Mixing Generated by Internal Waves Interacting with Rough Topography

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Abstract. The reflection of internal waves from a slope where the slope angle is close to the angle of the wave energy propagation has long been postulated as a mechanism for transfer of wave energy to smaller scales, leading to mixing through the enhancement of shear. Recent laboratory experiments and numerical simulations have examined the mixing generated when an internal wave reflects from a uniform slope at the critical angle. Here we perform simulations of interactions between internal waves and variable slope, using a novel numerical model which combines 3-dimensional topographic variations and non-hydrostatic dynamics.

Introduction

In regions of complex topography, observed diapycnal mixing [Polzin et al., 1997] is several orders of magnitude greater than observed in the ocean interior [Ledwell et al., 1993; Toole et al., 1994]. This enhanced diapycnal mixing plays an important role in the ocean general circulation [Munk, 1966; Munk and Wunsch, 1998] as well as being of local significance for the biogeochemistry of coastal regions. Hence a greater understanding of the factors which determine the distribution and magnitude of topographically generated mixing are of great importance for both local and global ocean prediction.

One mechanism for generating diapycnal mixing at topographic features is through the interaction between internal waves and topography. When internal waves are reflected from sloping topography the angle \( \theta \) between the propagation direction and horizontal is preserved. If the topography slopes at an angle \( \alpha \) close to \( \theta \), the reflected wave has a higher energy density than the incident wave, the vertical length scales are reduced, and shear is increased. Richardson numbers may be sufficiently reduced that instability results, leading to overturning and mixing. Mixing generated by the interaction of an internal wave with a uniform slope at the critical angle (\( \alpha = \theta \)) has been studied in the laboratory [Cacchione and Wunsch, 1974; Ivey and Nokes, 1989; De Silva et al., 1997] and through numerical simulation [Slinn and Riley, 1996]. It is yet not understood how 3-dimensional variations in slope may modify the mixing efficiency. In particular variable topography may confine the mixing to particular locations.

The mixing on the sloping topography can have a significant effect on the interior of the ocean only if mixed fluid is exported away from the boundary, and the mixing region replenished with stratified fluid. Such export mechanisms include the large-scale secondary circulation [Ivey and Nokes, 1989] and the periodic replenishment of the boundary layer fluid by the oscillating current itself [Slinn and Riley, 1996]. In the presence of rotation, baroclinic instability of the along slope currents established by mixing [Ivey, 1987] may contribute to the export process. If mixing is spatially variable, for example, in response to variable topography, lateral transports of mixed fluid may be enhanced. Hence the focus of our study is on the effects of variable topography on the mixing generated by internal wave reflection.

Numerical model formulation

Our tool for this study of internal waves interacting with variable topography is a numerical model which combines both non-hydrostatic physics (essential to capture the instability and overturning which lead to mixing) and arbitrary 3-dimensional topography [Figure 1]. Previous numerical studies have employed models which either parameterize the mixing [Holloway and Barnes, 1998] or explicitly resolve mixing in a model limited to uniform slope [Slinn and Riley, 1996]. We em-
ploy the MIT Ocean Model [Marshall et al., 1997a,b], a 3-dimensional non-hydrostatic Boussinesq model which includes topography through a finite-volume formulation [Adcroft et al., 1997]. The variant of the model which we use includes open boundary conditions and a free-surface: we force internal waves at the off-shore boundary and allow them to propagate toward the sloping topography.

Preliminary results: comparison between critical wave reflection on linear, concave and convex slopes

To demonstrate the capability of this model to include subtle topographic variations, and as a first step to examining the role of variable topography in determining mixing location and strength, we have carried out 3 simulations of an internal wave reflecting from a slope. In all three cases the parameters are identical apart from the curvature of the slope - in one case the slope is linear, and at the critical angle, while the other two cases are concave and convex respectively, with the same average slope as the linear case. These preliminary calculations are carried out in only 2-dimensions, with all derivatives in the y-direction set to zero. The stratification is initially uniform, with $N^2 = 10^{-6}$ s$^{-2}$. The total depth of the volume is 200 m, and the gravest mode internal wave is forced at $x = 0$ at the $M_2$ tidal frequency $1.41 \times 10^{-4}$ s$^{-1}$, with an velocity amplitude of $U_{max} = 0.024$ m s$^{-1}$. The coriolis parameter $f = 10^{-4}$ s$^{-1}$. A uniform resolution of 3.33 m is used in the vertical, while in the horizontal a stretched grid is applied, with a total of 640 grid points. Maximum horizontal resolution occurs over the slope, where $\Delta x = 7$ m. The calculations are carried out for a total of 9 tidal cycles.

As shown in previous studies, when the internal wave encounters the critical slope a bore-like feature develops in the density field (Figure 2), which leads to overturning and mixing, as dense fluid is pulled up the slope over less dense fluid. We average the fields over 6 tidal cycles, following the encounter of the wave with the slope, and compare the time averaged fields for the 3 different cases (Figure 3). All three cases have features in common - an Eulerian averaged flow which flows downslope right adjacent to the boundary, and upslope a short distance away. The along-slope velocity, generated as a result of finite rotation, is strongly sheared, with negative flow adjacent to the slope and positive flow just above. The temperature field has been strongly modified, with isotherms dragged upwards at the slope, bent downwards a short distance away, particularly toward the bottom of the slope, and bent upwards further in the interior and towards the top of the slope. The net effect of these isotherm distortions is a weakening of the
Figure 3. The fields at the slope, averaged over 6 wave-periods. (i) The cross-slope velocity field $U$, (ii) the along-slope velocity field $V$, (iii) the temperature field $T$, (iv) the change in the vertical gradient of the temperature field $dT/dz$, shown for (a) linear slope, (b) convex slope, (c) concave slope. Contour spacing is (i,ii) 0.0025 m s$^{-1}$, (iii) 0.01 K, (iv) $5 \times 10^{-5}$ K m$^{-1}$. Negative values are indicated by dashed contours, positive values by solid contours.
stratification in a broad band parallel to the slope (indicated by dashed contours), in the uniform slope case, with narrower regions of increased stratification at the slope itself and further in the interior (solid contours). Hence most of the mixing appears to take place not at the boundary, but a short distance into the interior.

Despite these broad similarities, there are differences between the 3 cases, particularly in the net changes in temperature and stratification. The concave slope simulation shows much less distortion of the isotherms than the other two cases, while the convex slope case has weakened stratification concentrated near the top of the slope, where the slope is critical. Note that the concave slope is critical toward the bottom of the slope, where the temperature perturbations associated with the 1st mode internal wave are small.

Discussion

These preliminary calculations show that variations in the slope can have a significant influence on the location and magnitude of mixing generated by internal wave reflection. Naturally the importance of such variations must depend on the relative scales of the topography variations compared to the wave motions: higher vertical modes might be less influenced by the broad topography variations studied here than the gravest mode.

Later calculations in 3-dimensions will examine along-slope variations in topography and the exchange of fluid between the boundary regions and interior initiated by 3-dimensional motion. Our ultimate aim is the development of parameterizations of mixing and its dependence on the topography and waves.

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References


