DISLOCATIONS

I Main Topics
A Dislocations and other defects in solids
B Significance of dislocations
C Planar dislocations
D Displacement and stress fields for a screw dislocation (mode III)

II Dislocations and other defects in solids
A Dislocations
  1 Originally, extra (or missing) planes or partial planes of material
     (e.g., atoms)
  2 Surfaces across which displacements are discontinuous
  3 Evidence for dislocations from electron microscopy
B Point defects
  1 Originally, extra (or missing) volumes (e.g., atoms)
  2 Displacements are discontinuous across point defects

III Significance of dislocations
A They account for permanent plastic deformation in crystals
B They account for the low observed strength of crystals relative to
   theoretical predictions
B They provide useful quantitative description of relative motions
   across surfaces across a broad range of scale (crystals [10^{-6} m] to
   plate boundaries [10^{6} m]) – ~12 orders of magnitude!
C They induce tremendous stress concentrations and account for large
   deformations under small “average” stresses
IV Planar dislocations
A Represented mathematically as infinitely long cut with a straight edge
B **Relative** displacement (of one side of the dislocation relative to the other) across a dislocation is called the Burger's vector $b$.
C Screw dislocation
   1 Accommodate a tearing motion
   2 Displacement is exclusively parallel to the dislocation edge
   3 Analogy: a lock washer or a 360° spiral staircase
   4 Macroscopic geologic use: to model faults
D Edge dislocation
   1 Accommodate opening or sliding motions
   2 Displacement is exclusively perpendicular to the dislocation edge
   3 Displacement can be parallel or perpendicular to the dislocation plane
   4 Analogy: an extra row of corn kernels on a cob of corn
   5 Macroscopic geologic use: to model dikes or faults

V Displacement and stress fields for a screw dislocation (mode III)
A Displacement parallel to the dislocation edge increases uniformly along a spiral-like circuit from one side of the dislocation to the other (for a right-handed screw dislocation, point your right thumb along the dislocation edge; displacement parallel to the edge increases in the direction your fingers curl.
B Angular position: $\theta = \tan^{-1}(y/x)$
C Expressions for displacements and strains
   1 Cartesian displacements:  
     \[ u = u_x, \quad v = u_y, \quad w = u_z \]
   2 Normal strains:  
     \[ \varepsilon_{xx} = \frac{1}{2} \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial x} \right), \quad \varepsilon_{yy} = \frac{1}{2} \left( \frac{\partial v}{\partial y} + \frac{\partial v}{\partial y} \right), \quad \varepsilon_{zz} = \frac{1}{2} \left( \frac{\partial w}{\partial z} + \frac{\partial w}{\partial z} \right) \]
   3 Shear strains:  
     \[ \varepsilon_{xy} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right), \quad \varepsilon_{yx} = \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right), \quad \varepsilon_{zx} = \frac{1}{2} \left( \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} \right) \]
   4 Cylindrical displacements:  
     \[ u_r, \quad u_\theta, \quad u_z = w \]
   5 Normal strains:  
     \[ \varepsilon_{rr} = \frac{1}{2} \left( \frac{\partial u}{\partial r} + \frac{\partial u}{\partial r} \right), \quad \varepsilon_{\theta \theta} = \frac{1}{r} \left( \frac{\partial u}{\partial \theta} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right), \quad \varepsilon_{zz} = \frac{1}{2} \left( \frac{\partial w}{\partial z} + \frac{\partial w}{\partial z} \right) \]
   6 Shear strains:  
     \[ \varepsilon_{r \theta} = \frac{1}{2} \left( \frac{1}{r} \frac{\partial u}{\partial \theta} + \frac{\partial u}{\partial r} - \frac{u_\theta}{r} \right), \quad \varepsilon_{\theta r} = \frac{1}{2} \left( \frac{\partial u}{\partial \theta} + \frac{1}{r} \frac{\partial u}{\partial \theta} \right), \quad \varepsilon_{r z} = \frac{1}{2} \left( \frac{\partial u}{\partial r} + \frac{\partial u}{\partial r} \right) \]
Polar coordinates

\( u_r = 0 \)

\( u_\theta = 0 \)

\( w = b \frac{\theta}{2\pi} \)

Cartesian coordinates

\( u = 0 \)

\( v = 0 \)

\( w = b \frac{\tan^{-1} y}{x} \)

2 Strain

Polar coordinates

\( \varepsilon_{r\theta} = \varepsilon_{\theta r} = 0 \)

\( \varepsilon_{r\theta} = \varepsilon_{\theta r} = \frac{b}{2\pi r} \)

\( u_{rz} = u_{zr} = 0 \)

\( \varepsilon_{rr} = 0 \)

\( \varepsilon_{\theta\theta} = 0 \)

\( \varepsilon_{zz} = 0 \)

Cartesian coordinates

\( \varepsilon_{xy} = \varepsilon_{yx} = 0 \)

\( \varepsilon_{yz} = \varepsilon_{zy} = \frac{b}{2\pi (x^2 + y^2)} = \frac{b}{2\pi r^2} \)

\( \varepsilon_{xz} = \varepsilon_{zx} = -\frac{b}{2\pi (x^2 + y^2)} = \frac{-b y}{2\pi r^2} \)

\( \varepsilon_{yy} = 0 \)

\( \varepsilon_{yy} = 0 \)

\( \varepsilon_{zz} = 0 \)

3 Stress (\( G = \text{shear modulus} \))

\( \sigma_{r\theta} = \sigma_{\theta r} = 0 \)

\( \sigma_{r\theta} = \sigma_{\theta r} = \frac{Gb}{2\pi r} \)

\( \sigma_{rr} = \sigma_{\theta\theta} = 0 \)

\( \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = 0 \)

\( \sigma_{yy} = \sigma_{yy} = \sigma_{zz} = 0 \)

4 Key points

a) Only the shear stresses acting on or in the z direction are non-zero

b) The stresses are singular (i.e., go to infinity) near the dislocation end: a powerful stress concentration exists there.

c) This theoretical singular stress concentration exists no matter how small the relative displacement \( b \) is.
SUPERPOSITION OF TWO (INFINITE) SCREW DISLOCATIONS (A,B) TO FORM A FINITE DISPLACEMENT DISCONTINUITY (C)
(View along the -z direction)

\[ w_A = \frac{b\theta_A}{2\pi} = \frac{b}{2\pi} \tan^{-1}\left(\frac{y'}{x'}\right) = \frac{b}{2\pi} \tan^{-1}\left(\frac{y}{x-a}\right) \]

\[ w_B = \frac{b\theta_B}{2\pi} = \frac{b}{2\pi} \tan^{-1}\left(\frac{y''}{x''}\right) = \frac{b}{2\pi} \tan^{-1}\left(\frac{y}{x+a}\right) \]

\[ w_C = \frac{b(\theta_A - \theta_B)}{2\pi} = \frac{b}{2\pi} \left[\tan^{-1}\left(\frac{y}{x-a}\right) - \tan^{-1}\left(\frac{y}{x+a}\right)\right] \]

\[ w_C (\theta_A = -\pi, \theta_B = 0) = -\frac{B}{2} \]
\[ w_C (\theta_A = 0, \theta_B = 0) = 0 \]
\[ w_C (\theta_A = \pi, \theta_B = 0) = \frac{B}{2} \]
\[ w_C (\theta_A = \pi, \theta_B = \pi) = 0 \]