Stresses

Exercise 1
1a Write down the expressions for all the stress components in the $x'y'$ coordinate system in terms of the stress components in the $xy$ reference frame. The expression for $\sigma_{x'x'}$ is given as a guide.

$$\sigma_{x'x'} = a_{x'x'}a_{x'x'}\sigma_{xx} + a_{x'x'}a_{xy'}\sigma_{xy} + a_{x'x'}a_{y'x'}\sigma_{yx} + a_{x'x'}a_{y'y'}\sigma_{yy}$$

$$\sigma_{x'y'} =$$  

$$\sigma_{y'x'} =$$  

$$\sigma_{y'y'} =$$  

1b Using the pictures below, describe in words what terms $a_{x'x'}, a_{x'y'}, a_{y'x'}, a_{y'y'}$ mean.  

1c On the boxes below draw positive normal stresses and positive shear stresses using the tensor ("on-in") convention for positive shear stresses.  

(30 pts total this page)
Exercise 2: Suppose $\sigma_{xx} = -1\text{MPa}$, $\sigma_{xy} = 8\text{MPa}$, $\sigma_{yx} = 8\text{MPa}$, $\sigma_{yy} = -31\text{MPa}$.

2a On a separate piece of paper, draw a square box about 2" on a side (This is about the size of the square on the previous page) showing the stresses with arrows acting on the sides of the square. Show arrows in the positive direction, and label them with the correct magnitudes, which can be either positive or negative. Put a north arrow on your box parallel to the y-axis. Make sure the box will be in equilibrium.  

2b On the same piece of paper, draw a new square box about 2" on a side showing the tractions $(t_{xn}, t_{xs}, t_{yn},$ and $t_{ys})$ with arrows acting on the sides of the square. Remember that the n- and s-directions for a traction are normal and parallel, respectively, to the side of the box the traction acts on. For each side of the box, show arrows in the positive n- and s-directions, and label them with the correct magnitudes, which can be either positive or negative. Put a north arrow on your box parallel to the y-axis.

2c Inside the box draw a line representing a vertical fault that strikes N70°E.

2d Guess whether this fault will tend to slip left-laterally or right-laterally.

2e Plot a Mohr circle describing this state of stress, labeling the point on the Mohr circle corresponding to the TRACTION COMPONENTS (see 2b) that act on the face that has a normal along the x-axis (i.e., $t_{xn}, t_{xs}$) and the point on the Mohr circle corresponding to the TRACTION COMPONENTS that act on the face that has a normal along the y-axis (i.e., $t_{yn}, t_{ys}$).

2f Using the Mohr circle, find the magnitude of $t_1$, the most tensile (i.e., most positive) traction, and $t_2$, the least tensile (i.e., most negative) traction.

2g Mark on the Mohr circle the points corresponding to $t_1$ and $t_2$.

2h Re-label the point marked “$t_1$” as “$t_{x'n'}, t_{x's'}$” while retaining the original “$t_1$” label. Re-label the point marked “$t_2$” as “$t_{y'n'}, t_{y's'}$” while retaining the original “$t_2$” label. Use the Mohr circle to find the negative double angles between the x-axis and the x'-axis, and between the x-axis and the y'-axis, and label them $-2\theta_{xx'}$ and $-2\theta_{xy'}$, respectively.

2i On a new sheet of paper draw new x, x', y and y' axes in their correct orientation. Then draw a 2" square with sides normal to the x' and y' axes. Then show the principal stresses $\sigma_{x'x'}$ and $\sigma_{y'y'}$ acting on the sides of the square.

2j Using the angle of (2h) and the formulas on the first page, calculate $\sigma_{x'x'}, \sigma_{y'y'}, \sigma_{y'x'}$, and $\sigma_{y'y'}$ to check your answer of (2f) – are they consistent?  

2k Inside this box draw a line representing a vertical fault that strikes N70°E.

2l Make a guess as to whether this fault will tend to slip left-laterally or right laterally. Is this guess the same as your first guess?  

2m On your the box of (2i) draw an x" axis parallel to the fault and a y" axis perpendicular to the fault; make sure they are right-handed. This will give a total of 3 sets of axes, so be neat.

2n Measure the angle between the x-axis and the y" axis and label it $+\theta_{xy''}$.

2o Using the corresponding negative double angle, plot on the Mohr circle the point representing $(t_{y''n''}, t_{y''s''})$. Label the negative double angle $-2\theta_{xy''}$. Will the fault slip left-laterally or right laterally? How does this compare with (2d) and (2l)?  

2p Use the procedure of (2j) to calculate $\sigma_{y''y''}$ and $\sigma_{x''x''}$ to check your results.

2q Use the Matlab command “[V,D] = eig” and the stress components $\sigma_{xx} = -1\text{MPa}$, $\sigma_{xy} = 8\text{MPa}$, $\sigma_{yx} = 8\text{MPa}$, $\sigma_{yy} = -31\text{MPa}$ to find the principal stresses to check your results (2). Draw a picture showing the principal stresses acting on the sides of a square (2) and the two sets of direction cosines defining the eigenvectors that give the orientation of the two principal stresses (2+2). Include a printout of your Matlab results.

(70 pts total this page)