \frac{\int \Delta u \, dx}{2a} = \frac{\pi}{4} (\Delta u_{\text{max}}) \]

IV. Seismic moment \( M_o \)

A. \( M_o = \mu \Delta u_{\text{ave}} A \)
\( \mu = \text{shear modulus; } A = \text{rupture area} \)

B. \( M_o \) has dimensions of energy; measures deformation

C. A larger area of rupture or a larger slip \( \Rightarrow \) larger earthquake

D. \( \mu \) has been measured; geologists can estimate \( \Delta u_{\text{ave}} \) and \( A \)

E. Seismic moments ("earthquake size") \( M_o \) can be predicted

V. Radiated seismic kinetic energy \( E_s \) (From Scholz, 1990)

\[ E_s + \Delta E_{\text{strain}} + \Delta E_{\text{friction}} + \Delta E_{\text{fracture surface}} + \Delta E_{\text{chemical}} = 0 \]

A. Kinetic energy (\( E_s \)) in seismic waves from a dynamics viewpoint

1. \( E_s \) varies with amplitude and \( L \) wavelength

2. Waves of a frequency of zero ("ultra-long wavelength") correspond to a static situation ("permanent" deformation).

3. Seismic moment, which describes the "permanent" deformation after an earthquake, should be related to seismic energy release.

4. Empirical relationship of seismic energy (\( E_s \)) to magnitude (\( M_w \)):
\[ E_s \text{ (joules)} = 10^{(4.8 + 1.5 M_w)} \]

B. Strain energy (\( \Delta E_{\text{strain}} \))

1. Energy in a linear spring
   a. Force in a spring (\( F \)): \( F = kx \); \( k = \text{spring constant}, x = \text{displacement} \)
   b. Strain energy (\( \Delta E_{\text{strain}} \)) equals area under a force-disp. curve:
\[ \Delta E_{\text{strain}} = \int_0^x F \, dx = \int_0^x kx \, dx = \left[ \frac{1}{2} kx^2 \right]_0^x = \frac{1}{2} kx^2 \]
\[ \Delta E_{\text{strain}} = \text{("Average force on spring") (total displacement of spring)} \]
2 Energy of deformation ($\Delta E_{\text{strain}}$) in an earthquake (Method of Reid)
   a The change in strain energy = work of faulting
   b Consider energy needed to restore rock to pre-quake conditions
   c $\Delta E_{\text{strain}} = (1/2)(\text{Peak shear stress})(\text{Area of rupture})(\text{slip})$
     (If the shear stress after slip occurred is zero, the relevant average stress $\tau$ is half the maximum stress. This is where the factor of 1/2 comes from.)
   d $\Delta E_{\text{strain}} = \tau_A \Delta u_{\text{ave}}$
     Note the similarity between this and $M_o$ (eq. IV.A)
   e $\tau = (\text{Shear strain})(\text{Shear modulus of rock})$
   f Example: San Francisco, 1906
     $\tau = (1/1500)(2 \times 10^{10} \text{ J/m}^2) = 1.33 \times 10^7 \text{ J/m}^2$
     $\Delta E_{\text{strain}} = (10^7 \text{ N/m}^2)(20 \times 10^3 \text{ m})(435 \times 10^3 \text{ m})(1/2 \times 4 \text{ m})$
     $\approx 10^{17} \text{ J}$
     For comparison, $\Delta E_{\text{Bikini}}, 1946 = 10^{12} \text{ J}$

C Heat due to friction ($\Delta E_{\text{friction}}$)
   1 $\Delta E_{\text{friction}} = ([\tau_{\text{friction}}][A][\Delta u_{\text{ave}}])$
   2 Assuming $\tau_{\text{friction}} = \tau_2$, $\Delta E_{\text{friction}} = [\tau_2][\Delta u_{\text{ave}}][A]$
D Energy in seismic waves ($E_s$) from a mechanics viewpoint
   1 $E_s + \Delta E_{\text{strain}} + \Delta E_{\text{friction}} + \Delta E_{\text{fracture surface}} = 0$

   The strain energy in the earth decreases after a quake, so $\Delta E_{\text{strain}} < 0$. Energy appears in the form of heat, so $\Delta E_{\text{friction}} > 0$.

   The energy to create fracture surfaces is assumed to be negligible.

   2 $E_s \approx -(\Delta E_{\text{strain}}) - (\Delta E_{\text{friction}})$

   3 $E_s \approx \frac{1}{2} [\tau_1 + \tau_2] [\Delta u_{\text{ave}}] [A] - \frac{1}{2} [\tau_2] [\Delta u_{\text{ave}}] [A]$??

   4 $E_s \approx \frac{1}{2} [\tau_1 - \tau_2] [\Delta u_{\text{ave}}] [A] = \frac{1}{2} [\Delta \tau] [\Delta u_{\text{ave}}] [A]$??

   5 So the seismic kinetic energy depends on the strength change on the fault $\Delta \tau$.

VI Formulas relating seismic energy release ($E_s$), moment ($M_o$), and moment magnitude ($M_w$)

   A) $M_w \approx \frac{2}{3} \log M_o - 6.067$
      $M_o$ in Nm

   B) $E_s \approx 10(4.8 + 1.5 M_s)$
      $E_s$ in joules

   C) $E_s \approx M_o / 20,000$
      $E_s$ in joules, $M_o$ in Nm

   D) $\log E_s \approx \log M_o - 4.3$
      $E_s$ in joules, $M_o$ in Nm

   E) $\log M_o \approx 1.5 M_w + 9.1$
      $M_o$ in Nm
Energy Release vs. Magnitude

The empirical relationship between energy content in radiated seismic waves and magnitude is (Richter, 1958; Bolt, 1989):

\[ E_s \text{ (joules)} = 10^{(4.8 + 1.5 M_s)}. \]

\( M_s \) is the surface wave magnitude.

Consider two earthquakes, where \( M_{s1} = 1 + M_{s2} \). Then

\[ \frac{E_{s1}}{E_{s2}} = \frac{10^{(4.8 + 1.5 [M_{s2} + 1])}}{10^{(4.8 + 1.5 M_{s2})}} \]

\[ = \frac{(10^{4.8})(10^{1.5 [M_{s2}]})}{(10^{4.8})(10^{1.5 M_{s2}})} \]

\[ = 10^{1.5} \approx 31.6 \]

A unit increase in magnitude corresponds to (a) a factor of 10 increase in amplitude of shaking, and (b) a factor of 31.6 increase in energy release. One magnitude 8 quake releases the energy of 1000 magnitude 6 quakes.

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Relationship between moment magnitude (\( M_w \)) and seismic moment (\( M_o \))

\[ M_w = \frac{2}{3} \log M_o - 6.067, \] where \( M_o \) is measured in Nm \hspace{1cm} p. 249 of Bolt

This empirical relation has been setup to dovetail with the surface wave magnitude (i.e. \( M_s = M_w \)). What is the relationship between \( M_o \) and \( E_s \)?

\[ E_s = 10^{(4.8 + 1.5 M_s)} \text{ where } E_s \text{ is in joules} \hspace{1cm} p. 179 of Scholz

\[ = 10^{(4.8 + 1.5 [2/3 \log M_o - 6.067])} \]

\[ = 10^{(4.8 + \log M_o - [1.5][6.067])} \]

\[ = 10^{(4.8 + \log M_o - 9.1)} \]

\[ = 10^{(\log M_o - 4.3)} \]

\[ = [10 \log M_o][10^{-4.3}] \]

\[ E_s = M_o / 20,000 \]

Alternatively, \[ E_s = [\Delta \sigma/2][\Delta u_{ave}][A] \]

\[ M_o = [\mu][\Delta u_{ave}][A] \]

\[ E_s/M_o = \{[\Delta \sigma/2\mu][\Delta u_{ave}][A]\} / \{[\mu][\Delta u_{ave}][A]\} = [\Delta \sigma/2\mu] \hspace{1cm} p. 179 of Scholz \]

Typically \( \Delta \sigma \approx 3 \text{ MPa}, \) and \( 2\mu = (2)(3 \times 10^4 \text{ MPa}), \) so \[ E_s/M_o = 1/20,000 \]