Decadal Variability in Millennium Integrations
with Coupled Atmosphere-Ocean General Circulation Models

Jin-Song von Storch
Institute of Meteorology, University Hamburg, Germany

Peter Müller
Department of Oceanography, University of Hawaii, Honolulu, Hawaii

Abstract.
Different types of decadal variability obtained from millennium integrations performed with coupled general circulation models are discussed. The first type, which is characteristic for large-scale atmospheric variations, represents the white-noise extension of short-term fluctuations. This variability is characterized by a spectral plateau over a wide range of time scales, extending from the longest time scale of the atmospheric fluctuations of about one year to time scales of decades and longer. The second type of decadal variability, which is associated with the baroclinic Rossby waves in the upper ocean, represents the variability involving mechanisms, which operate on decadal time scales. The spectra of such variability are characterized by a high-frequency $\omega^{-2}$ power law that levels off at time scales of decades. The third type of decadal variability, observed for the deep-ocean mass transport, appears as part of a low-frequency power law. The power, which varies from about minus one in the intermediate depth to about minus two near the bottom of the ocean, indicates the non-stationary character of the deep-ocean variability. All three types of decadal variability have to be considered as part of a general variability which spreads over a continuum of time scales. Nevertheless, dominant time scale, in particular that associated with a propagation of anomalies in space, can also be identified. For instance, the decadal variability related to baroclinic Rossby waves in the Pacific reveals a propagation of anomalies of sea level and ocean temperature on time scales of decades.

1. Introduction

In the recent years, several integrations over time periods of more than one thousand years have been performed using coupled atmosphere-ocean general circulation models (GCMs) (Manabe and Stouffer, 1996; Tett et al., 1997; Voss et al., 1998 and von Storch et al., 1997). These long integrations form a basis for detailed analysis of the decadal variability in the climate system. In particular, it is possible to obtain a relatively complete description of the decadal variability from these runs. Furthermore, after having identified the dominant characteristics, further studies concerning the mechanisms of the decadal variability can be carried out.

This paper concentrates on the description of the decadal variability in millennium integrations performed with coupled GCMs. The most intuitive way to describe this variability is to consider time series of climate variables. A few examples are given in Figures 1 and 2. Figure 1 shows the yearly time series of the maximal overturning in the Atlantic, characterizing the Atlantic meridional mass transport in four 1000-year integrations with different coupled GCMs (von Storch et al., 1998). Figure 2 shows the yearly anomaly time series of the ocean temperature at 150 m depth near 15°N and 130°E, as obtained from the ECHAM1/LSG millennium integration (von Storch et al., 1997). All time series shown reveal episodes marked by large / small values which are able to last persistently over a time period of decades, indicating the existence of variability on decadal time scales. However, the time series shown also vary on other time scales. In general, they are irregular in time. A given segment of a time series never exactly repeats itself during the considered 1000-year time interval, even when it is produced by the same model.

The situation is further complicated by the fact that a climate variable is generally a multivariate variable with its dimension being determined by the number of grid points needed to represent the variable in the physical space. Two examples of multivariate time series are given in Figures 3 and 4. They show yearly sequences of anomaly fields of zonally averaged stream function in a vertical and meridional section of the Atlantic (Figure 3) and of ocean temperature at 150 m depth (Figure 4). The sequences are obtained from the integration with the coupled ECHAM1/LSG model. Both the Atlantic stream function and the oceanic temperature at 150 m depth reveal notable irregular year-to-year variations in spatial structures. The model variability is therefore
accompanied not only by temporal but also spatial irregularities.

Because of these irregularities, the consideration of an individual event, e.g., defined as large positive values occurring persistently during a certain time interval at a certain location, cannot provide conclusive statements about the variability. A statistical description is required instead. Such a description concerns "averaged" quantities, such as variances and covariances, or their frequency decompositions, i.e., spectra and cross spectra.

Section 2 gives a short introduction to coupled GCMs used for millennium integrations. The time-mean states produced by these models are discussed in section 3. The temporal and spatial features of decadal variability are studied in sections 4 and 5. Since the spatial variability is observed as time passes, a strict separation of the temporal variability from the spatial variability is impossible. In general, the spatial and temporal variability are jointly described by the multivariate second moments. Because of the complexity of these statistics, the univariate spectral features, i.e., the spectral features of one pattern or of a variable at one location, are first considered in section 4. Statistical techniques used to study the multivariate spectral features are discussed in section 5. While the analysis of section 4 is carried out for various global atmospheric and oceanic variables, the consideration of section 5 is confined to the variability in the Pacific only. The large portion of the results is derived from the 1260-year integration with the coupled ECHAM1/LSG model of the Max Planck Institut (Hamburg, Germany) (von Storch et al., 1997). Some conclusions concerning the variability of the deep ocean are drawn from the 1000-year integrations (von Storch et al., 1998) performed at the Geophysical Fluid Dynamics Laboratory (Princeton, USA), the German Climate Computer Center (Hamburg, Germany) and the Hadley Centre (Bracknell, UK). The run of the German Climate Computer Center is carried out using the same ocean GCM, but a different version of the atmospheric
GCM and a different spin-up procedure. The three additional integrations will be referred to as the GFDL, ECHAM3/LSG and HadCM2 runs, respectively. Conclusions and discussions are given in the final section.

2. Model description

A coupled GCM of the atmosphere-ocean system consists of several model components. One represents the atmosphere and the other the ocean. These two components are formulated based on the conservation principles of momentum, heat and mass (not only for the total air/water mass, but also for its components such as moisture in the atmosphere and salt in the ocean). The two models are coupled to each other through fluxes of momentum, heat and fresh water at the interface between the atmosphere and the ocean. In addition to these, two other model components dealing with sea
data. Rather, we concentrate on the description and mechanism of the variability as seen in coupled GCMs and simplified models.

(a) Resolution

Due to limited computer capacity, the coupled GCMs used for millennium integrations have generally coarse resolutions. In the case of the ECHAM1/LSG model, the horizontal resolution of the atmospheric model (ECHAM1 model) is limited by a triangular spectral cut-off to a total wave number of 21 representing a 64 longitude by 32 latitude grid. The ocean model has an effective horizontal grid size of 4° by 4°.

(b) Parameterizations

When such coarse-resolution GCMs are used to study the large-scale variability, one has to be aware of the existence of the subgrid processes. Unfortunately, the parameterizations, which are designed to represent the subgrid processes, are tuned to reproduce the observed time-mean state. How appropriate these parameterizations are to represent the effect of the subgrid processes on the variability is not known and has not been studied systematically yet.

In addition, the ECHAM1/LSG model is not equipped with the newest parameterization package, nor are the other coupled GCMs, e.g., the coupled GCM used at the Geophysical Fluid Dynamics Laboratory in Princeton (USA) (Manabe et al., 1991). This is particularly true for land processes (including glacier calving) and sea ice dynamics.

The oversimplified and partly ad hoc parameterizations in the coupled GCMs may have consequence, such as that the total mass is not conserved, or the hydrological cycle is not closed. This type of problems become apparent, e.g., in terms of salinity trends, when the models are integrated over one thousand years. To what extent do these inappropriate parameterizations affect the variability is unknown.

(c) Coupling with flux corrections

The atmospheric and oceanic model components are coupled together through the fluxes of heat, momentum and fresh water at the surface. The heat flux for the atmosphere is a function of the sea surface temperature, while the fluxes of momentum and fresh water for the ocean are functions of the surface wind and the rate of precipitation and evaporation. However, the time-mean values of these fluxes produced by each model component separately are generally not identical to those derived from the observations. If the two model components were coupled together, the coupled system would tend to reach an equilibrium state different from the present climate state, a phenomenon known as climate drift.
In order to avoid the climate drift, flux corrections accounting for the differences between the simulated and the observed time-mean fluxes are applied (Sauesn et al., 1988). The implementation of flux corrections is equivalent to coupling the atmosphere and the ocean with anomalies of the fluxes computed relative to the observed equilibrium states of the two uncoupled subsystems.

(d) Spin-up of the deep ocean

The procedure to bring a system to its equilibrium state is called spin up. Different from the atmosphere, which can be spun up within a few years, the time required for the deep ocean to reach its equilibrium is of the order of $10^3$ to $10^4$ years. In order to reduce the computational expense, the deep ocean is therefore normally first spun up in an uncoupled mode over a long time period using prescribed surface fluxes (Cubasch et al., 1992, Manabe et al., 1991 and Voss, 1996). When then coupled to the atmosphere, the deep ocean may experience a shock. As an alternative, the coupled spin up of the deep ocean, designed to avoid the coupling shock, is used at Hadley Centre. However, instead of allowing the deep interior ocean to approach freely its own equilibrium state, the deep ocean is essentially tightened to the present-day state by prescribing the interior distribution of temperature and salinity. There is evidence suggesting that the circulation resulting from the prescribed climatological distribution of temperature and salinity is not necessarily superior to the circulation which is freely approached (Toggweiler et al., 1989). Von Storch et al. (1998) suggested that the different spin-up procedures lead to different time-mean states and therefore different variability of the deep ocean.

3. Time-mean state

The large-scale features of the time-mean circulation in the millennium integrations considered in this paper are consistent with the observational evidences. Two examples, concerning the simulated time-mean state of the atmosphere and the ocean, are considered below.

Figure 5 shows the distribution of sea level pressure (SLP) in northern winter (December-February) and in northern summer (June-August) averaged over the last 810 years, during which the coupling shock is essentially diminished. For the comparison, climatological distribution of the 1986-1995 ECMWF analysis is also shown. The Aleutian low during DJF in the model is about 5 hPa deeper than in the observations. The Iceland low is underestimated and shifted about 20° southwards relative to the observed one, causing a southward shift of the Atlantic gyre system. The mean SLP values over the Arctic are up to 15 hPa overestimated during northern winter. In northern summer, the model reproduces the observed surface pressure relatively well, with slightly underestimated high pressure systems over the northern oceans.

![Figure 5](image_url). Mean sea level pressure in northern winter DJF (a) and in northern summer JJA (b) averaged over the last 810 years of the coupled control run. For comparison, climatological distribution based on ECMWF analysis in 1986-1995 are shown in (c; DJF) and (d; JJA). Contour interval is 5 hPa.
In the Southern Hemisphere the subtropical highs are about 5 hPa weaker than observed in both seasons and the subpolar lows around Antarctica are 10-20 hPa weaker than in the observations. As a result, the meridional pressure gradient in 30-50°S in Figure 5a,b is strongly underestimated in the model compared to the present climate. Another feature which is evident in both the ECHAM1/LSG model and some other low resolution uncoupled atmospheric models is the too high values of SLP over Antarctica.

Despite the differences between the simulated and the observed distributions, Figure 5 suggests that the dominant large-scale features of the time-mean SLP fields, i.e., the distribution of highs and lows, are captured by the ECHAM1/LSG model.

The second example of the simulated time-mean circulation is given in Figure 6, which shows the 1000-year means of the zonally averaged stream functions in the vertical-meridional section of the Atlantic, as obtained from the GFDL, ECHAM3/LSG, ECHAM1/LSG and HadCM2 integrations.

Although the strength and the location of the circulation differ from integration to integration, the very gross features of the Atlantic overturning, i.e., northward flow in the upper 1000 to 2000 meters, the subsequent deep-water formation in the North Atlantic and southward spreading in deeper Atlantic, and antarctic inflow from still deeper layers, are produced by all models. Using observational data in the Atlantic, Roemmich and Wunsch (1985) estimated that 17±4 Sverdrups of relatively warm thermocline and intermediate water flow northwards at 24°N; and below this 20±5 Sverdrups, identified as North Atlantic Deep Water (NADW), flow southwards. Further below, about 3±3 Sverdrups of Antarctic Bottom Water (AABW) flow northwards. Figure 6 suggests that the transport of the NADW in the GFDL and the HadCM2 runs is close to the lower bound of the estimates of Roemmich and Wunsch and that in the two ECHAM/LSG runs is close to or above the upper bound of these estimates. The transport of northward AABW is underestimated in the ECHAM1/LSG. The numbers produced by the ECHAM3/LSG run are more comparable to the values of Roemmich and Wunsch than those produced by the ECHAM1/LSG run. It is noted that the discrepancy between Figure 6 and the results of Roemmich and Wunsch may also be related to the fact that the observed values do not represent the time-mean circulation as defined by 1000-year average in this paper.

4. Spectral features on decadal time scales

In this section, the univariate spectral features on decadal times scales are studied for the atmosphere and the upper and deep ocean, respectively. Instead of isolating the variability on decadal time scales, the overall spectra, i.e., the distribution of variance on all resolvable time scales, are considered first. From these spectra, the decadal spectral features are then derived. Such a consideration is motivated by the temporal irregularities described in the introduction. Because of the irregularities, the variability spreads over a continuum of time scales. In this sense, decadal variability should not be understood as something which varies only on decadal time scales, but something which varies on both decadal and intra- and inter-decadal time scales. The consideration of spectra provides not only an estimation of the strength of the decadal variability relative
to the variability on other time scales. It is also the indispensable first step toward the understanding of the interactions between variations on different time scales and of the origin of the decadal variability.

4.1. Atmospheric spectra

This paper assumes that large-scale motions have more low-frequency variance than small-scale motions and concentrates on spectra of time series of large-scale atmospheric patterns. Such patterns are defined by the leading EOFs of the global anomaly fields of various variables. A more detailed discussion on EOFs is given in section 5.

There are at least two ways to study the spectral features of defined patterns. One can either plot the individual spectra and discuss the variations of spectral shapes by comparing the plots. Or, since many coefficient time series of the patterns considered can be approximated by a first order auto-regressive (AR(1)) process, one can also study the spectral shape of various large-scale patterns using an objective measure defined through an AR(1)-process. Such a measure is considered below.

Let \( \alpha \) be the parameter of an AR(1) process, which is simply the lag-1 correlation of the process, and \( \Gamma_n \) the spectrum of the driving noise, then the spectrum \( \Gamma \) of the process is given by

\[
\Gamma(\omega) = \frac{\Gamma_n(\omega)}{1 - 2\alpha \cos(2\pi \omega \Delta) + \alpha^2}
\]

where \( \Delta \) is the time increment of the process and \( \omega \) the frequency measured in units of \( 1/\Delta \). For \( \omega \ll 1/(2\pi \Delta) \), the above equation reduces to

\[
\Gamma(\omega) = \frac{\Gamma_n(\omega)}{(1 - \alpha)^2 + \alpha(2\pi \Delta \omega)^2}
\]

For a white driving noise with \( \Gamma_n(\omega) = \text{const.} \), the shape of the spectrum is characterized, respectively for high and low frequencies, by

\[
\Gamma(\omega) \propto \begin{cases} 
\frac{1}{\alpha(2\pi \Delta)^2} \omega^2 & \text{for } \omega \gg \omega^* \\
\frac{1}{(1 - \alpha)^2} & \text{for } \omega \ll \omega^*
\end{cases}
\]

(1)

with \( \omega^* \) being defined as \( \omega^* = \frac{k}{2\pi \Delta} \).

\( \Gamma(\omega) \) increases with decreasing \( \omega \) at the rate of \( 1/\omega^2 \) for high frequencies with \( \omega \gg \omega^* \) and reaches a constant level at low frequencies with \( \omega \ll \omega^* \). The frequency at which the spectrum bends to merge the constant level is given by \( \omega^* \). A smaller \( \omega^* \) (i.e., a larger \( 1/\omega^* \)) leads to a wider frequency band over which the spectral energy increases at the rate of \( \omega^{-2} \) and a higher spectral level at frequency \( \omega \ll \omega^* \). As long as the considered time series can be approximated by an AR(1)-process, the spectral shape is completely determined by the frequency \( \omega^* \), or the time scale \( T^* = 1/\omega^* \).

For the atmospheric variables in the ECHAM1/LSG, the spectral shapes of coefficient time series of the EOFs (also referred to as the principal components, i.e., PCs) were studied in von Storch (1998). The analysis was concentrated on the stream function and the velocity potential, which completely describe horizontal motions. In order to additionally capture vertically varying motions, the stream function and velocity potential at 200, 500 and 850 hPa are considered. The EOFs are calculated from 500-year monthly unfiltered anomaly fields. Anomalies are obtained by subtracting the 500-year means of each calendar month.

The time scale \( T^* = 1/\omega^* \) of the first 10 leading PCs for the stream function and velocity potential at the chosen levels is shown in Figure 7. For velocity potential, the only large value is obtained from the sixth PC at 850 hPa. The corresponding EOF pattern (not shown) shows large anomalies near the Antarctic, indicating that this mode is related to the initial drift of the sea ice during the coupled integration (von Storch et al., 1997). Apart from this value, \( T^* \) of the stream function (Figure 7b) is noticeably larger than that of the velocity potential (Figure 7a). The largest \( T^* \) values, up to about 11 months, result from the first two PCs of the stream function. The result suggests that the non-divergent flow is redder than the non-rotational flow.

Figure 8 shows the spectra of the first four PCs of the 200-hPa stream function. Since the values of \( T^* \) (which are of the order of a few months) are generally poorly resolved in monthly data, due to the sample errors occurring at the time scale near the sampling time scale (i.e., a month), the spectra in Figure 8 are derived from 200-year daily PCs (obtained by projecting the daily data onto the EOF patterns of monthly data). Figure 8 shows that the shapes of the spectra, characterized by a high frequency \( \omega^{-2} \) slope and a flat plateau at low frequencies, are consistent with equation (1). The time scales at which the spectra become flat are comparable with the \( T^* \) values shown in Figure 7. Due to the logarithm representation, the difference between \( T^* \) of the first two leading PCs and the high order PCs are less dramatic in Figure 8 than in Figure 7.

The spectra shown in Figure 8 are flat with no significant peaks on time scales beyond one year. The decadal variability of the large-scale atmospheric variations in the ECHAM1/LSG run can therefore be considered as a white-noise extension of short-term fluctuations. The processes responsible for short-term fluctuations operate on a time scale no longer than one year.

4.2. Spectral features of the upper ocean

An important part of the variability in the upper a few hundred meters of the ocean is associated with baroclinic Rossby waves. By considering the baroclinic pressure that describes these waves, Frankignoul et al. (1997) constructed a simple linear model of extratropical ocean driven with stochastic wind stress forcing. The baroclinic spectra predicted by the model were then
compared with the North Atlantic pressure variability in the ECHAM1/LSG millennium integration.

Figure 9 shows spectra of the ECHAM1/LSG baroclinic pressure at 250 m and 30°N for various longitudes. The spectra are red with a high-frequency $\omega^{-2}$ increase that levels off at decadal time scales. These time scales correspond to the time needed for a long baroclinic Rossby wave to propagate across the ocean basin.

Figures 8 and 9 demonstrate the difference between the decadal variability in the atmosphere and that in the upper ocean. In the atmosphere, the spectra become
flat at time scales longer than one year, indicative of the short time scales of atmospheric processes. In contrary, the spectra of the baroclinic pressure level off at decadal time scales. This suggests that the decadal time scales are the time scales at which the process responsible for the baroclinic variability in the upper ocean operates.

4.3. Spectral features of the deep ocean

The situation changes again when considering the zonally averaged stream function in the deep ocean. This is a variable which excludes the effects of waves and describes the meridional and vertical mass transports. The spectra of the zonally averaged stream function can be piecewise described by power laws:

$$\Gamma(\omega) = \alpha \omega^{-\beta}$$

for $\omega \in \Delta \omega$, where $\Delta \omega$ indicates a frequency range. The spectrum (2) reveals a linear slope in a log-log plot. $\beta > 0$ indicates a slope which increases with decreasing frequency. $\beta \sim 0$ represents a spectrum which is independent of frequency. In general, a power law describes invariate relation against the time scale. The spectra discussed in sections 4.1 and 4.2 can also be described by such power laws, with the low-frequency power being zero and high-frequency power being minus two.

An example is given in Figure 10, which shows the spectra of zonally averaged stream function in the deep (i.e., at 4000 m) and intermediate (i.e., 2000 m) Atlantic, respectively. The solid straight lines indicate the fitted spectral slopes. The low-frequency range, over which equation (2) is fitted to the spectrum, extends from about a couple of decades to a few centuries. The fitted power law is closed to $\omega^{-2}$ (indicated the upper dashed line) in the deep ocean and to $\omega^{-1}$ (indicated by the lower dashed line) at the intermediate depth.

Figure 11 shows the low-frequency power $\beta$, for the zonally averaged stream function at all positions in the ECHAM1/LSG Atlantic. Again the fitting is carried out for the frequency range extending from about a couple of decades to a few centuries. In the tropical and subtropical oceans, the low-frequency spectral power increases with increasing depth. The values vary
from about zero near the surface to about one at intermediate depth and to about two near the bottom.

Similar spectra were obtained from the integrations with the GFDL and ECHAM3/LSG models (1998). The variability in the HadCM2 run, however, deviates from that in the GFDL and the two ECHAM/LSG runs. The values of $\beta_i$ in the Atlantic of the HadCM2 run decreases from about one in the upper 1000 m to about zero in a layer extending from 2000 to 3000 meters. Von Storch et al. (1998) suggested that these different spectral behaviors result from the differences in the time-mean state. This state crucially depends on the way how the deep ocean was spun up. The deep ocean in the GFDL and the two ECHAM/LSG runs was spun up by allowing it to freely approach its equilibrium state, whereas the deep ocean in the HadCM2 run was essentially tightened to the prescribed distribution of temperature and salinity. Toggweiler et al. (1989) suggest that the variability in the GFDL and the two ECHAM/LSG runs is superior to that in the HadCM2 run.

Figure 12 shows the spectra of the coefficient time series of the first two leading EOFs of the zonally averaged stream function in the Atlantic, as obtained from the ECHAM1/LSG run. Since these EOFs describe essentially the recirculation involving water masses in the intermediate depth, the corresponding spectra reveal, consistent with the large area with $\beta_i \sim 1$ in Figure 11, a slope close to $\omega^{-1}$. Since the leading EOFs represent the dominant large-scale variations of the considered variable, Figure 12 demonstrates the prominence of the $\omega^{-1}$ power law for the variability of mass transport in the ECHAM1/LSG run.

The spectra shown in this subsection suggest that there exists a third type of decadal variability in the climate system, which differs from that discussed in sections 4.1 and 4.2. The low-frequency part of this variability is characterized by power laws with the powers being of the order of minus one to minus two. The resulting spectra increase with decreasing frequency at the same rate for both the variability on time scales of

Figure 10. Spectra of the time series at grid points in the Atlantic of the ECHAM1/LSG model where $\beta_i$ is close to two and one. The spectra are estimated from 2 consecutive chunks. Such obtained spectral values are $\chi^2$ distributed with 4 degrees of freedom for all frequencies (except for the highest and the lowest resolvable frequencies $\omega = 0.5$ and $\omega = 2 \times 10^{-3}$). The grey band shows the 95% confidence band of the $\chi^2$ distribution. The solid straight line is obtained by least-square fitting equation (2) to the low-frequency part of the spectrum. The vertical dashed lines mark the upper bound of the low-frequency band. The two other dashed lines are proportional to $\omega^{-1}$ and $\omega^{-2}$.

Figure 11. Low-frequency spectral power $\beta_i$ of zonally averaged stream functions in the Atlantic, obtained from four coupled millennium integrations. The black, dark and light grey areas indicate the areas where values of the spectral power $\beta_i$ are close to two, one and zero, respectively. White areas with dashed lines indicate areas with $\beta_i < -0.5$. $\beta_i$ is fitted into spectra derived using two consecutive chunks.
Figure 12. Spectra of time series of the first two EOFs of the zonally averaged stream function in the Atlantic, as obtained from the ECHAM1/LSG integration.

several decades and that on time scales of a few centuries. The continual increase of spectral energy with decreasing frequency is indicative of the non-stationary character of the deep-ocean mass transport.

5. Multivariate spectral features in the Pacific

5.1. Methodology

Spatial variability is normally described in terms of a grid. Denote the considered climate state variable, such as the ocean temperature at 150 m depth, by

$$v(t) = \begin{pmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_M(t) \end{pmatrix}$$

where the subscript $i$ indicates the $i$-th grid point, $M$ is the total grid points, and $t$ is time. The parameter most frequently used to characterize the spatial variations is the covariance matrix. Let $\langle \rangle$ be an average operator and $v'(t)$ the deviation to the mean $\langle v \rangle$. The covariance matrix is defined as

$$\Sigma = \langle v'(t)v'(t)^\dagger \rangle$$

where $\dagger$ denotes the vector transposition.

The $i$-th diagonal element of $\Sigma$ is the variance of $v_i(t)$. A map of diagonal elements of $\Sigma$ displays the geographical distribution of the variance of $v(t)$. The off-diagonal elements of $\Sigma$ describe covariances of $v(t)$ at two different locations. When $v'(t)$ is normalized by the square root of local variances, a map of the $i$-th column of $\Sigma$ represents a teleconnection map, describing how variations at the $i$-th grid point is related to variations at other grid points. The most effective way to subtract information about the spatial variations of $v(t)$ from $\Sigma$, however, is to consider the eigenvectors of $\Sigma$, known as the empirical orthogonal functions (EOFs). The leading EOFs $e_i$, defined as the eigenvectors with the largest eigenvalues, are most efficient in representing the spatial and temporal variability of $v(t)$. If $v(t)$ is projected onto a subspace spanned by $N$ arbitrary vectors, the representation using the first $N$ EOFs is the one which makes the smallest mean square error.

On the other hand, since $\Sigma$ contains only simultaneous covariances, neither teleconnection maps nor EOFs can describe temporal variations of several patterns, i.e., the multivariate spectral features. In order to deal with this aspect, lagged covariance matrices

$$\mathcal{V}(\tau) = \langle v'(t)v'(t + \tau)^\dagger \rangle$$

with $\tau$ being the time lag have to be considered. The matrices $\mathcal{V}(\tau)$ and $\Sigma \equiv \mathcal{V}(0)$ (or the cross-spectrum matrices, which are the Fourier transform of $\mathcal{V}(\tau)$) represent the complete multivariate second moments of $v(t)$.

Since $M$ is generally very large, a systematic consideration of matrices $\mathcal{V}(\tau)$ is extremely difficult. The problem can be, however, simplified by approximating $v'(t)$ by a multivariate AR(1) process

$$v'(t) = \mathcal{A}v'(t-1) + r(t)$$

where the matrix $\mathcal{A}$ is the multivariate analog of the parameter $\alpha$ of a univariate AR(1) process and $r(t)$ is a multivariate white noise.

With the above approximation, $\mathcal{V}(\tau)$ equals $\mathcal{A}^\tau \Sigma$. The complete multivariate spectral features can then be estimated from $\mathcal{A}$ and the covariance matrix $\Sigma$ (for more detailed discussion, see von Storch, 1996). In particular, the space spanned by the eigenvectors of $\mathcal{A}$ can be used to decompose the multivariate second moments. These eigenvectors are called POPs (Principal Oscillation Patterns). The analysis technique based on the approximation (3) is referred to as the POP analysis.

Since $\mathcal{A}$ is not symmetric, its eigenvectors and eigenvalues can be real and complex. A real eigenvector describes a direction, in which the variability is univariate, i.e., all spatial points vary in the similar manner. Examples of such variability is given in section 4. A complex eigenvector, on the other hand, describes a subspace in which the variability is multivariate and has therefore to be described by at least two directions. A typical example in physical space corresponds to the situation that the variability at one location has a fixed phase relation to variability at another location, resulting a propagation in space. In order to describe such a propagation, at least two patterns are needed. The real and imaginary part of a complex POP, $p^r$ and $p^i$, are two such patterns. If $p^r$ and $p^i$ describe anomalies of similar amplitudes at different locations, the propagation is
characterized by the sequence
\[ \cdots \mathbf{p}^r \rightarrow \mathbf{p}^r \rightarrow -\mathbf{p}^r \rightarrow -\mathbf{p}^r \cdots \]

The anomalies at one location observed in \( \mathbf{p}^r \) are moved after a quarter of period to another location described in \( \mathbf{p}^r \) and so on.

5.2. An example

In order to study the propagating features within the Pacific basin, a POP analysis was performed for the sea level in the ECHAM1/LSG run (von Storch, 1994). Since the focus was on the propagations on decadal time scales, sea level data are filtered by removing variability on time scales shorter than 4 years and longer than 60 years.

A POP mode with oscillation period of about 10 to 20 years was identified. The imaginary part of the POP (Figure 13a) is characterized by positive sea level anomalies in the tropical central and eastern Pacific and negative anomalies northwest and southwest to the positive ones. After a quarter of the period, the tropical anomalies have moved westward and appear more pronounced north of the equator (Figure 13b), while negative anomalies have propagated northward in the North Pacific and southward in the South Pacific.

By searching for patterns of other variables which have similar temporal variations as the POP, it was found that the mode described by Figure 13 is associated with changes in the circulation and thermal structure of the upper ocean, in particular the layer of the first one to two hundred meters. In the subtropical North Pacific, the positive sea level anomalies in the east are accomplished by sinking of isotherms in the same region. After a quarter of the period, the isotherms deepen in the upper few hundred meters across almost the entire subtropical Pacific as the positive sea level anomalies move into the central Pacific. Thus, parallel to the evolution of the sea level anomalies, temperature anomalies also move westward with the largest anomalies being located at 150 to 250 meters depth in the west and at 75 to 150 meters in the east which corresponds to the upper part of the model thermocline.

The vertical structure of the temperature anomalies and the time interval needed for sea level anomalies to cross the subtropical Pacific suggest the active role of the first baroclinic Rossby waves in the evolution of the mode. The suggestion is consistent with the theoretical consideration of Frankignoul et al. (1997). However, it is noted that the time scale discussed in section 4.2 and the oscillation period associated with the POP mode considered here describe two different, but related, aspects of the decadal variability of the upper ocean. One represents the time scale on which the responsible mechanism operates, and the other is the time scale connected to a propagation in space. The former can be identified using a univariate spectral analysis, since the mechanism can leave its imprint at a location.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure13}
\caption{Imaginary and real part of the complex POP derived from sea level anomalies in the first 325 years of the ECHAM1/LSG integration. The upper diagram appears about one quarter of the period earlier than the lower one. For details, see von Storch, 1994.}
\end{figure}

The latter, in contrary, has to be studied using a multivariate technique. These two different aspects are also reflected in Figures 2 and 4. In the first diagram in Figure 4, small positive temperature anomalies are found near the coast of North America at 30°N. These anomalies move one year later into the central eastern Pacific. Such a movement, although not permanently present in the same strength, defines the oscillation period of the POP mode. On the other hand, when the spectrum of one pattern or the spectrum of temperature at one location, e.g., that shown in Figure 2, is calculated, one obtains a spectrum which bends at decadal time scales.

6. Discussions and conclusions

This paper studies the decadal variability of the climate system using millennium integrations performed
with coupled GCMs. The decadal variability in these runs is marked by temporal and spatial irregularities. In order to obtain conclusive statements about this variability, a statistical approach, which focuses on the spectra and covariances, is adapted here.

Several different types of decadal variability are identified from a univariate spectral consideration. The first one, observed in the model atmosphere, represents the white-noise extension of short-term fluctuations on time scales of decades or longer. With the longest time scale of the short-term atmospheric fluctuations being about one year, the white-noise extension is characterized by a spectral plateau over time scales longer than one year.

The second type of decadal variability, observed in the upper ocean, represents the variability involving mechanisms, which operate on decadal time scales. The spectra of such variability are characterized by a high-frequency \( \omega^{-2} \) increase that levels off at decadal time scales. Frankignoul et al. (1997) suggested that the underlying mechanism involves the dynamics of baroclinic Rossby waves which are excited by fluctuating atmospheric wind stress forcing.

The third type of decadal variability, observed for the deep-ocean mass transport, should be considered as part of a low-frequency power law. The power varies from about minus one in the intermediate depth to about minus two near the bottom of the ocean. The large non-zero power implies an increase of spectral energy with decreasing frequency, indicating the non-stationary character of the deep-ocean variability. The time-scale invariance indicates that the decadal variability and the variability on time scales of centuries may be generated by a similar process.

The analysis of the deep-ocean mass transport in the millennium integrations raises the question of whether the non-stationary character will disappear when longer time series is considered. Or in other words, whether the spectral energy will stop to increase with decreasing frequency for still lower frequencies not resolvable from a millennium integration. The question cannot be answered by the present integrations with coupled GCMs yet. However, an evidence about the non-stationary nature of the deep-ocean variability is found in an integration over 15000 years with the simplified coupled ECBILT model (Opsteegh et al., 1997 and Sellin et al., 1997). The oceanic component of this coupled model is a simplified version of the GFDL ocean GCM with flat bottom. The atmospheric model is a global quasigeostrophic model. Figure 14 shows the time series and the spectrum of the maximum stream function in the

![Figure 14. Time series and spectrum of the maximum of the zonally averaged stream function in the Atlantic, obtained from a 15000-year integration with the simplified coupled ECBILT model. The straight line in b) marks the fitted power law, which is close to \( \omega^{-1} \).](image-url)
Atlantic, which characterizes the northward mass transport of the Atlantic overturning circulation in the upper 1000 meters or so; 6000 years of the 15000-year integration, which does not obtain obvious drifts (Figure 14a), are considered. Since two chunks are used to estimated the spectrum, the longest resolvable time scale increases from 500 years for the millennium runs to 3000 years for the ECBILT integration. The spectrum (Figure 14b) can be approximately described by the power law \( \omega^{-1} \) over the time-scale range extending from a few decades to 3000 years. Figure 14b confirms the tendency of the continuous increase of spectral energy with decreasing frequency observed for deep-ocean mass transport in integrations with coupled full GCMs.

All three types of decadal variability have in common that they are not related to a single time scale. Instead, a spread of variance over a continuum of time scales, which reflects the temporal irregularity discussed in the introduction, is observed. Nevertheless, dominant time scale can be identified, in particular for variability associated with a propagation of anomalies in space. It is shown that the variability related to baroclinic Rossby waves in the Pacific reveals a well-defined spatial propagation on decadal time scales.

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