Internal Tide Generation at Open Ocean Topographies

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Abstract. The generation of internal tides is considered when semi-diurnal barotropic currents encounter three-dimensional, open ocean topographies characteristic of the Hawaiian Ridge. A fully three dimensional, free surface, nonlinear, hydrostatic model (the Princeton Ocean Model) is used to simulate the internal tide generation. Two types of bathymetry are used: idealized Gaussian-shaped seamounts and islands which provide a simplified view of the generation at isolated topographic features, and realistic bathymetric data for the Hawaiian Island region. The generated wave signal for the Gaussian topographies is characterized by tidal beams which emanate from the flanks near the top of the feature. Vertical motions induced by the sloping topography lead to the generation. The strength of this forcing varies considerably depending upon the manner in which the barotropic tide interacts with the three dimensional topography. Preliminary results using the realistic topography show regions of enhanced generation, particularly in the shallow channels between the islands.

Introduction

Recent satellite altimetry [Ray and Mitchum, 1996]) and acoustic transmission data [Dushaw et al., 1995] analyses indicate that semidiurnal band internal tides are generated at the Hawaiian Ridge. Ray and Mitchum detect outward propagating waves with mode 1 length scales within 1000 km of the ridge axis. Given that the ridge is not a uniform feature but a chain of largely isolated seamounts and islands (Figure 1), it is plausible that the internal tide signal is a superposition of waves originating from various source topographies rather than a plane wave emanating from a contiguous ridge.

Given this 3-dimensional topography, the question of how the internal waves are generated remains an open issue. For the Hawaiian Ridge, there are several candidates for transferring energy from the barotropic tidal flow to internal tides: the advection of stratified water over sloping bathymetry, tidal flow amplification through the channels causing lee disturbances, and flow convergences around the islands. The former is the primary mechanism for the generation of internal tides at continental slopes [Baines, 1982], and is of particular interest for the Hawaiian Ridge because steep topographic slopes are near-critical for M2 frequency internal waves at many depth ranges (Figure 1).

In this paper we summarize numerical results obtained using the Princeton Ocean Model (POM) which support the notion that barotropic tidal flow over sloping topography is the primary generation mechanism for the semidiurnal internal tide at the Hawaiian Ridge. These and other POM results are discussed in more detail in Holloway and Merrifield [1999]. We also present new POM results using realistic Hawaiian Island topography.

Model description

A fully three dimensional, nonlinear, free surface, hydrostatic, sigma-coordinate, primitive equation model, incorporating a Mellor Yamada level 2.5 turbulence closure scheme (the Princeton Ocean Model, Blumberg and Mellor [1987] is used in this study). The model is forced at the horizontal boundaries following Flather [1987] which yields a deep water barotropic tidal current in the east-west direction of approximately 1 cm s⁻¹ peak amplitude and of 12.42 hour period. The model runs
are over 4 days, with the forcing gradually increased over a 24 hour ramp period, and the last 24 hours used for the output tidal analysis.

The Gaussian-shaped topography is specified by

\[
h(x, y) = H - h_o \exp\left(-\left[\frac{(x-x_o)^2}{a^2} + \frac{(y-y_o)^2}{b^2}\right]\right)
\]

(1)

where \( H \) is the deep water depth, \( h_o \) is the height of the topographic feature above the sea bed, \( h(x, y) \) is the water depth and \( (x_o, y_o) \) is set at the center of the model domain. The values of \( a \) and \( b \) define the width of the feature. If \( a=b \) the feature is referred to as a seamount, or if \( h_o > H \) this becomes an island. When \( b > a \), the feature becomes a ridge, orientated across the deep water tidal velocity which is in the x-direction. In each run \( H=4500 \) m, the domain is 600 km in the east-west (x) and north-south (y) directions with a grid spacing of 2 km over the topographic feature (an area 100 by 100 km) which is stretched to 5 km in the deep water and to 10 km near each open boundary. In all runs, 81 vertical levels are used, evenly spaced except for the bottom 5 points that are logarithmically spaced to gain better resolution near the seabed.

The realistic topography is specified by the Smith and Sandwell [1997] bathymetry database with a uni-

Figure 1. Regions where the topographic slope, in the direction parallel to the deep ocean \( M_2 \) tidal current, is within 2° of the \( M_2 \) internal wave characteristics as determined from mean stratification data from Levitus. Regions that are located above/below 200 m depth are indicated in blue/red.

Figure 2. The average stratification profile for the Hawaii region used in the model simulations.

form horizontal grid spacing of 4 km and 61 vertical levels. The semi-diurnal tidal forcing is chosen to simulate the amplitude of the realistic deep ocean tidal flow and its near-normal direction relative to the ridge axis, however, realistic tidal forcing based on a larger-scale model has not been used in these simulations. Such a model run is currently being implemented and will be the subject of a later study.

Stratification for all runs is defined from annual average temperatures and salinities measured in the Hawaiin region (Figure 2). A latitude of 20° N is used to define the Coriolis parameter for all runs.

For each model run, time series of horizontal and vertical (mapped from the sigma to z coordinate) velocities and surface elevations are analyzed. The barotropic velocity is defined as the depth-averaged flow; the baroclinic horizontal velocity is the total velocity minus the barotropic component. These time series are harmonically analyzed Foreman [1978] to calculate \( M_2 \) amplitudes and phases of barotropic and baroclinic tidal velocities and vertical displacements. Energy fluxes are then calculated from the tidal values as described by Holloway [1996] and Holloway and Merrifield [1999].

Simplified Topography Runs

We begin with a simple Gaussian-shaped seamount to illustrate the structure of the generated internal tide and how changes to this topography affect the am-
amplitude of the wave signal. We select a feature submerged 200 m below the surface with width scales of $a=b=23$ km. These values were chosen using a seamount which by inspection seemed to typify the seamounts just north of the main Hawaiian Islands. After 4 days simulation, a cross section of the instantaneous east-west flow field is presented from the center of the domain (top of the topography) looking eastward toward the model boundary (Figure 3a). A dominant tidal beam emanates from the topography with reflections off the sea surface (near 350 km and 500 km) and seabed (425 km). High wave number spatial variability occurs within the first surface-bounce of the main beam. Changing the depth of the seamount to 50 m and 500 m below the surface does not significantly alter current variances from the 200 m run (within 10%).

![Figure 4. Depth integrated M2 energy fluxes plotted against distance eastward from the top of the topographic feature ($x=y=300$ km) for the 200 m deep seamount and ridge, and the symmetric ($a=b=23$ km) and long ($a=23$ km, $b=69$ km) island model topographies.](image)

**Figure 4.** Depth integrated M2 energy fluxes plotted against distance eastward from the top of the topographic feature ($x=y=300$ km) for the 200 m deep seamount and ridge, and the symmetric ($a=b=23$ km) and long ($a=23$ km, $b=69$ km) island model topographies.

![Figure 3. East-west sections (along $y=300$ km) of the u-component of the total velocity after 4 days simulation for (a) a symmetric seamount ($a=b=23$ km), and (b) an elongated ridge ($a=23$ km, $b=69$ km). Both topographies are submerged 200 m below the surface.](image)

**Figure 3.** East-west sections (along $y=300$ km) of the u-component of the total velocity after 4 days simulation for (a) a symmetric seamount ($a=b=23$ km), and (b) an elongated ridge ($a=23$ km, $b=69$ km). Both topographies are submerged 200 m below the surface.

The internal tide signal does change significantly when we elongate the seamount in the north-south direction. We hold the east-west scale of the feature fixed at $a=23$ km and alter by a factor of 3 ($b=69$ km) the north-south scale creating an asymmetric shape which we call a ridge (Figure 3b). Using the same barotropic current forcing as in the seamount run, the resulting tidal beams are much more energetic for the ridge than the seamount. Again, varying the depth of the feature (50, 500 and 1000 m) results in order 10% changes in current variance.

The contrast between the internal tide amplitude generated from the ridge and seamount runs is readily apparent in the eastward depth-integrated energy flux computed along the center axis of the features (Figure 4a). The internal tide from the ridge topography has nearly an order of magnitude higher peak energy flux, 1000 W m$^{-1}$ compared to 150 W m$^{-1}$ than the symmetric seamount.

Similar model runs are made for surface-piercing topographies, or islands, with symmetric ($a=b=23$ km, $h_0=5000$ m) and asymmetric ($a=23$ km, $b=69$ km, $h_0=500$ m) shapes. Unlike the seamount model tests, the barotropic current now cannot flow over the top of the feature. Again, the elongated shape or long island is a much more efficient internal tide generator than the symmetric shape (Figure 4b) and the energy fluxes are comparable, although weaker, than the submerged test cases. As with the submerged topography runs, the internal tide is characterized by a dominant beam originating at the shallower depths of the island flanks.

As discussed in Holloway and Merrifield [1999], the differences in these model runs is attributed to the manner in which the topography perturbs the barotropic tidal flow. To illustrate this point, we examine the forcing term for baroclinic motions caused by oscillatory barotropic flow over a sloping bottom as described by Baines [1982]
\[ F(x, z) = \frac{N^2(z)w_1(x, z)}{\omega} \]  

(2)

where \( N \) is the buoyancy frequency, \( w_1 \) is the vertical velocity that results from the bottom boundary condition of no flow through the sloping boundary, and \( \omega \) is the frequency of oscillation. Here, \( F \) is given as the peak amplitude of the forcing term over a semi-diurnal cycle. In terms of the peak barotropic mass flux, \( Q \), \( w_1 \) can be expressed as

\[ w_1(x, z) = -Qz(h^{-1})_z \]  

(3)

where \( h \) is bottom depth and the subscript denotes a partial derivative with respect to \( x \). For the model topographies, \( y \) derivatives are zero at the center line of the feature \( (y=300 \text{ km}) \). The specification of \( Q \) is often made on a regional basis under the assumption that the barotropic length scales are much longer than the topographic scales. In such circumstances, \( F \) is specified from the bottom slope and stratification in addition to the large-scale \( Q \).

For the seamount and ridge topography runs, \( Q \) (in the open ocean away from the topography), \( N \), and \( h \) along \( y=300 \text{ km} \) are identical. Yet, \( F \) is an order of magnitude larger for the ridge than the seamount (Figure 5). This seemingly contradictory result is due to the variation of \( Q \) near the topography. The ridge shape is apparently more effective than the symmetric seamount in causing the barotropic flow to cross isobaths, thus leading to vertical velocities which generate the internal tide. In Figure 6, the barotropic flow is seen to flow relatively undisturbed around the seamount compared to the more dramatic velocity changes induced by the ridge. The same holds true for the island runs. Thus, the details of the barotropic flow field at the topography are essential for determining the baroclinic response.

**Realistic Topography Run**

Internal tide generation over realistic topography is investigated for the Hawaiian Islands at the southern end of the chain. The islands are separated by shallow channels (100-500 m) in the center of the group. The islands of Hawaii and Kauai are separated from the central island cluster by deeper passages (1000 m). We force a semi-diurnal tidal current oriented perpendicular to the island chain (along the \( y \) axis in Figure 7) with a deep ocean speed of approximately 1 \( \text{cm s}^{-1} \).

We measure the strength of the generated internal tide by the depth integrated energy flux (Figure 7).

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Figure 5. The peak forcing term, \( F_r \), in equation 2, for the 200-m deep seamount and ridge. Units are \( \text{cm s}^{-2} \).

The strongest generation occurs in the Kauai Channel (200-300 km distance on the \( x \)-axis) between the islands of Kauai and Oahu. The strongest energy flux vectors originate from the shallow ridge adjacent to Oahu. Peak amplitudes reach nearly 5000 \( \text{W m}^{-1} \) near the ridge and fall off below 1000 \( \text{W m}^{-1} \) after approximately 100 km distance. Weaker internal tides originate from the Kaiwi Channel between Oahu and Molokai \( (x=350 \text{ km}) \) and in the Alenuihaha Channel between Maui and Hawaii \( (x=550 \text{ km}) \). Strong flux amplitudes but with variable direction occur along the flanks of the island of Hawaii.

This simulation demonstrates that appreciable semi-diurnal internal tide energy can be generated in the island region and that the forcing is highly variable in space. A more realistic simulation will require better boundary conditions for the barotropic forcing based on a regional tide model.

One consideration is whether the internal tide generation for the realistic bathymetry occurs in the same manner as the idealized topography runs, namely the advection of stratified water over sloping bathymetry. The occurrence of large baroclinic energy fluxes in the channels raises the possibility of amplified tidal flows in the constricted passages with associated lee waves. To estimate this effect, we consider the Froude number, \( F_r = u/c \) with \( u \) the barotropic tidal flow speed and \( c \) the internal wave phase speed. In the model simulation, \( u \) is approximately 10 \( \text{cm s}^{-1} \) through the Kauai Channel, an order of magnitude larger than the open water speed. Assuming a water depth in the channel of 100 m and a
constant stratification of 0.007 s$^{-1}$, $F_r$ is approximately 0.4. A value in this range may be conducive for weak lee wave generation [e.g., Maxworthy, 1979], although the wave amplitude will be sensitive to the details of the stratification and water depth. Further study using realistic tidal forcing is needed to determine the extent to which lee waves contribute to the generated signal in this region.

Summary

The model simulations suggest that tidal flow past a single seamount or island in the Hawaiian Ridge can generate an appreciable internal tide that propagates away to the open ocean. The interaction of the barotropic tide with the three-dimensional topography strongly affects the internal tide amplitude. Using the open ocean current with the local topographic gradient, a method that has been used for more two-dimensional continental slope topographies, can give misleading results when applied to open ocean features. For example, elongating a symmetric seamount to a horizontal aspect ratio of 3:1 increases the energy flux of the generated internal tide by an order of magnitude. The perturbation of the barotropic tidal flow in the near-field of the topography accounts for this difference.

Given the importance of determining the local vertical velocity, two limiting factors for modeling the internal tide at open ocean topographies are the accuracies of the bathymetric data and the regional barotropic tidal models. Recent satellite observations have produced significant improvements in both areas [Smith and Sandwell, 1997; Egbert, 1997]. Preliminary model simulations of the Hawaiian Islands using realistic topography with an idealized semi-diurnal tidal forcing indicate internal tide generation occurring preferentially in the major channels between the islands. The possibility of amplified channel currents leading to weak lee wave effects requires further study.

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References

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Figure 6. Plan views of the instantaneous barotropic (depth averaged) velocity vectors after 3 days simulation for (a) the 200-m submerged ridge, (b) the 200-m seamount, and (c) the 200-m submerged ridge with the Coriolis parameter set to zero. Depth contours in meters are also shown.


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