Internal Waves on the Continental Shelf

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Abstract. The Garrett-Munk spectral description of the deep-ocean internal wave field is re-scaled for straightforward application to the shallower water of the continental slope and shelf. Two new parameters are introduced to replace some of the GM parameters: spectral level $E_0$ and vertical waveguide scale $D$. The new scaling is applied to some observed spectra, as a first step toward a climatology of internal waves on the shelf. The consistency of the latitudinal scaling of the GM spectrum is also discussed, leading to a modified frequency dependence.

1. Introduction

Oceanic internal gravity waves can be generated by a myriad of forcing mechanisms. The nature of these internal waves will depend on the spatial and temporal characteristics of the forcing. Sometimes it is possible to relate the observed waves to a specific forcing mechanism. For example, the barotropic tide generates a periodic internal tide, and traveling storm fronts generate near-inertial waves. However, because of the complexity of nonlinear interactions and the general confusion of three-dimensional motion, the relationship between the observed wave field and the source is often obscured.

In the early 70s Garrett and Munk (1972; 1975) made a remarkable observation: Oceanic fluctuations over a wide range of time and space scales are strikingly stationary and homogeneous, and can be described statistically as an ensemble of random waves which are consistent with the kinematics governing internal waves. This description of the variability is often referred to as the Garrett-Munk (GM) spectrum. Although the GM spectrum does not explain the dynamics affecting the wave field and does not specifically include the tidal or near-inertial bands, it remains a useful description of the internal wave field in the open ocean. If asked to predict the basic statistics of internal waves at a particular location, the GM spectrum would be the prescription of choice.

On the continental slope and shelf the applicability of the GM spectrum is uncertain. However, there are many reasons why an internal wave description in shallow water would be useful. One reason is that differences from the deep ocean wave field may provide clues about which processes are most important in maintaining oceanic internal waves. Another reason to understand the shallow water internal wave field is because of its potential importance in generating turbulence. Knowledge of the role of internal waves in mixing could lead to improved parameterizations of turbulent processes in numerical models of coastal circulation. Over the steep topography of the slope, mixing enhanced by internal waves may play an important role in mixing the abyssal interior of the ocean (Munk and Wunsch, 1998). For a variety of applied concerns, it is important to know how the internal wave field on the shelf affects acoustic propagation and optical properties.

The purpose of this paper is to look briefly at some observations of internal waves on the shelf and suggest a statistical framework with which to view these observations. We begin with a review of the GM spectrum and suggest modifications and re-scalings needed to formulate a spectral description suitable for shallow water, called Aha99. Several other issues not included in this framework are also discussed.

2. Garrett-Munk spectrum: re-scaling from the deep ocean to the shelf

The GM spectrum is based on observations in deep water. To construct a spectrum applicable for shallow water, we start with the GM deep-water parameterizations and attempt to re-scale (Figure 1). There are many forms of the GM spectrum in the literature; we choose the one called GM79 presented in Munk (1981). The essence of the GM spectrum is contained in the expression for the energy distribution of the wave field written here in frequency $\omega$ and mode number $j$ space:

$$ S_{GM79}(\omega, j) = Eb^2 N_0^2 \frac{N(z)}{N_0} B(\omega) H(j) \quad (1) $$

where

$$ B(\omega) = \frac{2}{\pi} \frac{f}{\omega^2 - f^2}^{1/2} \quad (2) $$

and

$$ H(j) = \frac{1}{J} \frac{1}{(j^2 + j_0^2)} \quad (3) $$

1
where \( \int_{\omega}^{N} B(\omega) \, d\omega = 1 \) and \( J \) is a constant such that
\[
\sum_{j=1}^{N} H(j) = 1.
\]

This distribution function is derived assuming a random wave field with vertical symmetry, horizontal isotropy, and temporal stationarity. All useful spectral quantities can be derived from this function. The collection of constants \( E \, b^2 \, N_0^2 \) has the dimensions of energy / mass \([\text{J/kg}]\) and sets the energy level of the wave field. The constant \( E \) is nondimensional; the scales for length and time are given by \( b \) [m] and \( N_0 \) [s\(^{-1}\)] and are linked by GM to an exponential form describing the profile of buoyancy frequency:
\[
N(z) = N_0 \, e^{-z/b}
\]

The functions \( B \) and \( H \) describing the frequency and mode number dependence are separable—this is an assumption of GM, but is not essential to the re-scaling proposed below. Although discrete vertical modes are used in specifying the wavenumber energy distribution \( H \), the depth dependence is given simply as \([N(z) / N_0]\), following a simplified WKB approximation where the energy scales as \( N(z) \) relative to some reference \( N_0 \).

The length \( b \) is used also to scale the dimensional vertical wavenumber as \( m(z) = \frac{j\pi N(z)}{b \cdot N_0} \), which is a slowly varying function of \( z \). It would, of course, be more realistic to use actual modal wavefunctions to describe the vertical dependence instead of WKB, but for this discussion we retain the simplicity of the WKB scaling.

Two new parameters are defined to aid in re-scaling for shallow water: \( E_0 \), the dimensional energy / mass and \( D(\omega) \), the scale of the vertical waveguide (Table 1).

The function \( D \) is the effective depth range over which internal waves can propagate as free waves, i.e., where \( \omega < N(z) \). In GM the length of the waveguide is a constant \( b \) which makes a re-scaling for shallow water unclear. This is because the depth of the ocean does not appear explicitly as an independent parameter in the GM formulation—which is satisfactory as long as the depth of the ocean is much greater than \( b \). Using another parameter \( D \) is more straightforward and accommodates variations in the vertical waveguide due to both stratification and ocean depth. For example, suppose that the same exponential form of stratification exists in both shallow and deep water (Figure 2). This is a reasonable first-order description since the ocean gets "shallower from the bottom up" (M. Briscoe, personal communication). In this example \( b \) is the same at each location as it describes the form of the stratification, but \( D \) is not, as it is affected by the depth of the ocean.

<table>
<thead>
<tr>
<th>Table 1. Parameters re-scaled from GM79 to Aha99.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quantity</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>Energy/mass</td>
</tr>
<tr>
<td>Vertical wavenumber</td>
</tr>
</tbody>
</table>
3. Observations

To determine if this re-scaled form of the GM formulation is useful for shallow water, we look to the data. The results shown here are based on a limited number of observations, and therefore this note should be considered a preliminary discussion.

Examples of rotary velocity spectra from the Mid-Atlantic Bight (MAB) and the Oregon Shelf (OS) are compared with the GM spectrum in Figure 3. The details of each data set are given in Table 2 and in Boyd et al. (1997) and Pillsbury et al. (1974).

Table 2. Estimates of $E_0$ from MAB and OS.

<table>
<thead>
<tr>
<th>Depth, m</th>
<th>Average $N(z)$, s$^{-1}$</th>
<th>$E_0$ (J/kg) / 10$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.026</td>
<td>1.6</td>
</tr>
<tr>
<td>15</td>
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<tr>
<td>23</td>
<td>0.025</td>
<td>0.6</td>
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<tr>
<td>31</td>
<td>0.018</td>
<td>0.7</td>
</tr>
<tr>
<td>35</td>
<td>0.013</td>
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<td>43</td>
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<td>1.0</td>
</tr>
<tr>
<td>51</td>
<td>0.011</td>
<td>1.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Depth, m</th>
<th>Average $N(z)$, s$^{-1}$</th>
<th>$E_0$ (J/kg) / 10$^{-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.022</td>
<td>3.1</td>
</tr>
<tr>
<td>40</td>
<td>0.015</td>
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<td>60</td>
<td>0.010</td>
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<td>80</td>
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<td>3.1</td>
</tr>
<tr>
<td>95</td>
<td>0.007</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Figure 3. Rotary spectra from OS and MAB (solid lines) compared with GM shape (dashed lines). Clockwise component (CW) is bold; counter-clockwise component (CCS) is thin.

There are maxima in both the trend and $f$ and a whitening of the spectrum at high frequencies forming a spectral shoulder. The level of the GM spectrum was shifted subjectively to fit the observations in the continuum range above the trend and below the high frequencies. The shape of the GM continuum reasonably follows the observations, and the ratio of the CW and CCW components is also fairly consistent. Estimates of $E_0$ can then be made from equation (1) for various depths at MAB and OS using local values of $N(z)$ (Table 2). Representative values for MAB and OS are 1 and 3 mJ/kg respectively.

How does this compare with the energy level of GM79 before re-scaling: that is how does $E_0$ compare with $Eb^2N_0^2$? In attempting to answer this question one realizes the difficulty in using the existing GM parameterizations—what should be used for $b$ and $N_0$? Should one use the $b$ and $N_0$ that fit the shelf stratification? This would not make sense physically as $b$ and $N_0$ in this expression really only provide dimensional scales. It is more straightforward not to link the parameters specifying the stratification ($b$, $N_0$) with the absolute energy level ($E_0$). Using canonical values for the deep ocean for $E = 6.3 \times 10^5$, $b = 1300$ m, $N_0 = 5.2 \times 10^{-3}$ s$^{-1}$, yields an $E_0$ of 2.9 mJ/kg. This is relatively close to the levels of the shelf data shown here, i.e., within about a factor of 3. This preliminary result suggests the universality of $E_0$ in both deep and shallow internal wave fields—without the need to consider the universality of the stratification parameters $b$ and $N_0$. Note that since $E_0$ on the shelf is similar to the deep ocean level and the depth of the shelf is much shallower, the total energy per unit area on the shelf is much less than the deep ocean.

The effect of re-scaling the vertical wavenumber by replacing $b$ with $D$ is most dramatically demonstrated by looking at coherences. In the GM spectrum the vertical coherence is independent of frequency and is essentially a monotonic function of the scaled vertical separation $\Delta z/b$ (Desaubies, 1976). However, for large vertical separations, $\Delta z > b$, there is evidence of mode 1 dominance as the coherence increases with vertical separation and becomes out of phase (Blumenthal and Briscoe, 1995). On the shelf the stratification scale $b$ is not very different from the deep ocean, but the vertical coherence scale is. The observed coherence pattern is consistent with a mode 1 dominant
wave field, but with a vertical scale, $D \sim$ water depth, that is significantly shorter than the stratification scale $b \sim 1300$ m (Figure 4). Although not demonstrated here, the best fit is given by a $j_* \sim 1$, rather than 3, which is typical of the deep ocean.

The GM spectrum is horizontally isotropic, a property that is confirmed in observations of the deep ocean. However, on the shelf there is evidence of horizontal anisotropy. This is not surprising given the strong directionality provided by the nearby continental slope. Horizontal anisotropy is easily handled by the GM formalism of random waves—one needs only to parameterize the directional properties of the wave field (e.g., Schott and Willebrand, 1973). The anisotropy may be an important feature that distinguishes the shallow from the deep water wave field. There are clear examples of anisotropy due to domination of cross-shelf propagation (e.g., Winant et al., 1987; Gordon, 1978). This is demonstrated by vertical coherence that is much stronger in the cross-shelf velocity component. However, the universality of the horizontal anisotropy is still under study. Observations from MAB indicate some horizontal anisotropy, but it is not as profound as in these other examples.

4. Other Issues

While the re-scaled GM statistical framework is useful in quantifying some of the properties of the shallow water internal wave field, there are many other issues that are missing in this formulation. In fact it may be that these other topics are the most essential to understanding the dynamics of the internal wave field. Perhaps the GM spectrum merely represents the relatively uninteresting background random sea. A few of these other issues are discussed here.

One important issue is temporal variability. While nearinertial motion can be often be associated with storm events in the upper ocean (e.g., D'Asaro et al., 1995), the fluctuations of the energy in the internal wave continuum are more elusive and subtle (e.g., Frankignoul and Joyce, 1979). The wave field has been homogenized to such an extent that tracking observed waves back to their sources is not possible. Somehow the sources are steady enough on average and the nonlinear interactions are strong enough to maintain a fairly consistent spectral sea of internal waves. Some variability, such as seasonal fluctuations, might be expected as both surface forcing and upper ocean stratification vary. Internal wave energy in the Sargasso Sea shows relatively weak seasonal variation by a factor of 2 to 3 that decrease with depth (Briscoe and Weller, 1984). In shallow water stronger seasonal effects are anticipated as the entire ocean can be considered upper ocean.

Another issue complicating the internal wave field on the shelf is the presence of nonlinear internal wave (NIW) packets. Energetic packets of internal waves are generated owing to the nonlinearity of the shoaling internal tide. NIW have been observed on shelves throughout the world (e.g., Ostrovsky and Stepanyants, 1989), and are sometimes referred to casually as solitary waves, solitons, or solibores. Evidence of these waves can often be seen at the sea surface.
Figure 6. Examples of velocity spectra from different latitudes (Roith et al., 1981; equator (Eriksen, 1985); 9° (Käse et al., 1978); 16°N (Tarbell et al., 1977), 28°N (Briscoe, 1975). Spectra have been scaled based on local N, equation (1).

Figure 7. Velocity spectra from the equatorial Pacific Ocean at 140°W measured by NOAA’s Tropical Atmosphere Ocean (TAO) array. The spectral slope is less steep than GM below M2, suggesting a modified spectral form called Aha99.

visually or by surface radar as alternate bands of smooth and rough water. Internally the velocity structure is usually dominated by a mode 1 shape. The packet itself often consists of a collection of "pulses" or solitary-like waves (Figure 5). Nonlinear dynamics tend to create waves of solitary form from any initial wave shape. Despite the regularity of the barotropic tide, the characteristics of NIWs vary in time. The number, amplitudes, and shapes of the individual waves within a packet can vary dramatically from one packet to the next. Spectrally, much of the energy of the NIWs is concentrated at high frequency and contributes to the shoulder in the spectrum. However, the signal from the NIWs is spread over a significant bandwidth—the spectrum is not the best tool to analyze these nonlinear, non-sinusoidal phenomena. It appears that the spectral shoulder is not only due to NIWs, but may be a common feature of the upper ocean (e.g., Levine et al., 1983). Besides obscuring the spectrum of the background internal waves, NIWs may affect dynamically the generation and propagation of other internal waves.

Another interesting issue is the latitudinal scaling in the GM spectrum. While this is not only an issue on the shelf, it does affect the re-scaling of the GM formalism and is worth mentioning here. The spectral level in GM79 scales as \( f \), which comes from the normalization of \( B \) (equation 2). However, observations indicate that the spectral level does not scale with \( f \). Spectra from different latitudes are shown in Figure 6 using WKB to adjust for different \( N(z) \). The levels of the continuum are within a factor of 3 without applying any latitudinal scaling; \( f \)-scaling actually increases the spread in the data significantly. In a footnote GM79 recognizes this observation and suggests that the \( f \) dependence in \( B \) be changed to a constant \( f_p \) at say 30°, to carry the units. While this eliminates the \( f \) scaling of the spectral level, the integral of \( B \) from \( f \) to \( N \) is then no longer constant, but is a function of \( f \). This has implications on other statistical measures. For example, the level of the vertical wavenumber spectrum \( S(\beta) \), which is a measure of the vertical shear, is set by the integral of \( B \) from \( f \) to \( N \). In GM79 since \( B \sim f \), \( S(\beta) \) is not a function of \( f \). This appears to be consistent with observations (Gregg et al., 1993). If one instead uses a spectral level constant with \( f \), as observations of frequency spectra suggest, then \( S(\beta) \) goes like \( 1/f \) and increases dramatically in low latitudes. Both the fre-
frequency spectral level and the total energy integrated over all frequencies cannot be independent of $f$. Hence, the function $B$ needs to be modified.

Looking carefully at observed frequency spectra at low latitude, it is evident that the GM spectral slope does not fit the data at low frequencies (Figure 7). This has been commented on by Eriksen (1985) and even at a previous 'Aha 'Hulikoa (Levine, 1991). It appears that there may be a change in spectral slope at the semidiurnal frequency. This perhaps suggests a dynamic link between the high frequency internal wave field and the tide. If the tide is a significant energy source of high frequency internal waves, then this might help explain the remarkable stationarity and homogeneity of the internal wave field, as the tide is everywhere and steady. Recently, there has been renewed interest in the role of the internal tide in mixing the abyssal ocean (Munk and Wunsch, 1998). It may, however, be fortuitous to associate the spectral break with the tidal frequency, but clearly there is less energy below the M2 frequency than in the GM formulation. To fit the observations at low latitude more accurately (Figure 7), a modified functional form $B'$ is proposed for $B$:

$$B' = B(\omega) \begin{cases} 1, & \omega > M2 \\ \omega/M2, & \omega < M2 \end{cases}$$

Although the fit is not perfect between M2 and $f$, this simple analytical form does describe the essential shape. The total energy/mass over all frequencies then becomes a function of latitude, and therefore the $S(\beta)$ shear spectrum is still a function of $f$. However, since the spectral slope is less steep below M2, the $f$ dependence is not as pronounced and is perhaps more realistic. The function $B'$ at least is a consistent compromise to explain the lack or weak $f$ dependence of both $S$ and $S(\beta)$. Further refinement and verification of this modified form for $B$ is needed.

5. Conclusions

The Garrett-Munk spectrum remains a valuable first-order description of the oceanic internal wave field. To obtain an analogous description for the continental slope and shelf, we present the Aha99 form by re-scaling the GM spectrum and defining two new parameters: $E_0$ and $D$. The parameter $E_0$ is the total energy/mass, replacing the collection of constants $E b^2 N_0^2$; the length $D$ scales the vertical waveguide, replacing the $b$ scale. In GM the spectral level and vertical scale are both a function of $b$, the stratification scale. In the Aha99 these two scales are independent. The waveguide $D$ can then vary with water depth without affecting the spectral level or the depth scaling of the stratification.

Preliminary results on the shelf indicate that the spectral level $E_0$ of the continuum (between the tide and high frequency) is not very different from the deep ocean. The wavenumber bandwidth in terms of equivalent modes $j$ appears to be smaller than the deep ocean. Horizontal anisotropy is more pronounced on the shelves.

Although the apparent universality of the spectral continuum is interesting, other aspects of the wave field may be more important to understanding the dynamics. Other features affecting the climatology of the shelf internal wave field include temporal variability, horizontal anisotropy, and nonlinear internal wave packets. Although not an issue restricted to the continental shelf, the consistency of the latitudinal scaling of the spectrum through $f$ is explored. Observed spectra at low latitude indicate a whiter spectral slope than the GM $\omega^2$ at frequencies below M2. To follow these data more closely, a modification of the frequency dependence of the GM spectrum is suggested. The break in the spectral slope at M2 hints at a dynamical relationship between the internal wave continuum and the tide.

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References


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