EXPERIMENTS WITH A HYBRID STATISTICAL MECHANICS/OCEAN CIRCULATION MODEL

Michael Eby and Greg Holloway
Joint Program for Ocean Dynamics
Institute of Ocean Sciences, Sidney, BC, V8L 4B2, Canada, and
Centre for Earth and Ocean Research, University of Victoria, V8W 2Y2, Canada

ABSTRACT

A hybrid statistical mechanics / ocean circulation model is tested. A conventional
ocean model was revised to include a tendency for model streamfunction to relax toward a
maximum entropy configuration which depends on the shape of topography. The tendency
is called “topographic stress”. Comparisons are made between three cases; a control case
with streamfunction relaxation toward rest and two implementations of topographic stress
(differing by their functional dependence on total depth). The two topographic stress cases
perform similarly, but they differ from the control case in several regards. Topographic
stress strengthens equatorward tendencies in deep western boundary currents, sustains a deep
Alaska Stream, and leads to poleward eastern boundary undercurrents which are absent in
the control. In the upper water column, where direct wind and buoyancy forcing dominate,
the influence of topographic stress is slight.

INTRODUCTION

Oceanic general circulation models (OGCM) are used to advance our understanding of
physical processes in the ocean. Increasingly, OGCMs are being coupled to atmospheric models
and used to predict climate change. Most models, however, are not capable of simulating present
day ocean climatology accurately enough to provide a confident basis for predictions. We are
motivated to search for systematic defects which afflict these models. In particular, OGCMs
which are global in their domain and used for prediction over decades or longer are necessarily of
relatively coarse resolution. Oceanic eddies on length scales of tens of kilometers are either not
resolved, or are only marginally resolved in ways that may corrupt their dynamics. It is therefore
important to find a representation of unresolved eddies which is of sufficient skill to better
recover present day ocean climatology, providing an improved basis for climate change studies.

It has been suggested by Holloway (1992) that eddies interact with bottom topography
to generate pressure-slope correlations, possibly exerting large systematic forces (topographic
stress) upon mean circulation. The usual eddy parameterizations in terms of bottom drag or eddy
viscosity move a model towards a state of rest, whereas topographic stress may be a driving
force behind mean flows. It is suggested that a more skillful representation of unresolved eddies
may be given by the tendency toward higher system entropy.

Statistical dynamical tendencies were examined by Salmon et al. (1976) in the context
of ideal quasi-geostrophic dynamics. Among their simplest results is the expectation that,
on scales larger than the first deformation radius, motion should tend to be barotropic and
given by a streamfunction satisfying

\[(\alpha/\beta - \nabla^2) \psi \geq h\]  \hspace{1cm} (1)

where \(\nabla^2\) is the 2-dimensional Laplacian, \(\alpha/\beta\) is a ratio of Lagrange multipliers (due to dynamics which conserve energy and enstrophy), \(h = \int H/\delta H\) is the potential vorticity due to variation \(\delta H\) about mean depth \(H\), and \(f\) is the Coriolis parameter. This equation implies that an ocean with no external forcing, filled with random eddies (without mean motion), would tend to set up a mean flow \((\psi)\) that depends on the topography \((h)\).

In reality, the ocean has external forcing and internal dissipation, and thus is not a closed system to which maximal entropy solutions apply. The state of actual ocean circulation is achieved as a balance between entropy-increasing tendencies on account of eddy interactions and entropy-limiting tendencies due to forcing and dissipation. OGCMs already have modest skill to include large scale forcing, while internal dissipation is parameterized more haphazardly (in part due to poorly understood eddies). What OGCMs omit is the eddy tendency toward increasing entropy. We investigate the effects of modifying an OGCM such that the models would relax not toward rest, but rather toward a solution such as (1). There is a theoretical leap in applying a parameterization based on quasi-geostrophy to a primitive equation model. For this reason, as well as uncertainty in how to characterize the competition between forcing-dissipation and topographic stress, we do not know precisely how to proceed. What we do hope is that this study will help motivate further theoretical work and demonstrate that the inclusion of a relatively crude parameterization of topographic stress already improves the quality of model simulations.

IMPLEMENTATION

The model chosen to study the effects of topographic stress was the GFDL Modular Ocean Model (MOM) (Pacanowski et al. 1991) which is based on code originally formulated by Bryan (1969) and further developed by Semtner (1974) and Cox (1984). Versions of this three dimensional, primitive equation model are widely used (Killworth et al. 1991).

MOM calculates velocity as internal (baroclinic) and external (barotropic) modes. From a vorticity tendency, the model solves an elliptic equation for transport streamfunction from which it obtains the external mode of velocity. We will be using MOM at a grid resolution more coarse than the first deformation radius, hence at scales for which the maximum entropy solution is barotropic. Thus, we can introduce a simple relaxation of the model streamfunction toward that given by (1).

There is a further simplification as well as certain ambiguities which arise in application based upon (1). The ratio \(\alpha/\beta\) is not well defined in reality, since its theoretical motivation depends upon artifacts such as finite spectral truncation. However, \(\alpha/\beta = 1/L^2\) defines a length scale which is plausibly related to eddy length scales. In what follows, we treat \(L\) as an adjustable parameter on the order of 10 km. The model resolution we will use is much coarser than \(L\), so we may omit \(\nabla^2\) in (1), taking only \(\psi = L^2 h\).

Ambiguities arise also because (1) is based upon quasi-geostrophy whereas application will be made in primitive equation MOM. The range of variation of depth, expressed by \(h\), should be small under quasi-geostrophy. In fact we will use the full range of oceanic depth, making \(\psi = -f L^2 H/H_o\) where \(H_o\) is a reference depth. Under quasi-geostrophy, interpretation of \(\psi\) is
arbitrary; it may describe either a transport or velocity streamfunction. If we adopt the velocity streamfunction view, then \( \psi \) will be converted to a transport streamfunction for incorporation into MOM. Because variation in \( \psi \) is dominated by variation in \( H \), an approximation for the maximum entropy transport streamfunction is given by

\[
\Phi^* = -fL_vH^2 \quad \text{where} \quad L_v = \frac{\beta}{2\alpha H_o} \tag{2}
\]

If we adopt the transport streamfunction view of (1), we multiply through by \( H_o \), and the maximum entropy transport streamfunction becomes

\[
\Phi^* = -fL_t^2H \quad \text{where} \quad L_t^2 = \frac{\beta}{\alpha} \tag{3}
\]

Ambiguities such as the different functional dependences in (2) or (3) may seem unnerving. There should be no pretense to sophistication here. Simply, our aim is to use MOM to explore sensitivity, comparing differences under (2) or (3) with the results from traditional subgridscale relaxation to rest (\( \Phi^* = \) constant). Length scales \( L_v \) and \( L_t \) are treated as adjustable, with \( H_o \) absorbed into \( L_v \). Moreover, one may consider that these length scales exhibit some weak spatial dependence. In particular, we will consider that \( L_v \) or \( L_t \) vary with latitude. Clearly this invites parameter tuning. At present, our aim is only to observe sensitivity to such issues.

Finally, the transport stream function (\( \Phi \)) calculated by MOM is replaced at each time step with

\[
\Phi + \frac{\delta t}{\tau}(\Phi^* - \Phi) \tag{4}
\]

using either (2) or (3) for \( \Phi^* \). The model velocity time step \( \delta t \) is of order 1 hour and \( \tau \) is an adjustable relaxation time of order 25 days.

**MODEL SETUP**

A coarse-resolution global model was created with grid spacing 3.75° in longitude and 3.711° in latitude. This resolution closely approximates the spectral T32 grid used by the global atmospheric general circulation model of the Canadian Climate Centre. Fifteen levels were used with layer thicknesses ranging from 20 to 870 m. Topography was extracted from ETOPO5 (1986) using a raised cosine weighted average of the data within a grid cell. Four islands were included: Madagascar, Australia, New Zealand and Antarctica.

The model was forced with annual mean Hellerman and Rosenstein (1983) windstress and a 50 day relaxation of surface salinity and temperature to annual mean Levitus (1982). The domain was limited at 69° North to avoid high grid latitudes, and salinity and temperature were relaxed to Levitus values on the artificial northern boundary with a time scale of 3 years. Horizontal viscosity, horizontal diffusion, vertical viscosity and vertical diffusion were set to \( 2 \times 10^5 \text{ m}^2 \text{ s}^{-1} \), \( 4 \times 10^3 \text{ m}^2 \text{ s}^{-1} \), \( 2 \times 10^{-3} \text{ m}^2 \text{ s}^{-1} \) and \( 1 \times 10^{-4} \text{ m}^2 \text{ s}^{-1} \) respectively. Velocity time steps were 1 hour and the tracer time step was 2 days.

In exploratory integrations, we have allowed relaxation time \( \tau \) to vary from 10 to 200 days, and length scale \( L_t \) to vary from 10 to 30 km. Results overall were as expected — the
topographic stress parameterization caused larger model responses in cases with larger length scale or shorter relaxation times. These integrations also demonstrated that without a latitude dependence, topographic stress tendencies were relatively too strong at high latitudes. For longer integrations, we have assigned $\Phi^*$ a latitude dependence given by $1 - 0.9 \sin(\text{latitude})$. We have chosen relaxation time $\tau$ to be 25 days and the equatorial value of length scale $L_t$ to be 22 km.

To compare the two implementations of topographic stress, relative values of the length parameters $L_t$ and $L_v$ must be assigned. Equating the topographic stress solutions given by (2) and (3), length parameters are related by $L_v H^2 = L_t^2 H$. Within the model, $H$ varies discretely from 0 to 5.5 km; we equate both solutions at 5.5 km. Thus given a choice of 22 km for $L_t$, an equivalent $L_v$ is 88 km. Although the extreme values of the two implementations are equivalent, the barotropic velocities will be different. A topographic stress which is proportional to $L^2 H$, as in (2), will tend to produce relatively stronger barotropic velocities in shallow water compared to a topographic stress which is proportional to $L H^2$, as in (3). Maximum entropy (equilibrium) velocities corresponding to (2) and (3) are shown in Figure 1.

Integrations were carried out in parallel — two with relaxation to the equilibrium solutions described by (2) and (3), and a third with relaxation to zero (control). To keep the runs as similar as possible, the control case includes relaxation to zero streamfunction. This takes into account that topographic stress velocities are small compared with direct forced, upper ocean flows. Thus topographic stress is closer to relaxing streamfunction to zero than to no relaxation. As well, when including streamfunction relaxation, less explicit viscosity is required, so the model’s total transports are largely unchanged. With respect to the damping processes, we try to keep the control case as similar as possible to the topographic stress cases.

Integrations were started from horizontally averaged Levitus data. Interior relaxation to Levitus temperature and salinity was continued for 10 years with a relaxation time scale of 1 year. This method of start-up avoids shocking the model with observed data that is incompatible with the model physics (Semtner and Chervin 1988). The model was then released from any interior relaxation and integrated for another 190 years. Although the model will not have reached equilibrium after 200 years, comparisons of trends can be made between parallel runs.

RESULTS

From Figure 1 we can anticipate some of the effects of topographic stress. Equilibrium velocities are poleward on the eastern, and equatorward on the western, slopes of basins. These velocities suggest currents which are opposite to many of the well known surface currents such as the Gulf Stream, Canary, Brazil and Benguela Currents or the Kuroshio, California, East Australia and Peru Currents. Magnitudes of statistical dynamical velocities are small, however, compared to the magnitude of the wind-driven surface currents, so we expect that topographic stress will have little effect on the surface circulation. At greater depths, where velocities from directly forced flows have smaller magnitude, tendency toward higher entropy has relatively greater effect. One anticipates the development or strengthening of poleward undercurrents along eastern boundaries and equatorward undercurrents along western boundaries.
Figure 1. (a) Maximum entropy (equilibrium) velocities: (a) for the $L^2H$ case and (b) for the $LH^2$ case.

Velocities for the first few levels are dominated by the wind and buoyancy driven surface circulation. Since plots from the three integrations are visually indistinguishable, only one plot at 35 m (level 2) is shown in Figure 2. Total transports are also very similar for all three runs since much of the transport occurs in the upper ocean. Small differences in transport are noticeable along continental margins, but these differences are more clearly seen when comparing velocity fields. Results from the model integrations at greater depths will be discussed for three geographic areas: the North Pacific, the Mid-Atlantic and the Indian Oceans.

We will show results at two depth levels: 850 m (level 8) and 2750 m (level 12). Shallower levels are dominated by direct forcing. With the choice of parameters used here, the competition between direct forcing and statistical dynamics is such that the statistical dynamical tendencies become apparent in the lower main thermocline, roughly 850 m. At greater depths, statistical dynamical tendencies become more dominant.
**North Pacific**

Velocities at 850 m for the three implementations are shown in Figure 3. Differences between the control run (Figure 3a) and the topographic stress runs (Figures 3b and 3c) can be seen along the continental margins. The control run exhibits no California undercurrent and only a weak Oyashio. The control also has a strong Kuroshio extension cutting off a weak Alaska Stream.

Observational evidence for an undercurrent along the West coast of North America is extensive, including work by Hickey (1979) off the coast of Southern Washington, Freeland et al. (1984) off Vancouver Island and Chelton (1984) off California. Measurements indicate poleward flow up to 15 cm s\(^{-1}\), often with a width greater than 100 km, usually with a maximum at depths less than 700 m, but extending to more than 1000 m. Most of these studies were coastal in nature, thus the width and depth of the underflow has not been well established. The temporal and spatial persistence of the undercurrent is also not well known.

Observations by Warren and Owens (1988) indicate a deep Alaska Stream flowing westward, with mean velocities between 1 and 3 cm s\(^{-1}\), along the northern side of the Alcuitan Trench. They also report evidence for a deep, eastward jet which flows parallel to the trench, south of the Alaska Stream.

Figure 4 shows velocities at 2750 m for the three integrations. Coastal currents induced or strengthened by topographic stress at 850 m are seen more clearly at 2750 m, with a western boundary undercurrent now extending to the equator. Smaller differences between the control run and the topographic stress runs can be seen in the central Pacific.

Direct observations of deep western boundary currents in the North Pacific are few. Indirect inference from tracers such as silica (Talley and Joyce 1992) suggest northward deep flow along the western boundary. Current meter observations by Fukasawa et al. (1986) in the Shikoku Basin south of Japan (west of the region considered by Talley and Joyce 1992) show deep
mean currents of 5 to 10 cm s\(^{-1}\) toward the south-west (parallel to local isobaths). Northward flow of low silica water is contrary to that indicated by Figure 4, whereas the current meter observations are consistent with topographic stress.

Figure 3. Velocities at 850 m in the North Pacific Ocean: (a) for the control, (b) for the \(L^2H\) and (c) for the \(LH^2\) cases.

Figure 4. Velocities at 2750 m in the North Pacific Ocean: (a) for the control, (b) for the \(L^2H\) and (c) for the \(LH^2\) cases.

The deep, narrow trenches found in the Pacific are not resolved by this model, but could be important in setting up counterflows such as the one observed by Warren and Owens (1988) south of the Alaska Stream. At small scale the baroclinic influence should be taken into account, however, the barotropic formulations for topographic stress (formulae 2 or 3) suggest a tendency for opposing currents on opposite sides of a trench. One could imagine cyclonic shear above the trenches in mid-depth and abyssal waters, supporting northward and eastward transport of tracers in the western Pacific while current meters on the inshore side of trenches show southward or westward flow.

Differences between the two topographic stress runs are subtle. The effects (when compared to the control run) produced by the \(L^2H\) implementation (Figures 3b and 4b) tend to be stronger along the coast and slightly weaker in the central Pacific than with the \(LH^2\) implementation.
(Figures 3c and 4c). Since plots of the two implementations of topographic stress are so similar, only plots from the control run and the $L^2H$ run will be shown for the other regions.

**Mid-Atlantic**

Model velocities at 850 m are shown in Figure 5. Small differences between the control run (Figure 5a) and the topographic stress run (Figure 5b) can be seen along the continental margins. Topographic stress has weakened or reversed the control runs equatorward eastern boundary currents. The northward flow along the western margin is also weaker in the North Atlantic and stronger in the South Atlantic for the topographic stress run than for the control.

![Figure 5](image1.png)

**Figure 5.** Velocities at 850 m in the Mid-Atlantic Ocean: (a) for the control and (b) for the $L^2H$ cases.

![Figure 6](image2.png)

**Figure 6.** Velocities at 2750 m in the Mid-Atlantic Ocean: (a) for the control and (b) for the $L^2H$ cases.

Differences between the control run and the topographic stress run are more obvious at 2750 m (Figure 6) than at 850 m. The control run does not develop any poleward eastern boundary undercurrents. The deep, southward flowing western boundary currents are also stronger in the North Atlantic, and weaker in the South Atlantic for the topographic stress run compared to the control run.

Poleward undercurrents have been observed off the west coast of South Africa (Nelson 1989) and off the coast of North Africa (Mittelstaedt 1989). Poleward flow has also been described off the Iberian Peninsula (Barton 1989). Although spatial and temporal knowledge of poleward
undercurrents along the Eastern Atlantic is limited, direct measurements indicate a flow of up to 10 cm s\(^{-1}\) with a width of 30 to 100 km, often with a maximum at about 300 m, but extending to great depths.

The Peru-Chilean undercurrent is also present in Figure 6b. Observations of the Peru-Chilean undercurrent are summarized by Fonseca (1989). This current has been seen from the surface to below 300 m.

While topographic stress may be a dominant force in the deep western boundary currents of the North Pacific, thermohaline forcing is clearly important in the Atlantic. Because the model domain is truncated at 69° North, the thermohaline forcing has in part been provided by relaxation toward mean Levitus at all depths along the artificial northern boundary. To test the sensitivity of the model to this northern boundary condition, two further integrations were performed, for the control case and for the \(L^2H\) case, without interior relaxation on the boundary. Velocities at 2750 m are shown in Figure 7. Without relaxation, the western boundary current is largely absent in the control case (Figure 7), but remains present in the topographic stress case (Figure 7b).

![Figure 7. Velocities at 2750 m in the Mid-Atlantic Ocean without relaxation to Levitus along the northern boundary: (a) for the control and (b) for the \(L^2H\) cases.](image)

It is clear that when the nature of imposed forcing happens to agree with a maximum entropy configuration, topographic stress may not play a very significant role. The stress depends upon how far forced-dissipative flows are from ideal maximum entropy. A second observation concerns the climatic implications of these results. One may speculate that variation in thermohaline forcing of the North Atlantic could lead to abrupt alteration of the pattern of deep circulation. Comparison of Figures 6a and 7a indeed appears to support this speculation. However, when topographic stress is included, Figures 6b and 7b show a deep circulation which is rather insensitive to changes in thermohaline forcing. The indication is that inclusion or omission of topographic stress in coupled ocean-atmosphere climate models could have significant effect on the overall sensitivity of the coupled system.
Indian

Model velocities at 850 m are shown in Figure 8. Effects of topographic stress include a slight weakening of the West Australian and the Agulhas Currents, and a slight strengthening of both the undercurrent off the east coast of Australia and the cyclonic circulations in the Northern Indian Ocean.

Figure 8. Velocities at 850 m in the Indian Ocean: (a) for the control and (b) for the \( L^2 \) cases.

Figure 9. Velocities at 2750 m in the Indian Ocean: (a) for the control and (b) for the \( L^2 \) cases.

Figure 9 shows velocities for the two integrations at 2750 m. Differences between the two runs are more pronounced than at 850 m. Topographic stress induced effects include: an increase in the transport of deep Antarctic water northward into the equatorial Indian; a strengthened undercurrent along the south, west and east coasts of Australia; a strengthening of the circulation in the North Indian; and a slight weakening of the Circumpolar Current near the coast of Antarctica leading to the development of a countercurrent, west of the Kerguelen Plateau.

An undercurrent beneath the Leeuwin current has been observed to be equatorward (Church et al. 1989), contrary to the topographic stress tendency. Model results off the west coast of Australia demonstrate the competition between direct forcing and topographic stress. Figure 2 suggests a weak, poleward current which continues down to about 200 m, after which an equatorward undercurrent is present until about 1000 m (Figure 8). Although topographic stress opposes an equatorward undercurrent, other forces are overriding the
maximum entropy tendency. Between 1000 and 2000 m, both integrations generally show poleward flow. Differences between the two runs are most apparent between 2000 and 3500 m where slow, mixed flow is seen for the control run while a steady poleward flow is seen for the topographic stress run (Figure 9). Below 3500 m the flow is again equatorward for both integrations, although stronger for the control run.

SUMMARY

The GFDL Modular Ocean Model was used to explore proposed representations of topographic stress for large scale ocean modelling. We have sought to characterize the effect of unresolved eddy-topography interaction (topographic stress) in terms of a tendency for large scale flow to evolve toward a state of higher system entropy.

Two implementations of topographic stress were tested, differing in the assumed functional dependence of streamfunction upon depth. The two topographic stress cases perform similarly, but they differ from the control case in several respects. The most apparent effects of topographic stress are seen along continental boundaries, particularly in the development or strengthening of undercurrents. Integrations which include topographic stress produce many of the observed poleward eastern boundary undercurrents which the control run does not. Along western boundaries, the topographic stress tendency is equatorward. Differences from the control run are seen more clearly in the western Pacific than in the western Atlantic, since the Atlantic is also responding to stronger thermohaline forcing.

Acknowledgments

This work was carried out with the support of the Office of Naval Research (grants N00014-87-J-1262 and N00014–92–J-1775). The authors would like to thank Kelly Choo for his work on graphics.

REFERENCES


