AN ADAPTIVE INVERSE METHOD
FOR MODEL TUNING AND TESTING

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ABSTRACT

To determine the value of the adjustable parameters of an ocean model that are required to optimally fit the observations, an adaptive inverse method is developed and applied to a sea surface temperature (SST) model of the tropical Atlantic. The best-fit calculation is performed by minimizing the misfit between observed and simulated data, which depends on the observational and the modelization errors. An adaptive procedure is designed where the model that is being tuned is also used to construct a sample estimate of the observational error covariance matrix. Assuming idealized modelization errors, the procedure is applied to the SST model of Blumenthal and Cane (1989), yielding improved estimates for several model and heat flux parameters. The tuned model provides a better simulation of the mean annual SST, but the model's ability to represent the seasonal and the interannual variability is not improved, and the model-observation discrepancies remain too large. The existence of larger model deficiencies than was originally assumed in the model errors is confirmed by a statistical test of the correctness of the assumptions in the inverse calculation.

1. INTRODUCTION

All oceanic models contain parameterizations of such physical processes as convection and mixing. Surface forcing also depends on poorly known parameters. Parameterizations are based on physical ideas, but typically yield forms that contain parameters whose values are not known precisely. A parameter is often model dependent (e.g., mixing is a function of grid spacing), hence parameter tuning may be in part model dependent. In view of their inherent imprecision, the uncertain parameters should be tuned against observed data. At the same time, models should be consistent with known physics to within the tolerances allowed by the approximations made.

Particularly in the tropics where observations are sparse, both forcing and verification data are imprecisely known. Hence, the accuracy to be expected in model simulations is limited, even if the physics are perfectly represented, and data uncertainties should be taken into account in parameter tuning. Frankignoul et al. (1989) have developed a multivariate model testing procedure that provides an objective measure of the fit between ocean
model simulations and observations, taking into account the data uncertainties. By using a trial and error approach, the method can be used for model tuning (Duchêne and Frankignoul, 1991; Bracnot and Frankignoul, 1993). However, this requires that the number of adjustable parameters is small.

A more efficient tuning approach is that of Blumenthal and Cane (1989), who used inverse modeling procedures to determine the parameter values required to optimally fit sea surface temperature (SST) in a simplified tropical SST model. A priori knowledge constraining the parameter range was included in the calculation, but only a highly idealized model was used for the data errors. The error model enters the measure of the misfit between observed and predicted data which is minimized in the best-fit calculation. Thus, the atmospheric forcing uncertainties need to be properly represented, as they introduce large uncertainties in the model response.

As the forcing uncertainties have large and poorly known correlation scales, the error estimates are best derived from direct simulations. We have thus developed an adaptive tuning procedure, where the model that is being tuned is also used to construct the observational error model for the best-fit calculation. The tuned model is then tested against observations, and if it agrees with the data to within expected errors, it will be judged adequate. Such an adaptive technique combines the model tuning of Blumenthal and Cane (1989) and the model testing of Frankignoul et al. (1989). Although the procedure is developed in the context of a simplified tropical sea surface temperature model, it is general as long as the parameter dependence is linear. The adaptive procedure requires little computation and programming, and is much simpler to implement than the adjoint method. However, since the effective degrees of freedom of the error estimates is limited by the length of the sample, the number of parameters that can be tuned is limited.

The emphasis here is on the adaptive inverse procedure, although it is introduced in the context of a tropical SST model. An in-depth discussion of the results is given in Scoffier et al. (1993).

2. MODELING SEA SURFACE TEMPERATURE VARIATIONS

a. Ocean model and surface heat flux

The ocean model is that of Blumenthal and Cane (1989, hereafter BC). The velocity is predicted with a linear, multimode equatorial beta-plane model with a surface mixed layer of constant depth $h=35$ m, which adds a direct Ekman flow to the modal currents. The model has five vertical modes, which are characteristic of mean tropical Atlantic conditions and have gravity wave speed of 2.36, 1.38, 0.89, 0.69 and 0.53 m/s, respectively. The model basin extends from 30°N to 20°S and has a simplified geometry; its resolution is 1° in longitude and 0.5° in latitude and the time step is one week. The equations are solved in the longwave approximation, so that the model is only appropriate
away from the western boundary. In the following, we only consider the domain in Figure 1, which should not be affected by the model artificial boundaries.

The SST is uniform in the mixed layer and determined from the net balance of horizontal advection, upwelling, horizontal diffusion, and surface heat exchanges:

$$\partial_t T + u \partial_x T + v \partial_y T + \frac{\gamma w (T - T_d)}{h} = \kappa (\partial_{xx} + \partial_{yy}) T + \frac{Q}{\rho C_p h}$$  \hspace{1cm} (1)

where $w$ is the vertical velocity at the mixed layer base in case of entrainment and zero otherwise, $\kappa$ a horizontal diffusion coefficient and $Q$ the surface heat flux into the mixed layer, and $T_d$ the temperature below the mixed layer which is parametrized as a function of the thermocline depth. As in BC, the parameterization of $T_d$ is done in two parts: First the temperature at the mixed layer base is fit to the depth of the 20°C isotherm in the equatorial zone using observations, then the 20°C isotherm depth is fit to the model prediction of the thermocline depth. The upwelling term is usually written as $w(T - T_e)$, where $T_e$ is the temperature of the water entrained into the mixed layer, but the two forms are equivalent if

$$T_e = (1 - \gamma) T + \gamma T_d$$  \hspace{1cm} (2)

where the “entrainment efficiency” $\gamma$ is an adjustable parameter that should be less than one, because $T_e$ is between $T$ and $T_d$.

The surface heat flux parameterization is that of Seager et al. (1988, henceforth SZC), which was designed to avoid using either the (poorly measured) air-sea temperature differences found in the bulk formulae or the artificial feedback to a prescribed climatological air temperature often imposed in ocean simulations. This parameterization only makes use of wind speed $v^a$ and fractional cloud cover $C$ as measured variables:

$$Q = 0.94 Q_0 (1 - a_c \alpha + a_\alpha \alpha) - \rho C_p L v^a a_{rh} q_s(T) - a_f (T - T_r).$$  \hspace{1cm} (3)

The first term is the (usual) short wave radiation, where $Q_0$ is the clear sky solar flux reduced by the effects of a constant surface albedo and by the absorption and reflection of the atmosphere, which depends on $C$ and solar angle $\alpha$. The second term represents the latent heat flux, which is computed from the standard bulk formula using a fixed percentage $a_{rh}$ of the saturation humidity $q_s(T)$ as evaporation potential; this assumes that the moisture content of the air has equilibrated with the ocean temperature, which is a reasonable assumption sufficiently far from the coasts. To compensate for the loss of variability in using monthly winds, $v^a$ is not allowed to fall below 4 m/s. The smaller sensible heat flux and back radiation are simply modeled together in the last term as being proportional to $T$ minus a constant reference temperature $T_r$. 
In the SST equation and the heat flux formulation, there are a number of parameters that are not precisely known, but were assigned a "reasonable" value by SZC. Here we assume that seven parameters are adjustable within reasonable ranges: the entrainment efficiency \( \gamma \), the horizontal diffusion \( \kappa \), and the heat flux parameters \( a_c, a_a, a_h, a_f, \) and \( a_f T_f \) in (3), which we represent below by the seven-dimensional vector \( \mathbf{a} \). The a priori values of the tunable parameters, denoted by \( \mathbf{a}_p \), are those of SZC, namely \( \gamma = 0.5 \), \( \kappa = 2 \times 10^8 \) m\(^2\) s\(^{-1}\), \( a_c = 0.62 \), \( a_a = 0.0019 \), \( a_h = 0.3 \), \( a_f = 1.5 \) W m\(^{-2}\) K\(^{-1}\) and \( T_f = 273.15 \) K. The drag coefficient for the wind stress is not allowed to vary because its uncertainty is simulated explicitly.

b. Simulation of the tropical Atlantic SST climatology

After spin-up, the model is forced by a monthly wind stress derived from ship reports for the period 1964-1986 and described in Frankignoul et al. (1989, henceforth FDC). To simulate the drag coefficient uncertainty, we follow the Monte Carlo approach of Braconnot and Frankignoul (1993) and use five different, equally plausible drag coefficients in the bulk formula. They are calculated by prescribing a relative humidity of 80\% and using either a constant air-sea temperature difference of \(-1^\circ\)C (for the parameterization of Cardone (unpublished manuscript)), or a climatological monthly air-sea temperature difference derived from the COADS data (for the parameterizations of Liu et al. (1979), Large and Pond (1981), Isemer and Hasse (1987), and Smith (1988)). To avoid smoothing, the monthly mean wind stresses were corrected to insure that linear interpolation on the model time step would not alter the original means. Cloudiness data are of poorer quality, so that cloud cover is prescribed from the monthly climatology of Esbensen and Kushnir (1981), with an added normal noise of 0.1 standard deviation to crudely simulate its short space-time scale variability.

Ignoring the first year to eliminate the effects of the unknown initial conditions, we have five 22-year simulations of the SST whose dispersion is representative of both the interannual variability and the drag coefficient uncertainty. The mean cycle of simulated SST is warmer than the observations, as illustrated in Figure 1 for January, April, July, and October by a comparison with the mean SST over the same period calculated from the data of Servain et al. (1985).

The differences between the SST predictions and the observations are due to (a) errors in the atmospheric forcing (wind stress, cloud) and the SST observations, (b) model shortcomings due to over-simplification of the physics, or (c) poor choice of the model parameters. To assess the validity of the SST model, we must take (a) into account and minimize (c) by an optimal tuning. Remaining discrepancies should then point to the model deficiencies (b).
Figure 1. (left) Mean SST in °C during January, April, July and October for the period 1965-1986 as predicted using the a priori values of the model parameters. (center) Corresponding SST as derived from the observations by Servain et al. (1985). (right) Differences between simulations and observations.

Root-mean-square (rms) SST differences between model and observations on the 2° × 2° grid of the latter are given in Table 1 (left column), where we distinguish between annual mean, mean seasonal variations around the annual mean (hereafter the mean seasonal variability), and SST anomalies. The model-observation differences are large, particularly for the long term mean which is strongly affected by a 3.9°C mean bias.

A more quantitative estimation of the model performances taking into account some of the uncertainties in the oceanic observations and the atmospheric forcing, as well as their space-time correlations, has been made for the mean seasonal cycle obtained with Cardone’s drag coefficient. Following the multivariate approach of FDC, we calculate the misfit

\[ T^2 = (\vec{T} - \vec{T}_0)'D^{-1}(\vec{T} - \vec{T}_0), \]  

(4)
where $\bar{T}$ and $\bar{T}_0$ describe the mean seasonal cycle of modeled and observed SST, respectively, the vector space including all grid points (on the observational grid) and the twelve months. The overbar denotes the 22-year mean, the prime denotes the vector transpose, and $D$ is the error covariance matrix of $(\bar{T} - \bar{T}_0)$. In the calculation reported here, $D$ is estimated from the five 22-year samples, assuming for simplicity that each year is statistically independent. It takes into account the uncertainties in the mean seasonal variations that are due to interannual variability, non-systematic observational errors of SST, wind, and cloud cover. Not represented in $D$ are systematic observational errors (e.g., incorrect Beaufort scale), drag coefficient uncertainty, lack of high frequency variability, and limited resolution of the wind stress curl. As the dimension of the SST field is much larger than the degrees of freedom of $D$, the misfit (4) is calculated in a truncated space which is sufficiently small to calculate $D$ reliably while representing the main space-time patterns of $(\bar{T} - \bar{T}_0)$.

### Table 1: Rms difference in °C between observed and modeled SST before and after tuning in the 20°N-10°S region. The correlation between observed and simulated monthly anomalies during 1965-86 is given in italic.

<table>
<thead>
<tr>
<th>(SST_{mod} - SST_{obs})</th>
<th>before tuning</th>
<th>after tuning</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual mean</td>
<td>4.0</td>
<td>1.9</td>
</tr>
<tr>
<td>seasonal variability</td>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>anomaly correlation</td>
<td>0.13</td>
<td>0.10</td>
</tr>
</tbody>
</table>

If the SST fields are multinormal, the null hypothesis that the model response to the true forcing is equal to the true SST can be tested because the test statistic (4) is then Hotelling's $T^2$ statistic. As shown in Table 2, $T^2$ is much larger than the critical value at the 5% level, especially for the yearly mean difference. Although only part of the observational errors have been considered in the test, the data uncertainties are clearly insufficient to explain all the model-observation discrepancies, which must be mainly attributed to model shortcomings and poor parameter tuning.
Table 2: Misfit between model and observations in the 20°N-10°S region, before and after tuning. The critical values for rejecting the null hypothesis of no modelization error are given for the 5% level (right).

<table>
<thead>
<tr>
<th>Misfit</th>
<th>before tuning</th>
<th>after tuning</th>
<th>critical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>annual mean</td>
<td>906</td>
<td>277</td>
<td>4</td>
</tr>
<tr>
<td>seasonal variability</td>
<td>2012</td>
<td>1694</td>
<td>73</td>
</tr>
</tbody>
</table>

3. AN ADAPTIVE PROCEDURE FOR MODEL TUNING

a. Linear model corrections

To see how the tunable parameters enter the SST calculation, it is convenient to write equation (1) in matrix form

\[
\mathbf{L}(T) + \mathbf{M}(T)\mathbf{a}_p = 0
\]

(5)

where the vector \( \mathbf{T} \) represents temperature at all the points in space and time where a model solution has been obtained, \( \mathbf{a}_p = (\gamma, \kappa, a_x, a_y, a_h, a_r, a_r T_r) \) is the vector of a priori parameter values, \( \mathbf{M}(T) \) and \( \mathbf{L}(T) \) are linear operators determined at all space/time points by retaining the terms of the model equations (1) and (3) that are and are not affected by parameter changes, respectively. Specifically, the \( i \)\textsuperscript{th} row of \( \mathbf{L}(T) \) includes the contribution at space/time point \( i \) from

\[
\partial_x T + u \partial_x T + v \partial_y T - 0.94 Q_0,
\]

while the \( i \)\textsuperscript{th} row of \( \mathbf{M}(T) \) correspondingly represents the transpose of the terms

\[
\begin{bmatrix}
\frac{w(T - T_0)}{h} \\
-(\partial_{xx} + \partial_{yy})T \\
0.94 Q_0 C \\
-0.94 Q_0 \alpha \\
-\rho C_L \nu v q_i(T) \\
T \\
-1
\end{bmatrix}
\]
Both $L$ and $M$ depend on the atmospheric forcing, which is imperfectly known, so that even if the model was perfect and the uncertain parameters optimally chosen, the model predictions would differ from the observations.

Because SST is a relatively well-measured variable, we follow BC and estimate the “corrective heat flux” $\delta q$ that, for the a priori values of the uncertain model parameters, would be necessary to make the model SST match the observed SST exactly. To do so, we run the model using the observed SST, denoted by $T_o$, instead of the calculated one, after interpolation on the model grid. Equation (5) is then only satisfied by adding a “heat flux correction” $\delta q$:

$$L(T_o) + M(T_o) a_p + \delta q = 0 \quad (6)$$

As expected from the limited SST agreement, the heat flux correction $\delta q$ is rather large, and additional cooling would be needed for realistic simulations (Fig.2a).

Because $\delta q$ depends linearly on the tunable model parameters, the estimation of their optimal value can be formulated as the linear inverse problem

$$\delta q = M(T_o) \delta a, \quad (7)$$

where $\delta a = (\delta Y, \delta k, ..., \delta a_L T_p)$ represent the parameters changes that minimize the heat flux correction $\delta q$, yielding

$$\delta q_{\text{min}} = \delta q - M(T_o) \delta a, \quad (8)$$

A good estimator of $\delta a$ must take errors into account, as well as our knowledge of the expected parameter range.

There are many sources of errors in the estimates appearing in (7). The wind stress and the cloud data used to force the model have significant errors, resulting in model response uncertainties with large correlation scales, particularly in the equatorial waveguide. The observed SST is noisy as well, although to a lesser extent. When the best-fit calculation is based on a mean seasonal cycle as in this paper, there are also sampling errors which reflect the interannual variability and have large correlation scales. Finally, there are “irreducible” modelization errors inherent in the ocean model formulation, e.g., errors due to subgridscale phenomena or to the oversimplification of the ocean dynamics and the air-sea fluxes, which cannot be expected to be reduced by model tuning. The modelization errors (called system errors in the Kalman filter literature) thus represent the errors that would exist if there were no observational errors and the uncertain parameters were at their true value.

Using a Bayesian viewpoint, Tarantola (1987) discusses the general inverse problem in the case of an inaccurate theory. When the forward problem is linear as in (7) and there are Gaussian modelization errors in $M$, described by the covariance $C_T$, the solution of the
inverse problem takes a simple form if the observational errors in \( \delta q \) are Gaussian and statistically independent from the modelization errors. If the a priori value of the parameter correction \( \delta a \) is zero, as in the present case, the optimal solution is given by the minimum of the misfit function

\[
S(\delta a) = \frac{1}{2} \left( (M\delta a - \delta q)' C^{-1} (M\delta a - \delta q) + \delta a' C_a^{-1} \delta a \right)
\]

(9)

with \( C = C_r + C_d \), where \( C_d \) is the error covariance matrix of the observations \( \delta q \), and the covariance matrix \( C_a \) describes the a priori uncertainty of \( \delta a \). The solution is

\[
\delta a = (M' C^{-1} M + C_a^{-1})^{-1} M' C^{-1} \delta q.
\]

(10)

BC followed this formalism, assuming for simplicity that the observational noise only affected the model matrix \( M \), and the modelization error only the heat flux correction \( \delta q \). On the basis of order of magnitude estimates, they used a constant rms error of 35 W/m² (10 W/m²) with a simple exponential decay for the total (modelization) errors. There are a number of simplifications in this approach. As shown by (6), both \( \delta q \) and \( M \) depend on the input data (e.g., the surface wind stress affects both the heat exchanges and the ocean dynamics), hence they are both affected by data uncertainties and modelization errors. The errors in \( \delta q \) and \( M \) are thus not statistically independent, and the model matrix really is a stochastic regression matrix. Unfortunately, ordinary and generalized least squares estimators are in general not consistent in this case of nonlinear coupling between model and data errors. The error models used by BC are also highly idealized. Since the results of the tuning are sensitive to the assumed error models, we adopt a more elaborate strategy to achieve a refined estimate.

b. The adaptive procedure

The correlation scales of the model response errors due to forcing and SST uncertainties are large and complex, hence difficult to represent a priori. However, they can be estimated by using the different wind stress products and the long SST time series of section 2b, since many plausible realizations of the model seasonal response are available. We thus perform the optimization on the mean seasonal cycle, which is least noisy, and use the dispersion of the model seasonal responses as independent information to construct a more realistic model for the observational errors.

Assuming that the parameters do not vary in time, we can write for each year \( t \) (here \( t = 1, 22 \)) and for each forcing \( i \) (here \( i = 1, 5 \)), denoted by the upper index, that the linear model (7) holds:

\[
L^t,i(T_0^t) + M^t,i(T_0^t) a_p + \delta q^t,i = 0.
\]

(11)
Denoting long-term sample means by an overbar and the mean over the different forcing by an angle brace, we write relation (6) under the form
\[
< \Delta L(T_0) > + < \Delta M(T_0) > a_p + < \Delta q > = 0.
\] (12)

The errors in (11) and (12) are due to forcing and SST uncertainties, and to model inadequacies.

Let us write the parameter estimation as the linear statistical model
\[
< \Delta q > = < \Delta M > \delta a + < \bar{e} >
\] (13)
where \(< \bar{e} >\) represents the errors, which are assumed to be Gaussian, with zero mean and unknown true covariance matrix \(C\). Because of the statistical dependence between \(< \Delta q >\) and \(< \Delta M >\), an estimate of \(\delta a\) is required before one may estimate the random errors from the sample. Thus, an adaptive approach is used, where the estimates of the observational error covariance and the model parameters are updated as part of an iterative procedure. If we have a first estimate of \(\delta a\), say \(\delta a_0\), which we will take equal to zero, then we can estimate for each year \(t\) the mean error over the different forcing, \(<e_l^t >\), by
\[
<e_l^t > = < \Delta q^t > - < \Delta M^t > \delta a_0.
\] (14)

A first sample estimate of the error covariance matrix associated with the random wind, cloud and SST errors is
\[
S_{rl} = \frac{1}{21 \times 22} \sum_{t=1}^{22} (<e_l^t > - < \bar{e}_l >)(<e_l^t > - < \bar{e}_l >)'
\] (15)
where we have assumed for simplicity that observations are independent at yearly intervals. We can also estimate for each forcing \(i\) the long-term mean error, \(\bar{e}_l^i\), by
\[
\bar{e}_l^i = \overline{\Delta q}^i - \overline{M}^i \delta a
\] (16)
and a first sample estimate of the error covariance matrix associated with the drag coefficient uncertainties is
\[
S_{rl} = \frac{1}{4 \times 5} \sum_{i=1}^{5} (<e_l^i > - < \bar{e}_l >)(<e_l^i > - < \bar{e}_l >)'
\] (17)
A first sample estimate of the error covariance associated with the observational uncertainties, say \(S_{dl}\), can then be obtained by
\[
S_{dl} = S_{rl} + S_{rl}
\]
and it can be used to compute an estimated generalized least squares estimate of \(\delta a\), say \(\delta a_1\). As in (10), we incorporate the modelization errors and our a priori knowledge on the model parameters,
\[ \delta a_1 = (\overline{M'} S_1^{-1} \overline{M} + C_n^{-1}) \overline{M'} S_1^{-1} \overline{\delta q}. \]  

(18)

with

\[ S_1 = S_{d1} + C_T. \]  

(19)

The procedure is repeated by using \( \delta a_1 \) in (14) to get an improved estimate \( S_2 \), leading to the parameter correction \( \delta a_2 \), and so on. If \( \delta a_0 \) represents a reasonable first guess and if the inverses in (18) are well-conditioned, the procedure should converge rapidly. The end result is a data error structure consistent with the results of the multi-year model run, and thus presumably a better parameter estimation.

The error model \( S_n \) represents most of the nonsystematic data and model errors; it also includes such data errors as artificial trends in wind and SST data. The true interannual variability is not treated as an error since it appears in both \( \delta q^f \) and \( M^f \) in (14). The weighting in the least squares fit is therefore based on data noise and uncertainties and it takes into account, at least approximately, the lack of independence between \( \overline{M} \) and \( \overline{\delta q} \). On the other hand, the weighting is not affected by the systematic errors that recur every year; model deficiencies, or systematic data biases, must be dealt with explicitly.

Because of the limited sample, the error covariance matrix \( S_{dn} \) is of strongly reduced rank and the inverse of \( S_n \) dominated by unreliable information. Hence, the problem is ill-conditioned. To circumvent the difficulty, we strongly reduce the dimension of the fields and tune the model in the highly truncated space. The iterative method is implemented in reduced space: for each forcing, each individual year is projected onto the reduced base, thereby defining a reduced heat flux correction and a reduced model matrix. By projection, a reduced modelization error matrix is also constructed. The sample error covariance matrix associated with the observational uncertainties and the optimal parameter corrections are then directly calculated in reduced space, so that the computational costs are very limited.

c. Model testing

The correctness of the SST model and the main assumptions in the inverse calculation (e.g., modelization and data errors) can be checked by looking at the residuals after optimization, but this ignores useful information on correlation scales. To take the multidimensional aspects of the fields into account, we generalize a multivariate test derived by Tarantola (1987) and consider the minimum of the misfit function (9), given by

\[ 2S (\delta a_n) = \overline{\delta q} (\overline{MC_a \overline{M} + S_n})^{-1} \overline{\delta q} \]  

(20)
with $S_n = S_{dn} + C_T$. The null hypothesis that the only errors besides the observational ones are the modelization errors can be tested since the test statistic (20) is distributed as Hotelling's $T^2$ with degrees of freedom $\eta$ (the reduced dimension) and $\tau$ (the equivalent degrees of freedom of $S_n$). If (20) exceeds the critical value at a given level of confidence, then some of the assumptions are unlikely to be acceptable. Since, except for possible biases, the observational uncertainties are represented by an error model which is, by construction, consistent with the available observations, the most likely interpretation is that the model is not as accurate as it has been assumed, i.e., the modelization errors have been underestimated.

4. TUNING THE TROPICAL ATLANTIC SST MODEL

The monthly values of $\delta q^{i,j}$ and $M^{i,j}$ are first spatially smoothed with a $5^\circ \times 5^\circ$ running average. The fit is then done in the region between $10^\circ S$ and $20^\circ N$, by considering

![Figure 2a. (left) Mean heat flux correction in Wm$^{-2}$ during January, April, July and October for the period 1965-1986, when using the a priori values of the model parameters. Corresponding values of (center) the upwelling flux and (right) horizontal diffusion.](image_url)
January, April, July, and October, which are representative of the various SST regimes. The data dimension $p$ is $322 \times 4 = 1288$.

The mean heat flux correction $\langle \Delta q \rangle$ is represented in Figure 2a. The rms value is large (69 Wm$^{-2}$), and negative values in excess of -100 Wm$^{-2}$ are found off Africa and in the Gulf of Guinea, mostly where the largest SST differences are observed. The tuning can be viewed as determining the best fit of the heat flux correction vector in Figure 2a by the seven column vectors of $\langle M(T) \rangle$, which are represented in Figures 2a,b (units are arbitrary). The upwelling pattern (Fig. 2a, center) has a large signal in the Gulf of Guinea with maximum amplitude during the upwelling season in July; a smaller signal is seen in the ITCZ with maximum amplitude off Africa, except in April. The meridional scaled of the diffusion pattern (Fig. 2a, right) is slightly smaller than that of upwelling. The cloud pattern cloud pattern (Fig. 2b, left) has broader scales and its seasonal changes reflect

Figure 2b. (left) Cloud factor values during January, April, July and October for the period 1965-1986, when using the a priori values of the model parameters, (center) solar angle, and (right) latent heat flux.
those of $Q_0$ and $C$. The evaporation pattern (Fig. 2b, right) has a large meridional scale and strong zonal gradients. Additional patterns are the insolation pattern in Figure 2b (right), a constant, and the observed SST pattern shown in Figure 1.

The data compression is done by working in the space defined by orthonormalizing the eight vectors consisting of $<\overline{\delta q}>$ and the seven column vectors of $<\overline{M}>$. As the dimension $\eta$ of the subspace is the number of adjustable parameters plus one, the inverse problem remains formally overdetermined. As described in section 3, $\delta q^{t,i}$ and $M^{t,i} t$ are projected onto the reduced base for each year $t$, and the sample error covariance matrix directly estimated in reduced space at each iteration $n$. Because $S_{dn}$ has limited degrees of freedom, its elements are inaccurately known (large sampling errors) and the condition number of the matrix $S_n$ is very large. Lacking precise information on the modelization errors, we use BC's model, but double the rms error to 20 Wm$^{-2}$. This modelization error matrix is not sufficient to insure good conditioning, so a singular value decomposition is used to invert $S_n$ in (18). In practice, we apply a taper which is an estimate of the accuracy of the elements of $S_{dn}$.

For simplicity, we use zero for the parameter correction $\delta a_q$, but the results are similar when using a different initial value. Convergence is reached in two or three iterations, with the largest changes occurring after the first iteration. Figure 3 shows the a priori and a posteriori values of the adjustable parameters with twice their standard deviation (an approximation to the 95% confidence interval). Of the seven adjustable parameters, two strongly decrease to values that are positive, but not significantly different from zero at the 5% level: the upwelling efficiency $\gamma$ and the horizontal diffusion $\kappa$. Both parameters are well-resolved by the data set and independently resolved. However, such a small value for the upwelling efficiency is unlikely from a physical point of view. Although the changes in the cloud factor $a_c$ and the latent heat flux $a_{lh}$ are also well-resolved, they are not statistically significant at the 5% level, which suggests that the a priori choices were good, needing only little adjustment. However, the two parameters are not independently resolved and are anticorrelated, and correlated with the three remaining parameters, $a_\alpha$, $a_T$ and $a_T T_c$, which are poorly resolved by the data set.

Figure 4 shows the heat flux correction (8) after tuning. The amplitudes are smaller than in Figure 2: the rms value has dropped to 32 Wm$^{-2}$ and the space-time average to -7 Wm$^{-2}$, suggesting that the warm SST bias in Figure 1 should be mostly corrected. However, heat flux corrections larger than 100 Wm$^{-2}$ can still be seen off the North African coast during winter and in the equatorial upwelling region during summer. These are too large to be explainable by the data uncertainties and are associated with model deficiencies, as discussed by BC and Scoffier et al. (1993).
To verify the consistency of the inverse calculation, we apply the test of section 3c. Although the critical value of the test statistic (20) is difficult to establish as the total error covariance is the sum of a sample one and an (assumed to be) true one, upper and lower bounds can easily be found. For true covariances, the critical value, given by the $\chi^2$ distribution with 8 degrees of freedom (the dimension of the space), would be 16 at the 5% level (lower bound). For sample covariance matrices, it would be given by Hotelling's $T^2$ and equal to 32 (upper bound). The test is 385, which largely exceeds the latter value. This confirms that the modelization errors have been strongly underestimated. In particular, there are large modelization biases, not only random modelization errors as assumed.

Figure 3. Evolution of the parameter corrections as a function of the number of iterations. The error bars represent the 95% confidence intervals.
Figure 4. (left) Mean heat flux correction in Wm\(^{-2}\) during January, April, July and October for the period 1965-1986, after optimization; (center) Corresponding SST predictions; (right) Differences between simulated and observed SST.

Because the tuning minimizes the heat flux correction (more precisely a weighted form of it), it is of interest to verify whether the SST predictions have been improved by the parameter changes. The tuned model was thus run with the same forcing fields as before. As expected, a more realistic SST field is obtained (Fig. 4, center), although model-observation differences of a few degrees can still be seen in the upwelling region off Africa during the first part of the year and in the Gulf of Guinea during the second part (Fig. 4, right). Tables 1 and 2 suggest that the model improvements are limited to a decrease of the warm SST bias, although it still averages to 1.5°C. The mean seasonal variability and the observed SST anomalies are not significantly improved, so that the SST model remains largely inconsistent with the observations: the tuning is unable to compensate the model shortcomings.

The method is not very sensitive to the details in the calculation. The largest parameter corrections were obtained when working with low-passed seasonal data, because filtering
decreases the magnitude of the observational and modelization errors, thereby giving more weight to the observations in the best-fit calculation. Unfortunately, the increased resolution by the data set leads to vanishing upwelling efficiency, which is not acceptable. Although a larger upwelling efficiency could be obtained by constraining more γ, this stresses the inadequacy of the upwelling representation for the tropical Atlantic.

5. CONCLUSIONS

We have developed an adaptive inverse method to tune the adjustable parameters of a tropical SST model in a way that optimally takes into account the large uncertainties of the atmospheric forcing and the oceanic data, the expected modelization errors and our a priori knowledge of the parameter values. This is achieved by performing the model optimization for the mean seasonal SST cycle and using the dispersion of the model responses for each year and (equally plausible) forcing field as independent information to construct a sample estimate of the observational error covariance matrix. The procedure is more refined than that of BC in that the nonlinear nature of the inverse problem is taken into account and the large correlation scales of the forcing uncertainties are represented realistically. The method is general as long as the parameters enter the SST equation linearly, and it can be extended to the nonlinear case by using an iterative approach. Since the optimization is performed in a strongly reduced space, the computational cost is limited. However, the estimation of the observational errors requires that several multi-year model runs be available.

The method has been applied to tuning BC's SST model of the tropical Atlantic. The optimization reduces the warm SST bias of the model, but brings no significant improvement in its ability at representing the seasonal or interannual SST fluctuations. A statistical test of the correctness of the assumptions in the inverse calculation shows that the modelization errors are much larger than assumed. The model flaws are discussed in Scoffier et al. (1993), who show that the model's inability to properly represent SST cooling by upwelling is linked to the parameterization of $T_A$ in (1) and, as seen in Figure 2a (center), may result in SST heating by upwelling when the SST is low and the thermocline deep, which is not realistic.

Finally, it should be noted that the adaptive tuning procedure provides an alternative to imposing the "correction flux" that is often needed to avoid climate drift when coupling an SST model to an atmospheric model. Indeed, the decrease in mean SST bias should decrease climate drift in the coupled mode without introducing the drawbacks of the correction flux method, because the correction more properly takes place via model parameters, without altering the SST dynamics.
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REFERENCES


