On the Variability of Fine-Scale Shear

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Abstract. Most of the (non-turbulent) shear in the sea is associated with motions of vertical scale 1-100 m. The dynamics which govern this field are strongly non-linear and poorly understood. To document the nature of upper ocean variability, a geographic survey is conducted. Using Doppler sonar, the shear field is observed at five sites globally (with Arctic and equatorial stations repeated). Equatorial shears (2°S, 156°E) have the greatest variance. Arctic Beaufort Sea data are the least energetic. When shear spectra are normalized by the Vaisala frequency the distinction between the Beaufort Sea observations and all the rest is enhanced.

Classical spectral scalings based on convective instability, and Kelvin Helmholtz instability, and wave-current interaction are examined. With the exception of the Beaufort Sea, shear spectral levels tend to rise with increasing wavenumber, for wavenumbers less than \( k_* = \frac{N}{U_{rms}} \) (intrinsic waves) and fall at higher wavenumber (compliant waves). Here, \( k_* \) is the wavenumber of the wave whose horizontal phase speed equals the rms horizontal velocity, \( U_{rms} \), of the overall wavefield.

Introduction

Gregg (1989) has noted that the mean rate of dissipation of mechanical energy, \( \epsilon \), in the thermocline can be related to the square of the variance of 10-m scale shear. The general success of his relationship suggests that there is indeed a cascade of energy from large to fine (1-100 m) to micro (<1 m) vertical scales and that, with proper parameterization, the process can be modeled accurately (e.g., Polzin, 1995). As the new scaling arguments are applied to a wider range of issues (e.g., D'Asaro and Morison, 1992), it becomes appropriate to take a closer look at the fine-scale shear field itself.

There is clearly a need to have a realistic statistical model of the shear field which advances beyond the notion of "universality" to describe the changing spectral form as a function of overall field variance. Such models have been suggested for the atmosphere by Smith et al. (1987), Fritts (1991), and in the ocean by Duda and Cox (1989). In a previous Aha Huliko'a conference, Müller et al. (1991) presented a unified spectral model, synthesizing existing atmospheric and oceanic information (Figure 1).

In this work we provide observations in support of the unification effort. Shear data are presented from seven sites, ranging from 83°N to 2°S latitude. The observations span a factor of 100 in internal wave energy density, a factor of 30 in shear variance, and a factor of 30 in the inertial frequency, \( f \). The measurements are obtained in the upper ocean. Variation in the magnitude of the Vaisala frequency, \( N \), is slight, relative to studies such as Polzin et al. (1995).

Methods

Data were obtained primarily with a 161-kHz Doppler sonar developed at the Marine Physical Laboratory of Scripps in the winter of 1988-1989. The sonar transmitted repeat sequence coded pulses (Pinkel and Smith, 1992), with a bandwidth of 5-10 kHz. The four beams of the sonar were orthogonal in azimuth, and directed 30° off vertical. For the 1995 coastal California shear observations, a 140-kHz system was employed. This sonar was technically similar to the earlier device, although the acoustic beam widths were significantly narrower (±1.5° vs ±2.5°). The sonars typically achieved ranges of 300-400 m.

At the small scales of interest here, a variety of noises and distortions contaminate the signals. The dominant sources of uncertainty are the imprecision in the velocity estimate and geometric distortion resulting from the finite resolution of the measurements. Theoretical performance bounds have been derived for sinusoidal transmissions (Miller and Rochwarger, 1972; Terriault 1986) and broadband coded pulses (Edwards, 1979, Pinkel and Smith, 1992). Velocity imprecision results from the limited "dwell time" of the acoustic signal in individual resolution cells. This "noise" is correlated spatially, with the correlation scale set by the duration of the transmitted pulse. Secondary sources of velocity imprecision include electronic noise in the sonar and distortions that occur as the signal is amplified, digitized, and processed. Nearly white noise in the velocity spectrum (as limited by the finite receive bandwidth of the sonar, not the bandwidth of the transmitted pulse) results from these real-world electronics problems. Motions of the platform on which the instrument is mounted produce an error which is uniform in range.

A number of important error sources are instrument-independent, associated with the definition of "the" velocity. Swimming acoustic scatterers and time changes in velocity that occur during the period in which a given
Figure 1. Schematized vertical wavenumber spectra of the vertical shear of internal gravity waves in the ocean, troposphere, stratosphere, and mesosphere, scaled to a common value of the buoyancy frequency N. The wavenumber power law ranges are indicated. The transitional wavenumbers for the ocean are the bandwidth m*, the cut-off or roll-off wavenumbers m_A, the buoyancy or Ozmidov wavenumber m_B = \left( \frac{N^2}{\nu} \right)^{1/2}, and the Kolmogorov dissipation wavenumber m_K = \left( \frac{\nu}{\epsilon} \right)^{1/4}, with \epsilon being the kinetic energy dissipation rate and the molecular viscosity \nu. The analogous wavenumbers for tropo-, strato-, and mesosphere are indicated by superscripts t, s, and m, respectively. The oceanic spectrum consists of a large-scale part (m > m*) which is not well established, an intermediate-scale part (m* < m < m_B) which is well described by the Garrett and Munk spectral model; a small-scale or "saturation" range between m_B and m*. The variability of the spectrum is indicated by the shading. The "saturation" range is much less variable than the other ranges. The atmospheric spectra also show a saturation range with the same spectral slope and level as the oceanic spectra. The line N^2/m represents theoretical predictions by Lumley (1964) and Holloway (1983) for buoyant turbulence or nonlinear wave interactions (from Müller et al., 1991).

Describes the individual beam variance, as well as geophysical signals whose horizontal spatial scales are comparable to or smaller than the horizontal separation of the beams. This vertical velocity difference spectrum is a viable standard for estimating the signal-to-noise ratio as a function of wavenumber and frequency.

To minimize the influence of noise, elemental one- or two-minute averaged profiles of echo covariance (whose phase is proportional to Doppler shift) are further averaged in time. Noise variance decreases linearly with increased averaging. Variance associated with signals that are unchanging in time does not decrease. A 20-min profile averaging time was selected for this study. In the process of minimizing the influence of noise, motions of short horizontal and vertical wavelengths, which are seen at high encounter frequencies, are also filtered from the data.

The depth-time measurements from the sonar are used initially to estimate wavenumber-frequency spectra of shear. These are formed from profiles of slant velocity that are rotated into horizontal zonal and meridional components by combining estimates from the various sonar beams at like depths. Depth regions in the records below or between maxima in N^2 are selected. No attempt is made to "WKB stretch" the observations prior to processing the data. The component profiles are differentiated in depth over a distance comparable to the nominal spatial resolution c(T - \tau)/2, as well as first differentiated ("pre-whitened") in time. Here, c is the speed of sound, T is the duration of the transmitted pulse, and \tau is the lag of the autocovariance used to estimate velocity. A time-mean shear is then removed, a triangle window is applied, and the data are Fourier transformed in time. In each frequency band, the depth mean is removed. Fourier coefficients are windowed and transformed in depth. Spectral estimates are formed from the wavenumber-frequency Fourier coefficients. These are "recolored" in frequency and corrected for the finite difference approximation to the true shear. A modeled \kappa^2 noise spectrum is then removed from the shear spectral estimate, presumably accounting for effects of electronic noise. The spectrum is then response corrected through division by \text{sinc}^4(\kappa c(T - \tau)/2. Finally, a second \kappa^2 noise spectrum is removed, corresponding to the fundamental imprecision of the velocity estimate, as well as environmental noises.

The various corrections have little effect on the spectrum at vertical scales greater than 10 m, significant effect at smaller scales. The two-stage noise removal process employed here, while formally correct, produces a resulting estimate which differs but little from estimates corrected by a single stage process.

Measurement Sites

Broadband sonar measurements of shear were first obtained north of Svalbard (83°S, 13°E, CEAREX) during
April 1989. Western Arctic (Beaufort Sea) measurements were made in spring 1992 (75°S, 150°W, LEADEX) and fall 1993 (75°S, 150°W, SIMI). Midlatitude observations were obtained from the Research Platform FLIP off California in the deep sea (1990, SWAPP, 35°S, 127°W) and at a coastal site (1995, MBL, 35°S, 122°W). Equatorial data were obtained during the winter of 1992-93, as an aspect of the TOGA COARE experiment (2°S, 156°15'E). For this work, the sonar was mounted under the hull of the R/V John Vickers.

Representative profiles of squared Vaisala frequency are presented for the Toga COARE (Nov-Dec 1992, R/V John Vickers, leg 1), SIMI, and CEAREX sites in Figure 2. Associated profiles of shear variance, calculated as a 2-, 5-, and 10-m first difference are also given. In Figure 2a,b, the variance profiles appear congruent to their Väisälä counterparts. This relationship holds generally for all cruises except CEAREX, (Figure 2c), where significant shear variance is seen above the region of maximum $N^2$.

**Shear Dependence on Differencing Interval**

We can investigate the dependence of shear variance on vertical differencing interval explicitly, forming the normalized structure function of horizontal velocity

$$S^2(\Delta z) = \frac{(u(z) - u(z + \Delta z))^2}{\Delta z^2}.$$ 

The structure function is related to the vertical wavenumber spectrum of shear by

$$S^2(\Delta z) = \int_0^\infty E(k) \text{sinc}^2(\pi k \Delta z) dk.$$ 

A white shear spectrum corresponds to a $\Delta z^{-1}$ dependence in $S^2(\Delta z)$. Structure functions, further normalized by $N^2$ for each cruise, are presented in Figure 3.

The more energetic records tend to approach a $\Delta z^{-1}$ form at large separations. This replicates the behavior of finite difference strain (Pinkel and Anderson, 1992), and is consistent with a $k^0$ low wavenumber dependence for shear and strain, a $k^{-2}$ dependence for velocity and vertical displacement.

**Figure 2.** Profiles of shear variance obtained at 2, 5 and 10 m vertical differencing intervals (left) and Vaisala frequency (right) from (a) the first (November - December, 1992) leg of Toga COARE, (b) the Arctic experiments SIMI (Beaufort Sea, 1993), and (c) CEAREX (Amundsen Basin, 1989). The smoothed representation of the Väisälä frequency was used in normalizing data and displaying results.
Shear Dependence on Stratification

The general congruency in profiles of \( \langle S^2 \rangle \) and \( \overline{N^2} \) (Figure 2a,b) can be explored in greater detail. In Figure 4, shear (a) and velocity (b) variance are plotted vs. \( \overline{N^2} \). In the WKB approximation for linear internal waves, \( \langle S^2 \rangle \sim \overline{N^2}^{-3/2}, \langle u^2 \rangle \sim \overline{N^2}^{-1/2} \). The more common experience for shear is \( \langle S^2 \rangle \sim \overline{N^2} \). Reference lines for the latter two relations are plotted.

When viewed in detail the relationship between \( \langle S^2 \rangle, \langle u^2 \rangle \) and \( \overline{N^2} \) is surprisingly complex. In general, the velocity data are in a more consistent relationship with \( \overline{N^2} \) than the shear. This is perhaps surprising, in that the propagation of smaller vertical scale waves should be better described by a WKB model than the propagation of large scale waves. Perhaps these waves are responding to refractive influence in addition to \( \overline{N^2} \), such as mean currents. Currents affect small scale (slowly propagating) waves more than large. The CEAREX data (far left) show the least consistent relationship between \( \overline{N^2} \) and both shear and velocity. There is strong subinertial forcing at this site (Padman et al., 1992). Perhaps this has an effect (Polzin et al., 1995).

Vertical Scale Dependence on Stratification

One can estimate a scale vertical wavenumber from the ratio of rms shear to rms velocity. In the WKB approximation, the estimate should vary as \( \overline{N^2}^{-1/2} \). In Figure 5, estimates of scale wavenumber are presented for shear vertical differencing intervals presented for a vertical differencing interval of 6 m. The CEAREX results again appear at low \( \overline{N^2} \), the equatorial results at high. In general, the magnitude of the scale wavenumber varies inversely with the overall \( \langle S^2 \rangle / \overline{N^2} \) variance level, as given in Figure 2. The Beaufort Sea observations cluster at the highest values of scale wavenumber, corresponding to vertical wavelengths of 25 m.

The Vertical Wavenumber Spectrum of Shear

In Figure 6a, vertical wavenumber spectra of shear are presented for the seven sites. These have been derived from Eulerian wavenumber-frequency spectra through integration from 0.5f (or 1/3 cpd for the COARE measurements) to 1.5 cpd. The TOGA equatorial shears have the greatest variance density, a factor of 3-10 greater than typical midlatitude observations. The Beaufort Sea spectra are least energetic, nearly two orders of magnitude
smaller than their equatorial counterparts. They also appear shifted to significantly higher wavenumber, peaking at 10-20 m scales as opposed to the 30-50 m spectral maxima seen on the equator.

The low wavenumber portions of all spectra are limited by the finite aperture of the observation. Variance from large (>200 m) vertical scales leaks into these spectra in spite of steps taken to window the data properly. All spectra tend toward an approximate $k^{-1}$ slope in this region. This form is seen more reliably in the shear spectral estimates of Pinkel (1985) and Sherman and Pinkel (1991), where a lower frequency (~75 kHz) sonar with significantly greater range was used.

A sense of the representativeness of these estimates is given by the repeat equatorial (COARE I - Fall 92, COARE III - Spring 93) and Beaufort Sea (LEADEX - Spring 92, SIMI - Fall 93) observations. Significant differences are seen in the spectral estimates at low wavenumber, presumably associated with secular change in the fine-scale environment. These differences diminish with increasing wavenumber. The Beaufort Sea spectra are indistinguishable at all wavenumbers greater than the spectral peak ($k > 0.06$ cpm). The equatorial spectra become comparable at nearly the same wavenumber ($k > 0.06$ cpm). At this point, however, the spectral levels are 10 dB below peak level.

In Figure 6b, the shear spectra are normalized by the mean squared Vaisala frequency, $N^2$, over the analysis depth. These have been referred to as Froude spectra (Gregg and Kunze, 1991; Gregg et al., 1993) or inverse Richardson spectra (Munk, 1981). When so normalized, the Garrett-Munk model for the vertical wavenumber spectrum of shear (Munk 1981; Gregg and Kunze, 1991) is nearly invariant with depth. The normalization separates our global observations into two groups: the Beaufort Sea - and everywhere else. Indeed, the Arctic observations, CEAREX, LEADEX, and SIMI, bound the collection above and below.

Figure 5. Plots of scale vertical wavenumber $k = \left(\frac{\langle S^2 \rangle}{\langle U^2 \rangle}\right)^{1/2} / 2\pi$ vs $N^2$.

Figure 6. Vertical wavenumber spectra of shear for the seven sites (a, c), and $\left(\frac{\langle S^2 \rangle}{N^2}\right)$ (b). The solid curves in (c) are log normal functions, with variance, first and second moments identical to the observed spectra.
At high wavenumber, the spectral decay varies from less than $k^{-1}$ for the Beaufort Sea to greater than $k^{-2}$ in the Eastern Arctic. For the energetic spectra, the slope is significantly steeper than the canonical $k^{-1}$ first suggested by Gargett et al. (1981) and reflected in Figure 1. In part, this is due to the (extreme) 20-minute averaging time employed here to minimize the influence of instrument noise. During the averaging interval, the shear field is advected vertically by ambient motion in the thermocline. At fixed depth, an apparent time variability is seen, with smaller vertical scales associated with more rapid temporal variation. Thus, temporal averaging preferentially reduces the variance at small vertical scale, increasing the apparent spectral slope.

Gregg et al. (1993) suggest that the high wavenumber spectral slope steepens at low latitude. Their low latitude observations are significantly more energetic than the GM norm. When comparing the Beaufort Sea with other sites, the indication is that high wavenumber spectral slope steepens with increasing spectral level, independent of latitude. However, it is reasonable to conjecture that the Beaufort Sea operates in a significantly different dynamical regime than the rest of the world, at fine scale. Disregarding the Beaufort observations, there is still weak evidence for systematic variation of spectral slope with spectral level.

The non-Beaufort spectra tend to peak at scales between 30 and 100 m. The Beaufort Sea spectral peak is quite broad and generally shifted to shorter vertical scales (15-30 m). The shift is, to an extent, mirrored in the vertical scale of the dominant near inertial waves observed at the various sites. In general, Beaufort Sea near-inertial packets have short ($k \sim 0.04$ m$^{-1}$) vertical scales (e.g., Merrifield and Pinkel, 1996), midlatitude wave packets have larger scales (Pinkel, 1983), and equatorial packets have the largest. Is this a result of energy level or latitude? The 84°S high energy CEAREX data might hold the key here. Unfortunately, the wavefield is strongly forced at sub-inertial frequencies. It is far from a typical high latitude “open ocean” site.

In Figure 6c, we plot selected shear spectra on a linear scale. A “log-normal” model spectrum:

$$ \hat{E}(k) = \frac{A}{k} e^{-\left[\ln(k/k_0)\right]^2/\sigma^2} $$

is “blind fit” to the data by forcing the model variance, first and second moments to match the observations over the range $k = 0.01 - 0.1$ m$^{-1}$. While the fit is striking, the particular choice of a log-normal functional form is not significant, given the influence of the 20-minute averaging on our spectral estimates at high wavenumber. Figure 6c does suggest that there is room to improve the canonical model of the shear spectrum, which is constant at wavenumbers below $k = 0.1$, and of $k^{-1}$ form above (Gargett et al., 1981). The spectral forms here reinforce the suggestion of Pinkel (1975) that strain and shear spectra can be viewed as a distinct high wavenumber spectral shoulder on the wavenumber spectra of vertical displacement and velocity.

### Scaling the 1-d Shear Spectrum

Gargett et al. (1981) and Munk (1981) have noted that the vertical scale (about 10 m) at which the shear spectrum begins its decay roughly corresponds to the scale at which the Richardson’s function

$$ \bar{R}_i(k) = \frac{N^2}{k} \left[ \int_0^k E(k')dk' \right]^{-1} $$

approaches unity from above. The suggestion is that the high wavenumber decay of the spectrum is related to Kelvin Helmholtz or convective instability of the fluid. The Richardson function is a useful metric of these processes. Subsequently, Duda and Cox (1989) have found that this relation is maintained over a range of spectral levels.

Polzin et al. (1995) select a scale wavenumber $k_p$, where the Richardson function reaches the value 1.42. They normalize wavenumbers by $k_p$, and scale spectral levels such that $k_p E(k) = 0.7 \bar{R}_i(k_p)$. The normalization enforces uniformity of spectral level at low wavenumber, enabling comparison of the spectral details in the cut-off region.

With the present data, $k_p$ occurs at sufficiently high wavenumber that the spectral estimates are poorly known. We can explore the Polzin et al. scaling, redefining $k_p$ such that $\bar{R}_i(k_p) = 2.5$. The spectral estimates, so rescaled, are presented in Figure 7a. The scaling does not significantly decrease the variability of the energetic spectra, as seen in Figure 6b. It does serve to emphasize the difference between the Beaufort Sea observations and all the rest. Shear and convective instability presumably play a small role in the Beaufort Sea.

Smith et al. (1987) and Fritts (1991) have proposed an alternative scaling, with kinetic energy rather than shear taken as the governing factor. Convective instability is hypothesized to be the dominant mechanism. Smith et al. suggest a saturation threshold $E_F(k) = 2 \pi^2 N^2 / k$ based on the self-instability of isolated wave packets, of bandwidth $\Delta k = k$.

Their associated model spectrum is

$$ E_F(k) = \frac{4}{\pi} \frac{E_o}{k_F} \left[ \frac{k_F}{k} \right]^3 \left[ \frac{k_F}{k} + 1 \right] $$

where $k_F \equiv \left( \pi N^2 / 24 E_o \right)^{1/4}$ (Fritts, 1991).
Munk (1981) explores this same spectral scaling from a different perspective. As in Smith et al. (1987), he notes that the horizontal phase speed, \( c_p = \sigma / k_H \), of linear internal waves is given by \( c_p \sim N / k \) over frequencies removed from \( f \) and \( N \). Thus, vertical wavenumber can be used as a surrogate for phase speed. Rather than focus on convective instability per se, Munk emphasizes propagation issues. The vertical propagation of fine-scale waves is significantly impeded when fluctuations in background horizontal currents are of the same order as the phase speed of the wave (e.g., Booker and Bretherton, 1967). If the “background” currents are due to larger scale waves in the wavefield, propagation is impeded for waves of

\[
k \geq k_* = \frac{N}{U_{\text{rms}}}\]

where

\[
U_{\text{rms}} = \left( \int_0^\infty (2\pi k)^{-2} E(k) dk \right)^{1/2}
\]

Scaling vertical wavenumbers by \( k_* \) and the spectrum \( E(k)/N^2 \) by \( k_*^{-1} \), Figure 7c is obtained. The relative positions of the various spectra are unchanged from 7b. However, the energetic spectra are seen to peak at \( k/k_* = 1 \), the boundary between Munk’s intrinsic (\( c > U_{\text{rms}} \)) and compliant (\( c < U_{\text{rms}} \)) portions of the wavefield. At higher wavenumbers, wave propagation speeds are less than typical horizontal velocities. The spectrum decays. Again, the Beaufort Sea is in a separate class, with the waves, short as they are in vertical wavelength, propagating too fast to be impeded by wavefield currents. There is building evidence, however, that interaction with subinertial currents can influence this wavefield significantly (Merrifield and Pinkel, 1996).

If the “background waves” indeed propagate in accord with linear (shear-free) theory in the WKB approximation, then

\[
U_{\text{rms}} = \left( E_o N(z)/N_o \right)^{1/2},
\]

where \( E_o \) is an energy dependent constant that does not vary in depth (in contrast to the Fritts usage, where \( E_o \sim N \)). Thus, \( k_* \sim N^{1/2} \) and overall shear variance \( \langle S^2 \rangle \sim k_*^2 \langle U^2 \rangle \sim N^2 \langle z \rangle \).

This propagation oriented model provides a rationale for the observed \( \langle S^2 \rangle \sim N^2 \langle z \rangle \), without explicit reliance on a Richardson number based instability mechanism.

**Discussion**

Observations of upper ocean velocity and shear have been obtained at seven sites ranging from 83°S to 2° S. The individual observations demonstrate a factor of 100 variation in velocity variance, a factor of 30 variation in shear variance. There is generally poor correspondence
between profiles of shear variance and $\bar{N}^2(\sigma)$, in conflict with a simple WKB model of propagating waves. Yet when $\bar{N}^2$ is used to normalize the global collection of vertical wavenumbers spectra, it has a significant effect. The $\bar{N}^2$ scaled observations are divided into two groups: the Beaufort Sea and "everywhere else". The suggestion is that fine scale internal waves are refracted at least as much by current variations as by $\bar{N}^2(\sigma)$. However, the Väisälä frequency still plays a key role in other aspects of dynamics, such as wave instability.

The shear spectra generally exhibit a $k^4$ form at low wavenumber ($-0.005 < k < 0.02$), and a $k^2$ form at high ($k > 0.1$ cm). There is some indication that the high wavenumber slope steepens with increasing spectral level.

The high wavenumber slope is steeper than in most previous observations (e.g., Gargett et al., 1981) and is influenced by the 20-min averaging time imposed here. Much of the shear at scales smaller than 10 m is associated with high (encounter) frequency motions. Given our experience with shear in the Beaufort Sea (where advection effects are small) and with strain in mid-latitude oceans, we suggest that these motions have high intrinsic frequency as well.

The low wavenumber rise in the spectrum is frequently seen when the shear field is observed in both depth and time, and sub-inertial constituents of the field can be removed.

There is no clear signature of variation in the Coriolis frequency on the wave spectra.

Of the several spectral scalings examined, Munk's (1981) propagation-based model is most appealing. Shear spectral levels increase for $k < k_*$, $c/U_{rms}$ (intrinsic waves) and decrease for $k > k^*$ (compliant waves). Ever the exception, the Beaufort Sea spectra appear to decrease at larger scales than would be predicted by the $c/U_{rms}$ criteria. Clearly, more remains to be understood.

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