DIANEUTRAL ADVECTION

Trevor J McDougall

CSIRO Division of Oceanography, GPO Box 1538, Hobart, TAS 7001, Australia.

ABSTRACT

The nonlinear nature of the in situ density of seawater as a function of the potential temperature, pressure and salinity causes two vertical advection processes (thermobaricity and cabling) and also complicates the use of microstructure data to deduce upwelling velocities. In most of the ocean thermobaricity and cabling are weak, but where there is a significant epineutral gradient of potential temperature, the downwelling due to cabling is quite large. In the Southern Ocean, where the neutral surfaces have large slopes, thermobaricity causes a larger downwelling velocity than cabling and the two processes together cause a dianeutral velocity of about $-2 \times 10^{-7}$ m s$^{-1}$. The complementary roles of vertical mixing and vertical advection in achieving water-mass conversion are demonstrated, since maps of the dianeutral motion caused by vertical mixing are quite different to maps of water-mass conversion caused by the same process. This emphasizes the need to include both vertical advection and vertical mixing in ocean models. The method used to infer the upwelling velocity from microstructure dissipation measurements is also significantly affected by the nonlinear nature of the equation of state. The extra term that needs to be included in this method is a strong function of depth, changing sign at a depth of about 1500 m. The differences between interior mixing and boundary mixing are explored using the conservation statements for buoyancy and volume in the turbulent boundary layers. It is shown that the deep circulation driven by boundary mixing is quite different to that of Stommel and Arons. Interestingly, seamounts are, on average, sinks for the lateral flow, with downwelling in the turbulent boundary region surrounding each seamount.

INTRODUCTION

The starting point for this paper is the conservation equations for scalars. For salinity, $S$, and potential temperature, $\theta$, the conservation equations are

$$\frac{\partial \theta}{\partial t} + \left[ \mathbf{V}_n - \frac{1}{h} \mathbf{V}_n(hK) \right] \cdot \nabla_n \theta + e \theta_z = K \nabla_n^2 \theta + [D \theta_z]_z,$$  \hspace{1cm} (1)

$$\frac{\partial S}{\partial t} + \left[ \mathbf{V}_n - \frac{1}{h} \mathbf{V}_n(hK) \right] \cdot \nabla_n S + e S_z = K \nabla_n^2 S + [D S_z]_z,$$  \hspace{1cm} (2)

where $\mathbf{V}_n$ is the Lagrangian two-dimensional lateral velocity vector, $u \mathbf{i} + v \mathbf{j}$, $e$ is the velocity of fluid through a neutral surface (dianeutral velocity), $h$ is the height between adjacent neutral surfaces, $K$ is the epineutral diffusivity, $D$ is the dianeutral (or vertical) diffusivity and $\nabla_n$ is the lateral gradient operator in a neutral surface. These conservation equations have been
previously explained by McDougall (1987a); the only additional feature now is that the lateral variations of $h$ have been included. The two terms in these equations that contain the epineutral diffusivity, $K$, can be derived by taking the epineutral divergence, $\nabla_n \cdot \nabla_n$, of the total flux of heat between two adjacent neutral surfaces, $-hK \nabla_n \theta$. The spatial gradients of $\theta$ and $S$ along neutral surfaces and also their temporal derivatives are related through the thermal expansion coefficient, $\alpha$, and the saline contraction coefficient, $\beta$, so that

$$\alpha \theta_t \bigg|_n = \beta S_t \bigg|_n, \quad \text{and} \quad \alpha \nabla_n \theta = \beta \nabla_n S. \quad (3)$$

Combining Eqs (1) – (3), one finds

$$\begin{align*}
\theta_t \bigg|_n + \left[ \nabla_n - \frac{1}{h} \nabla_n (hK) \right] \cdot \nabla_n \theta &= K \nabla_n^2 \theta + D \frac{g}{N^2} \theta_z^2 \beta \frac{d^2 S}{d \theta^2} \\
+ K \frac{g}{N^2} \theta_z \left\{ \frac{\partial \alpha}{\partial \theta} \nabla_n \theta \cdot \nabla_n \theta + \frac{\partial \alpha}{\partial p} \nabla_n \theta \cdot \nabla_n p \right\}, \quad (4)
\end{align*}$$

and

$$\begin{align*}
\left[ e - D_z \right] \frac{N^2}{g} &= D \left( \alpha \theta_{zz} - \beta S_{zz} \right) - K \left\{ \frac{\partial \alpha}{\partial \theta} \nabla_n \theta \cdot \nabla_n \theta + \frac{\partial \alpha}{\partial p} \nabla_n \theta \cdot \nabla_n p \right\}, \quad (5)
\end{align*}$$

where $\partial \alpha/\partial \theta$ and $\partial \alpha/\partial p$ are given by Eq. (8) below. Equation (4) is sometimes called the water-mass transformation equation, since temporal changes of potential temperature on a neutral surface imply changes to the S-$\theta$ curve of a water-mass. An important point to note in Eq. (4) is that the total effect of the vertical diffusivity, $D$, on the $\theta$ conservation equation is proportional to $d^2 S/d \theta^2$, the "curvature" of the S-$\theta$ curve formed from a vertical CTD cast. As first pointed out by McDougall (1984), if either the dianeutral advective term or the vertical diffusive term in Eq. (1) is omitted in a diagnostic model, then the term in the conservation equations due to vertical turbulent mixing must take the form on the right-hand side of Eq. (4), namely, $DgN^2 \theta_z^2 \beta \cdot d^2 S/d \theta^2$. The corresponding equations for salinity, $S$, and another passive tracer, $C$, are obtained by eliminating $[e - D_z]$ between Eqs (5) and (2) (and similarly for $C$), obtaining

$$\begin{align*}
S_t \bigg|_n + \left[ \nabla_n - \frac{1}{h} \nabla_n (hK) \right] \cdot \nabla_n S &= K \nabla_n^2 S + D \frac{g}{N^2} \theta_z^3 \alpha \frac{d^2 S}{d \theta^2} \\
+ K \frac{g}{N^2} S_z \left\{ \frac{\partial \alpha}{\partial \theta} \nabla_n \theta \cdot \nabla_n \theta + \frac{\partial \alpha}{\partial p} \nabla_n \theta \cdot \nabla_n p \right\}, \quad (6)
\end{align*}$$

and

$$\begin{align*}
C_t \bigg|_n + \left[ \nabla_n - \frac{1}{h} \nabla_n (hK) \right] \cdot \nabla_n C &= K \nabla_n^2 C + D C_z \frac{g}{N^2} \theta_z^2 \beta \frac{d^2 S}{d \theta^2} + D \theta_z^2 \frac{d^2 C}{d \theta^2} \\
+ K \frac{g}{N^2} C_z \left\{ \frac{\partial \alpha}{\partial \theta} \nabla_n \theta \cdot \nabla_n \theta + \frac{\partial \alpha}{\partial p} \nabla_n \theta \cdot \nabla_n p \right\}. \quad (7)
\end{align*}$$
If the tracer $C$ is consumed or produced, either biologically or radioactively, a suitable term can be added to Eq. (7) in the normal way; for example, radioactive decay requires an additional term, $\lambda C$, on the left-hand side.

**THERMOBARICITY AND CABELLING**

The terms that appear in the conservation equations (4) to (7) that are proportional to $\partial \alpha / \partial \theta$ and $\partial \alpha / \partial \phi$ are the diabatic advection processes of cabling and thermobaricity respectively. The parcel-mixing argument of Fig. 1 illustrates that when two water parcels that initially lie on a neutral surface are brought together without mixing, the temperature-dependence of the compressibility causes the parcels to move vertically off the neutral surface. This vertical advection process is thermobaricity. Once the two fluid parcels mix intimately, cabling causes an increase in density and a further vertical motion through neutral surfaces, as indicated in the figure. Both processes are proportional to the lateral flux of heat, $-K \nabla_n \theta$, cabling always producing a downwelling velocity through neutral surfaces, while thermobaricity can cause either upwelling or downwelling. Interestingly, while the diabatic motion achieved by cabling and thermobaricity is quite significant, the diabatic diffusion, or mixing, is negligible (McDougall 1987a). The partial derivatives, $\partial \alpha / \partial \theta$ and $\partial \alpha / \partial \phi$ that appear in the above equations are in fact shorthand expressions for

$$\left[ \frac{\partial \alpha}{\partial \theta} + 2 \frac{\alpha}{\beta} \frac{\partial \alpha}{\partial S} - \frac{\alpha^2}{\beta^2} \frac{\partial \beta}{\partial S} \right] \quad \text{and} \quad \left[ \frac{\partial \alpha}{\partial \phi} - \frac{\alpha}{\beta} \frac{\partial \beta}{\partial \phi} \right]$$

respectively. These expressions are called the cabling and thermobaric parameters and are graphed in figure 9 of McDougall (1987a). Typical values are $10^{-5}$ $K^{-1}$ for the cabling parameter and $2.6 \times 10^{-8}$ $K^{-1}$ (db)$^{-1}$ for the thermobaric parameter.

Vertical mixing and advection processes are usually inferred from measurements of the dissipation rate of mechanical energy, $\varepsilon$. However, the magnitudes of thermobaricity and cabling are determined by the epineutral heat flux and are not limited by the strength of vertical mixing processes. Hence neither thermobaricity nor cabling has a detectable signature in $\varepsilon$ measurements, and the strengths of these diabatic advective processes must be inferred from information on the mesoscale lateral eddy field.

In order to map the cabling and thermobaric velocities in the ocean, a value of 1000 m$^2$ s$^{-1}$ was chosen for the epineutral diffusivity, $K$. Freeland et al. (1975) used the statistics of float displacements to find the lateral diffusivity of about this size at a depth of 1500 m in the relatively quiet MODE region, while Rossby et al. (1983) found $K$ to be 50% larger than this at a depth of 1300 m in the region of the Local Dynamics Experiment. Estimates nearer the sea-surface are generally larger. Hence we believe that by taking a value of 1000 m$^2$ s$^{-1}$ we have erred on the side of being conservative, so that the strengths of cabling and thermobaricity are probably underestimated here. The combined contributions to the diabatic velocity from cabling ($e^{cabb}$) and thermobaricity ($e^{tb}$), are (from Eq. 5),

$$e^{cabb} + e^{tb} = -\frac{g}{N^2} K \left\{ \frac{\partial \alpha}{\partial \theta} \nabla_n \theta \cdot \nabla_n \theta + \frac{\partial \alpha}{\partial \phi} \nabla_n \theta \cdot \nabla_n \phi \right\} \quad (m s^{-1}). \quad (9)$$
Fig. 1. A diagram illustrating the physical causes of thermobaricity and cabbeling. The upper surface depicts a neutral surface along which the pressure and potential temperature vary. A warm and a cool water parcel are moved together without doing any work against gravity and without exchanging heat or salt with their surroundings. When these two parcels meet they are allowed to mix completely. The lateral heat flux, $V_n \theta'$, can be imagined to be due to many such combinations of lateral stirring followed by intimate mixing. Along the path towards the central point, each water parcel has a different potential temperature to that on the original neutral surface at this location, and so the compressibility of the water parcel is different to that of fluid on the neutral surface. Because of the difference in compressibilities, the pressure excursions associated with the lateral motion cause the water parcels to move off the original neutral surface as shown. This process is called thermobaricity, as it is caused by the second-order partial derivative of in situ density with respect to potential temperature (thermo-) and pressure (baric). Once the two fluid parcels mix intimately, there is an increase of density and consequent dianeutral motion caused by cabbeling.
Fig. 2. The dianeutral velocity due to cabbeling and thermobaricity (assuming an epineutral diffusivity of $1000 \text{ m}^2 \text{ s}^{-1}$), in the North and South Atlantic Ocean and in the Southern Ocean in the Atlantic sector, on two neutral surfaces. Fig (a) is for the "27.25" neutral surface which has a potential density of 27.25 (referred to 0 db) in mid-ocean on the equator; its average pressure is about 650 db. Fig (b) is for the "27.75" neutral surface which has a potential density of 27.75 (referred to 0 db) in mid-ocean on the equator; its average pressure is about 1500 db.
Fig. 2 shows the combined dieneutral velocity from thermobaricity and cabling on two neutral surfaces in the North and South Atlantic, labeled "27.25" and "27.75", respectively. These and other figures in this paper were derived from the Levitus (1982) data set. The nomenclature ‘neutral surface ‘27.25’ ’ means that the surface has a potential density, referred to the sea-surface, of 27.25 in mid-ocean on the equator. At other locations the potential density is different, as explained by You and McDougall (1989). A large proportion of the low-latitude region of the North Atlantic on the "27.25" neutral surface has downwelling larger than 0.5x10^-7 m s^-1. On much of the mid- and high-latitude region of the North Atlantic's "27.75" neutral surface, the downwelling due to cabling exceeds 0.5x10^-7 m s^-1, while in the Southern Ocean on this figure the downwelling is primarily caused by thermobaricity and is as large as 5x10^-7 m s^-1. There is also a small region of net upwelling near 33°S where thermobaricity is larger than cabling and of opposite sign.

It is clear that thermobaricity and cabling need to be included in inverse models of tracer data in regions where these processes cause significant water-mass conversion. The rate of water-mass conversion is given by the dieneutral velocity multiplied by the vertical tracer gradient. It is interesting to note that general circulation models that carry salinity as well as temperature, and therefore use the full equation of state, automatically include cabling and thermobaricity. To date, the contribution of these processes to the total upwelling of Bottom Water has not been separately determined in these models. Note that as a general rule, thermobaricity and cabling will be unimportant in shallow water where N^2 is large (see Eq. 9).

It is very tempting to believe that cabling, or "densification on mixing", (and perhaps thermobaricity too) is partly caused by small-scale turbulent mixing as parameterized by the dieneutral diffusivity, D. This is not the case. That vertical mixing does not contribute to cabling or thermobaricity can be seen from the form of Eqs (4) to (7). In particular, from Eq. (5), the dieneutral motion caused by the vertical diffusivity, \( gN^{-2}D[\alpha \theta_z - \beta S_{zz}] \), is independent of the nonlinearity of the equation of state. Rather, this expression depends only on the local values of \( \alpha \) and \( \beta \). Similarly, in Eqs (4), (6) and (7), the terms multiplying \( D \) do not depend on the nonlinear terms of the equation of state. This means that neither cabling nor thermobaricity is assisted by vertical mixing processes to produce either upwelling (Eq. 5) or water-mass conversion (Eqs (4), (6) and (7)). If one could (by magic) make the equation of state twice as nonlinear as it actually is, the vertical mixing terms on the right-hand sides of Eqs (4) - (7) would not change.

DEDUCING DIENEUTRAL ADEWCTION FROM THE BUOYANCY FLUX

In this section, only that part of the dieneutral velocity that is caused by small-scale turbulent mixing is considered. The dieneutral fluxes caused by this mixing are parameterized by a dieneutral diffusivity, \( D \), and its contribution to dieneutral advection is

\[
D_z + DgN^{-2}[\alpha \theta_{zz} - \beta S_{zz}] .
\]

This can be expressed in terms of the vertical derivative of \( N^2 \) as

\[
eN^2 = [DN^2]_z - [DN^2] \frac{R_\rho}{K_\rho - 1} \left[ \frac{\alpha_z}{\alpha} - \frac{\beta_z}{\beta} \frac{1}{K_\rho} \right] ,
\]

(10)
or,
\[
N^{-2}N_z^{-2} - gN^{-2} [\alpha \theta_{zz} - \beta S_{zz}] = \frac{R_p}{R_p-1} \left[ \frac{\alpha_z}{\alpha} - \frac{\beta_z}{\beta} \frac{1}{R_p} \right] = \frac{[DN^2]_z}{DN^2} - \frac{e}{D}.
\]
(11)

Since \(N_z^2\) is not equal to \(g[\alpha \theta - \beta S_z]\), the dieneutral velocity is not simply proportional to the vertical divergence of the turbulent buoyancy flux, \([DN^2]_z\). The dissipation method of Osborn (1980) is commonly used to estimate the dieneutral diffusivity from microstructure measurements of the dissipation rate of mechanical energy, \(\varepsilon\), using the formula \(DN^2 = \Gamma \varepsilon\), where \(\Gamma\) is a constant, usually taken to be 0.2. Equation (11) shows that \(e/D\) is not simply equal to \(\varepsilon/\varepsilon\). The additional term is shown in Figs. 3(a) and (b) on the "27.25" neutral surfaces in the Atlantic and Indian oceans. Using a value for \(D\) of \(10^{-4}\) m\(^2\)s\(^{-1}\), a contoured value of 100 in these figures implies that the dieneutral velocity is \(10^{-7}\) m s\(^{-1}\) less than the estimate \(D \varepsilon/\varepsilon\) that one would infer from the microstructure data alone. It is seen that there are significant regions of the shallow neutral surfaces in each ocean where this correction term approaches or exceeds \(10^{-7}\) m s\(^{-1}\).

Figure 4 shows how the expression in Eq. (11) varies in the vertical at a variety of locations in different oceans. The nonlinear term in the above equations is positive and quite significant in the top kilometer of these profiles, thereafter changing sign in the deeper ocean. This can be understood by examining the bracket \(\left[ \frac{\alpha_z}{\alpha} - \frac{\beta_z}{\beta} \frac{1}{R_p} \right]\), which may be approximated by \(\left[ \frac{1}{\alpha} \frac{\partial \alpha}{\partial \theta} \theta_z + \frac{1}{\alpha} \frac{\partial \alpha}{\partial \rho} p_z \right]\), where \(p_z\) is simply \(-1\) dB m\(^{-1}\). Since the terms \(\frac{\partial \alpha}{\partial \theta}\) and \(\frac{\partial \alpha}{\partial \rho}\) are about \(1 \times 10^{-5}\) K\(^{-1}\) and \(2.6 \times 10^{-8}\) K\(^{-1}\) (db\(^{-1}\)) respectively, the nonlinear term in the above equations is zero when \(\theta_z \approx 2.6 \times 10^{-3}\) K m\(^{-1}\); a typical gradient of potential temperature at a depth of about 1500 m in the ocean. Deeper than this, the positive \(\theta_z\) term becomes dominated by the negative \(p_z\) term, and at a depth of 4000 m, Eq. (11) asymptotes to about \(-20 \times 10^{-5}\) m\(^{-1}\). McDougall (1988a) used a very simple model of one-dimensional upwelling in the deep ocean to demonstrate the importance of even this small negative value of Eq. (11). The vertical variation of \(N^2\) was taken to be exponential with an e-folding height of 1000 m, and Bottom Water was assumed to upwell at the depth-independent velocity of \(10^{-7}\) m s\(^{-1}\).

Using Osborn's (1980) formula for \(DN^2\), Eq. (11) becomes a first-order differential equation for \(e\). This equation was integrated vertically to find that the nonlinear terms caused a doubling of the dieneutral diffusivity between 1500 m and 4000 m.

It is emphasized that the contribution of these nonlinear terms of the equation of state to the dieneutral velocity is by way of interpretation only. As explained above, vertical mixing processes do not contribute to thermobaricity or cabling. It is only if one chooses to find the dieneutral velocity from the vertical variations of the turbulent buoyancy flux that the grouping of nonlinear terms appears in Eq. (11). A very similar nonlinear term must be added to the conservation equation of potential density if the flux divergence of potential density is represented by the normal term, \(\partial [D \partial \rho] / \partial z\). The full conservation equation of
Fig. 3. These figures illustrate the difference of the dianeutral velocity between (i) that estimated from the vertical divergence of the buoyancy flux, and (ii), the true dianeutral velocity, $e$, given by Eq. (5) in the Atlantic and the Indian Oceans. When multiplied by the vertical diffusivity, $D$, the values from these figures give this difference in dianeutral velocity.
potential density is given by Eq. (70) of McDougall (1988b) and ignoring thermobaricity, cabbeling and the fact that there is a non-zero epineutral gradient of potential density, the vertical advective diffusive balance is given by

\[
[w^d - D_z \left(- \frac{1}{\rho_\theta} \frac{\partial \rho_\theta}{\partial z}\right)] = D \left( \partial \theta_{zz} - \bar{\beta} S_{zz} \right)
\]

\[
= -D \frac{1}{\rho_\theta} \frac{\partial^2 \rho_\theta}{\partial z^2} - D \frac{\partial \alpha}{\partial \theta} \bigg|_{s, p = p', \theta^2_z}. \tag{12}
\]

Here the diapycnal velocity is \( w^d \) and the thermal expansion and haline contraction coefficients are evaluated at the reference pressure of the potential density variable, \( p_r \). The extra term that appears on the right-hand side of Eq. (12) is very similar to that discussed above; the difference being that now there is no contribution from \( [\partial \alpha / \partial \rho] p_z \). In the upper ocean the change in the diapycnal velocity due to the nonlinear term in Eq. (12) is very similar to that estimated from Figs. 3 and 4. It is concluded that serious errors are occurring in the conservation equation of potential density in present inverse models since the two terms on the right-hand side of Eq. (12) are of the same magnitude, both being responsible for a contribution to the diapycnal velocity of order 10^{-7} m s^{-1}.

One-dimensional Advection-Diffusion Balances

Gargett (1984) has suggested that the vertical diffusivity, \( D \), may vary as the reciprocal of the buoyancy frequency, \( N \), and that if \( N \) varies exponentially with depth, \( z \), mixing would drive an equatorward interior flow in the deep ocean, in contrast to the poleward flow of the Stommel-Arons (1960) theory. Here I wish to make two points in relation to this one-dimensional view of the deep ocean. Firstly, this argument has ignored the non-linear terms of the equation of state that appear in Eq. (11), and it can be seen from Fig. 4 that at a depth of 1000 m, these terms can cause a vertical gradient of the diapycnal velocity larger than 4x10^{-10} s^{-1}, or a change of the diapycnal velocity of 4x10^{-7} m s^{-1} over a depth change of 1000 m. At this magnitude, the non-linear terms dominate the contribution to \( e_z \) calculated from the vertical buoyancy flux. These terms become smaller with depth, but they are probably not negligible in the abyssal ocean. Secondly, I wish to point out that the argument is quite sensitive to the assumed vertical variation of the buoyancy frequency.

For simplicity of argument, the non-linear terms in Eq. (11) are ignored, and so the diapycnal velocity is given by \( eN^2 = [DN^2]_z \), and using Gargett’s suggestion that \( D = A N^{-1} \), where \( A \) is a constant, gives

\[
e_z = -A \left[N^{-1}\right]_{zz}. \tag{13}
\]

For an exponential \( N(z) \) profile, \( e_z < 0 \), implying an equatorward flow, but for other vertical distributions of \( N \), \( e_z \) can be positive. For example, the general \( N(z) \) profile that gives \( e_z = 0 \) is

\[
N(z) = \frac{1}{[a - bz]}, \tag{14}
\]

where \( a \) and \( b \) are positive constants. This functional form may be indistinguishable from an exponential shape over much of the ocean’s depth (see Fig. 5).
Fig. 4. Vertical mixing causes dianeutral advection to occur at the rate

\[ D_z + gN^{-2}D \left[ \alpha \theta_{zz} - \beta S_{zz} \right] \]

rather than \[ N^{-2}[DN^2] \], as one may guess at first. These vertical profiles show the difference between these two expressions (divided by \( D \)) at six different places in the world's oceans. The same terms are shown as contour maps in Figs. 3(a) and (b) on a neutral surface in each of the Atlantic and Indian Oceans.

Another aspect of a one-dimensional advective-diffusive balance is the influence of (i) the entrainment into sinking Bottom Water plumes and (ii) the variation of the ocean's area with depth, on the sense of \( e_z \) and hence of the sign of the poleward velocity, \( v \). Consider the situation depicted in Fig. 6 where a dense plume of Bottom Water sinks to the ocean floor, entraining fluid from the interior as it sinks. The downward volume flux of the sinking plume, \( Q(z) \), must equal the upwelling of interior fluid in our one-dimensional ocean of area \( A(z) \), so that

\[ Q(z) = e(z)A(z) \]  \hspace{1cm} (15)

Differentiating this equation with respect to \( z \), we find that

\[ e_z = \frac{Q_z}{A} - \frac{Q}{A^2}A_z \]  \hspace{1cm} (16)

Both parts of the right-hand side of this equation are negative, so that both physical processes argue for a deep interior ocean circulation in the opposite sense to that of Stommel and Arons (1960). A different flow pattern must exist very near the ocean floor where the flow must

Fig. 5. Sketch of the buoyancy frequency \( N(z) = \left[ a - bz \right]^{-1} \) (where \( a \) and \( b \) are both positive) for which the vertical diffusivity may vary as \( N^{-1} \) and yet \( e_z \) is zero.
leave the ocean boundary and fill the interior. Perhaps the Stommel-Arons circulation applies to the vertically-integrated flow, with most of the interior going in the opposite direction to the deep boundary layer. Note that the argument based on Fig. 6 and Eqs (15) and (16) does not depend on the magnitude or the vertical variation of the vertical diffusivity. Rather, the argument has relied only on the conservation of volume: the vertical diffusivity and the vertical density stratification must arrange themselves to be consistent with the volume fluxes in the steady state. Perhaps the weakest part of this argument is the fact that only one source of Bottom Water is considered, whereas in reality, many sources of various density must exist. The simple remarks presented here on one-dimensional advective-diffusive balances are meant mainly to reinforce our feeling of ignorance about the deep ocean circulation; we have not come very far since 1960!

![Fig. 6. Sketch of a plume of Bottom Water descending into an ocean of area A(z). The volume flux in the descending plume, Q(z), must be equal to that due to upwelling in the ocean interior.](image)

**DIANEUTRAL ADVECTION VERSUS DIANEUTRAL DIFFUSION**

Equation (5) shows that the contribution of vertical mixing processes, as parameterized by a vertical diffusivity, $D_z$, to dianeutral advection is equal to $gN^{-2}D \left[ \alpha \theta_z - \beta S_z \right]$. Fig. 7 shows $gN^{-2} \left[ \alpha \theta_z - \beta S_z \right]$ contoured on the "27.25" neutral surface in the Pacific Ocean. A vertical diffusivity, $D_z$, of $10^{-4}$ m$^2$ s$^{-1}$, together with a contoured number of 10 on this figure implies a dianeutral velocity, $e$, of the canonical value, $10^{-7}$ m s$^{-1}$. This is the contribution to the dianeutral velocity of the vertical turbulent mixing process. Other contributions occur through the vertical variation of the vertical diffusivity, $D_z$, cabling, thermodaricity, and other mixing processes that have not been included in our analysis (such as double-diffusive convection, double-diffusive interleaving and the dianeutral advection of submesoscale coherent vortices). Some of the patchiness in these maps is undoubtedly due to the difficulty in estimating second order vertical derivatives from the data set. There is however a striking pattern in Fig. 7 showing upwelling in the North Pacific and downwelling in the South Pacific. A similar clear pattern of positive and negative values is seen on the "27.25" surface in the Indian Ocean, with upwelling between 10°S and 30°S, and predominantly downwelling in the other regions (Fig. 8(a)).
Fig. 7. Contour map on the “27.25” neutral surface in the Pacific Ocean of the term that when multiplied by the dianeuutral diffusivity, \(D\), is the part of the dianeuutral velocity caused by vertical mixing processes.

The contribution of vertical mixing processes to water-mass conversion is given by \(D\) multiplied by \(gN^2 \theta_z^2 \beta d^2S/d\theta^2\). This term is the sum of the diffusive flux divergence of potential temperature and the advective term, all divided by \(D\), i.e.,

\[
\frac{g}{N^2} \theta_z^3 \beta \frac{d^2S}{d\theta^2} = \frac{(D\theta_z)_z - e\theta_z}{D} = \frac{\beta(\theta_z S_{zz} - S_{zz}\theta_z)}{(\alpha\theta_z - \beta S_z)},
\]

(17)

where the last part of this expression indicates how it is evaluated from the data, using first and second order vertical derivatives of \(S\) and \(\theta\). The sign of this term is governed by that of \(d^2S/d\theta^2\), as \(\theta_z\) is positive on the surfaces presented here. Water-mass conversion caused by vertical mixing on the shallow neutral surfaces in the Atlantic and Pacific tends to increase the potential temperature and the salinity, while deeper in the water column, vertical mixing has a cooling and freshening tendency. On the “27.25” neutral surface in the Indian Ocean, vertical mixing cools and freshens the water-mass in the equatorial region, while at mid-latitudes, the opposite tendency is evident (Fig 8(b)).
Fig. 8. (a) Contour map on the "27.25" neutral surface in the Indian Ocean of the term that when multiplied by the dianeutral diffusivity, $D$, is the part of the dianeutral velocity caused by vertical mixing processes. (b) Contour map on the "27.25" neutral surface in the Indian Ocean of the term that when multiplied by the dianeutral diffusivity, $D$, is the rate at which vertical mixing changes the potential temperature on a neutral surface (this is called "water-mass conversion").
Inverse models of the ocean circulation often take dieneutral advection as the only vertical process in the conservation equations for scalars. Fig. 8(a) and (b) allow us to estimate the error associated with this procedure. In the conservation equation of potential temperature, the ratio of the total water-mass conversion caused by the vertical diffusivity, $D$, to that caused by only the dieneutral advective term is

$$
\frac{D}{N^2} \frac{g}{N^2} \alpha \frac{\theta_z^3 \beta \frac{d^2 S}{d \theta^2}}{- \theta_z D \frac{g}{N^2} [\alpha \theta_{zz} - \beta S_{zz}]} = \frac{\{\text{Fig. 8(b)}\}}{- \theta_z \{\text{Fig. 8(a)}\}}.
$$

On virtually the whole of this neutral surface the quantities mapped in Figs. 8(a) and 8(b) have the same sign so that the ratio of Eq. (18) is negative. This implies that existing box inverse models cannot find the correct sign for both the interfacial volume flux (e times the horizontal area of a box) and the total effect of vertical mixing processes in the conservation equation for potential temperature. Since the dieneutral velocity affects the linear vorticity equation, it is clear that by using only interfacial advection, one cannot simultaneously model the influence of vertical mixing processes on both this equation and the heat equation.

Of equal concern are the errors caused by such over-simplified physics when more than one passive tracer is considered. For example, if conservation equations are written for both salinity and potential temperature, vertical advection affects these equations in the ratio $\theta_z/S_z$, whereas in reality, the total effect of vertical diffusion and advection occurs in the ratio $\beta/\alpha$ (compare the $D$ terms in Eqs (4) and (6)). The relative changes in the inverse model’s $\theta$ and $S$ equations are then in error by $R_\theta = \alpha \theta_z/\beta S_z$; a ratio that is about 2 in the Central Water and is often negative in the Intermediate Water and below. Similarly, the ratio of the relevant two terms in the $C$ equation, (7), to the corresponding term in the $S$ equation will not resemble $C_2/S_2$ which it is forced to do by ignoring dieneutral diffusion and considering only dieneutral advection.

These remarks apply particularly to inverse models such as that of Wunsch (1984a) where the aim was to find the strength of equatorial upwelling using radiocarbon and other tracer data. One cannot expect to even find the correct sign for such upwelling/downwelling if one ignores an inherent aspect of dieneutral mixing, namely dieneutral diffusion. Similarly, Wunsch (1984b), in an eclectic model of the North Atlantic, found a “highly energetic convective, or thermohaline, mode of motion”, displayed in the streamlines of the zonally averaged overturning flow in the meridional sections of his figures 8b and 9. The above discussion, and the corresponding figures for the Atlantic (which are not shown here) shows that if the dieneutral motion of these figures had the correct sign, the concomitant water-mass conversion is modelled with an incorrect magnitude, and also the wrong sign in some of the North Atlantic. In an inverse model of the Deep Water in the Eastern Atlantic, Schlitzer (1987) included both interfacial mixing and advection, but without an equation like Eq. (5) above to link them together. In one of his solutions, he finds a zero vertical diffusivity and a downwelling velocity through his $\sigma_4$ surface of $13 \times 10^{-7}$ m s$^{-1}$. Such a large interfacial velocity must be caused by a correspondingly large vertical diffusivity. It is hoped that the step-by-step derivation of the appropriate conservation equations (4, 6 and 7) will assist in the design of future inverse models so that the elegant mathematical machinery developed by Wunsch and others can be used in a consistent fashion. Equation (6) has already been used successfully by Bauer and Siedler (1988). This paper and one by Hogg (1987) demonstrate that the second order vertical derivatives that are required can be evaluated with sufficiently
little noise. Bauer and Siedler (1988) found that the diffusivities could be determined quite accurately, and in particular, both lateral and vertical diffusivities were positive even when the model was perturbed substantially by doubling the lateral advection term. This study provides real hope that once mixing processes are included in diagnostic models (including inverse models) in a consistent fashion, we may expect that not only will the strength of mixing processes be a well-determined model output, but that such mixing processes will provide useful constraints on the reference level velocities and hence the magnitude of the mean circulation.

Table 1 shows the relative roles of dianeutral diffusion and advection for a variety of mixing processes in the ocean. The columns labeled “Diffusion” and “Advection” are the two contributions of the mixing processes to the right-hand side of the conservation equation for potential temperature, Eq. (4), and the sum of these columns appears at the right of the table, labeled “Water-Mass Conversion”. The first row is concerned with small-scale turbulent mixing as parameterized by a vertical diffusivity, $D$, and this process has been discussed in

<table>
<thead>
<tr>
<th>Process:</th>
<th>Diffusion</th>
<th>+ Advection</th>
<th>= Water-Mass Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small-scale turbulent mixing</td>
<td>$(D\nabla^2)z - D_z\nabla^2 - \theta_g N^2 D (\alpha \nabla^2 - \beta \nabla^2)$</td>
<td>$D g N^2 \frac{\partial}{\partial z} \theta \frac{\partial S}{\partial z}$</td>
<td></td>
</tr>
<tr>
<td>Double-diffusive convection</td>
<td>$-F_z^g$</td>
<td>$-F_z^g \left( \frac{1}{\gamma - 1} \right) \frac{\nabla^2 p}{[R_p - 1]}$</td>
<td>$-F_z^g \left[ \frac{R_p}{R_p - 1} \right]$</td>
</tr>
<tr>
<td>Cabling</td>
<td>$\rightarrow 0$</td>
<td>$\theta_z K g N^2 \frac{\partial \theta}{\partial z} \cdot \nabla \theta \cdot \nabla \theta$</td>
<td>$\theta_z K g N^2 \frac{\partial \theta}{\partial z} \cdot \nabla \theta \cdot \nabla \theta$</td>
</tr>
<tr>
<td>Thermobaricity</td>
<td>$\rightarrow 0$</td>
<td>$\theta_z K g N^2 \frac{\partial \theta}{\partial z} \cdot \nabla \theta \cdot \nabla p$</td>
<td>$\theta_z K g N^2 \frac{\partial \theta}{\partial z} \cdot \nabla \theta \cdot \nabla p$</td>
</tr>
<tr>
<td>Double-diffusive interleaving</td>
<td>$-(B\nabla^2)z$ (negative sign)</td>
<td>$\theta_z g N^2 \alpha (B \theta_z)z$</td>
<td>$\frac{1}{[R_p - 1]} (B\nabla^2)z$ (positive sign)</td>
</tr>
<tr>
<td>SCV motion</td>
<td>small ?</td>
<td>significant ?</td>
<td>significant ?</td>
</tr>
<tr>
<td>Helicity of neutral trajectories</td>
<td>? , but small</td>
<td>proportional to $\int_A \nabla \theta \times \nabla p , dA$, small</td>
<td>? , but small</td>
</tr>
</tbody>
</table>

this section. The second is double-diffusive convection, where the ratio of the vertical flux divergences of heat and salt (in buoyancy units) is $\gamma$. This case has been discussed in detail by McDougall (1987a). Double-diffusive convection changes the potential temperature of a water-mass at a rate that is larger than the diffusive contribution, $-F_z \theta$, by the ratio $[R_p/\gamma - 1]/[R_p - 1]$ which is often greater than 3. Cabling and thermobaricity are the next entries in the table. As explained above, these processes are advective by nature and they do negligible amounts of vertical diffusion (see also McDougall, 1987a).
Double-diffusive intrusions are commonly observed at thermoclinic fronts and are expected to be responsible for a significant amount of water-mass conversion in these locations. The total dianeutral fluxes of properties by double-diffusive interleaving are due to (i) the "diffusive" and "salt-finger" fluxes across the quasi-horizontal double-diffusive interfaces, and (ii), the dianeutral advection of the intrusions themselves. Garrett (1982) and McDougall (1985) have shown that the dianeutral advective contribution is often as large as that directly caused by the driving interfacial fluxes. The vertical fluxes of $\Theta$ and $S$ due to salt-finger are down their respective vertical gradients, while for positive $\Theta_r$ and $S_r$ (such as in the Central Water of the World's oceans), the dianeutral advective fluxes are up their gradients. The advective contributions to the fluxes of $\Theta$ and $S$ almost balance in buoyancy terms, while the salt-fingers transport twice as much salt as heat, and there are also small vertical fluxes due to the "diffusive" interfaces. The total dianeutral flux of heat is up the potential temperature gradient while the effective vertical diffusivity of salt is smaller and may be either positive or negative depending on the exact conditions and on the model assumptions that are not now well-determined. This suggests that a convenient way to parameterize double-diffusive interleaving is to take the vertical salt flux to be zero and the vertical flux of potential temperature to be given by a negative vertical diffusivity, $-B$, where $B$ is positive. This simple parameterization scheme should preserve the flavour of what is now known about double-diffusive interleaving. Since dissolved substances are fluxed by salt-fingers in a similar fashion to salt (the fluxes are proportional to the respective vertical gradients), the effective vertical diffusivity of tracer $C$ should also be taken to be zero. With this parameterization of double-diffusive interleaving, the conservation equation (1) for potential temperature has the term $- (B \Theta_r)$ on the right-hand side. The contribution of double-diffusive interleaving to the dianeutral velocity is shown in the table and the total effect on water-mass conversion acts like a positive diffusivity, $B$, with a weighting function $\left( R_p - 1 \right)^{-1}$. The positive nature of this effective diffusivity arises because the advective contribution outweighs the negative diffusive contribution.

The last two processes in Table 1 are concerned with less well known dianeutral processes, namely the dianeutral motion of submesoscale coherent vortices (SCVs) and the vertical advection caused by the ill-defined nature of neutral surfaces. Submesoscale coherent vortices move vertically through both potential-density surfaces and neutral surfaces (McDougall, 1987b) and this contribution to the average dianeutral motion in an ocean basin will be significant if submesoscale coherent vortices make a significant contribution to the epineutral fluxes of scalars. Since this has not yet been proven, a temporary conclusion is that the dianeutral motion of submesoscale coherent vortices is probably small. Because neutral surfaces are formally ill-defined, neutral trajectories are helicoidal in nature, and the lateral motion along neutral trajectories necessarily involves vertical advection (McDougall and Jackett, 1988). This vertical advection does not rely on vertical mixing processes per se. Recent work (Jackett and McDougall, manuscript in preparation) has shown that this process does not cause significant vertical motion in any part of the main thermocline in any of the world's oceans.

**BOUNDARY MIXING VERSUS INTERIOR MIXING**

Bottom Water is formed each year and yet the ocean is not believed to become continually more dense so it is clear that there must be a mean upwelling of water through neutral surfaces. It is not yet known where this upwelling occurs. The Stommel-Arons (1960) theory for the circulation of the deep ocean is driven by an upwelling velocity field that is
uniform in latitude and longitude and increases linearly with height, \( h \), from zero at the ocean bottom. Partly fuelled by the lower-than-expected observed levels of ocean microstructure in mid-ocean, some authors have suggested that a large part of the required mixing activity may occur at the ocean’s boundaries.

Armi (1978) stresses that mixed layers on seamounts or continental boundaries are periodically injected into the ocean interior by unsteady meso-scale eddies, thereby increasing the efficiency of boundary mixing above the level that would apply if the ocean were steady, while Garrett (1979) has argued that the mixing in bottom boundary layers is often quite inefficient in that where the energy available for mixing is the largest, there is a vanishingly small density gradient (i.e. the flux Richardson number is small where the overall Richardson number is small). Eriksen (1985), and more recently, Garrett and Gilbert (1988) have studied a different type of “near-boundary” mixing that is based on the relaxation of the internal wave field to the Garrett & Munk spectrum after internal waves are reflected off the sloping ocean floor. These theories can give quite large vertical diffusivities and the elevated mixing activity occurs within 100 m to 200 m of the ocean floor, which is quite a large distance compared with the typical bottom mixed-layer depth of about 35 m.

Early research on boundary mixing concentrated on the along-slope flow produced because of the vanishing density gradient normal to the slope (Phillips, 1970, and Wunsch, 1970). However, Thorpe (1987) has considered a fully turbulent boundary layer structure and has shown that this process leads to net motion along the slope that is proportional to the vertical diffusivity outside the boundary layer. Phillips et al. (1986) have analyzed the motion adjacent to a sloping turbulent boundary layer in a stratified fluid and have shown that the diffusion of material along the slope is due to three quite separate processes. First, the diffusivity in the boundary region, second a counter-flowing velocity field in the boundary region, and third, a mean convergent flow toward a depth of maximum buoyancy frequency.

In their terminology, if \( \kappa_e \) is the diffusivity in the boundary layer, \( \theta \) is the angle of the slope to the horizontal and \( \delta \) the boundary layer thickness, the magnitude of the up- and down-slope velocities in the counter-flowing structure is \( v' = \delta^{-1} \kappa_e \cot \theta \) and the buoyancy perturbation between the counter-flowing cells is \( b' = N^2 \delta \cos \theta \). The vertical velocity of each cell is of magnitude \( w' = \delta^{-1} \kappa_e \cos \theta \) so that the effective vertical diffusivity of this counter-flowing structure is of order \( [w'b']/N^2 = \kappa_e \cos^2 \theta = \kappa_e \), so that this counter-flowing convection process is quite weak. Here we will not be concerned with the detailed processes that cause diapycnal fluxes of properties in the boundary or “near-boundary” region, nor the detailed structure of the boundary layer. Rather, boundary mixing or near-boundary mixing will be simply regarded as a normal vertical mixing process, parameterized by a vertical turbulent diffusivity, \( D \), which is simply enhanced near ocean boundaries.

Laboratory studies by Thorpe (1982) and Ivey (1987a) show that when a stratified fluid is mixed at a side boundary, there is a continual bi-directional exchange of fluid between the interior and the boundary layer. This exchange amounts to an efficient means of mixing passive tracers into the interior but does not cause significant diapycnal transport (Ivey and Corcos, 1982). This observation has led to the common belief that strong mixing in the relatively small area of the ocean near boundaries is equivalent to a much weaker mixing intensity distributed evenly throughout the ocean. This assumption is often written in the form

\[
D_i^{\text{eff}} = D_b \frac{A_b}{A_i},
\]
(see for example, Ivey, 1987a,b), where $D^\text{eff}_i$ is the effective interior diapycnal diffusivity caused by the boundary mixing, $A_i$ is the area of the ocean measured along a particular isopycnal and $A_b$ is the area of this same isopycnal surface that is inside turbulent boundary regions where the diapycnal diffusivity is $D_b$ (see Fig. 9). In this section of the paper, it will be shown that if most mixing happened near boundaries, the circulation of the deep ocean would be profoundly affected. Interior and boundary mixing may have similar effects on the distributions of passive tracers but the mean circulation driven by the two types of mixing could hardly be more different.

![Region of near-boundary mixing](image)

**Fig. 9.** A schematic cross-section showing four near-boundary mixing regions of total area $A_b$, and the total lateral area, $A_i$, of a particular isopycnal.

**A Single Side-wall**

A steady-state ocean is considered in which there is boundary mixing but no interior mixing. Initially mixing is considered to be present at only one side boundary, and then the discussion is broadened to include the more general cases of many actively mixing boundaries and also the implications of mixing at seamounts. In a steady-state, the depths of all the neutral surfaces are constant and so the volume between pairs of neutral surfaces is constant. In an ocean with no interior mixing, there can be no dianeutral motion and so there is not net exchange of volume between the turbulent boundary region of Fig. 10 and the interior. All the upwelling of Bottom Water occurs in the turbulent boundary region. In the steady state, the area of the boundary region, $A_b$, the vertical density gradient and the mixing intensity, $D$, must have adjusted so that the buoyancy flux and the volume flux in the boundary region are both independent of height. The side-walls in Fig. 10 need not be vertical. This simple case demonstrates that a totally quiescent interior ocean is consistent with no interior vertical mixing.

**Several Side-walls**

The situation with two separate turbulent boundary regions is shown in Fig. 11. Since there is no dianeutral motion and the volume between neutral surfaces is constant (under the steady state assumption), there is still no nett flow into or out of the interior. However there can now
be flow in the ocean interior as fluid moves from one boundary region to the other. Where there is increased mixing activity at a particular depth in one boundary layer, the vertical buoyancy flux is locally enhanced, leading to a negative value of $e_z$ in the boundary region and a consequent loss of fluid from the boundary layer to the quiescent interior. This is indicated graphically in Fig 12. In a steady state, the volume flux lost from one boundary is gained by another. The implication of these simple arguments of buoyancy and volume conservation is that a deep ocean that is mixed only at its boundaries will not display the smoothly varying mean flows of the Stommel-Arons type. Rather, mean flows will occur in a more random pattern and probably with a smaller amplitude. These mean flows have the rather passive job of simply moving fluid from one boundary layer to another: all the action occurs in the boundary regions.

Seamounts

Seamounts are worthy of special consideration because the area available for boundary mixing is a strong function of height. The conservation equations for buoyancy and volume are

$$AeN^2 = \left[ ADN \right]_z$$ \hspace{1cm} (20)

and

$$Cu = - \left[ Ae \right]_z ,$$ \hspace{1cm} (21)

where the subscript $z$ has been dropped from the area, $C$ is the circumference around the seamount on a neutral surface, and $u$ is the horizontal velocity leaving the boundary layer and entering the non-turbulent interior ocean. (Of course, the non-linear equation of state term from Eq. (11) should also appear in Eq. (20), but is excluded here for convenience).

Assuming that the dominant contribution to $\left[ ADN \right]_z$ at a seamount comes from the comparatively rapid variation of $A$ with $z$, one finds that

$$Ae = DA_z ,$$ \hspace{1cm} (22)

and

$$Cu = - \left[ DA_z \right]_z = -DA_{zz} .$$ \hspace{1cm} (23)

The area available for mixing at a seamount is a decreasing function of height, $z$, and so from Eq. (22), the vertical motion in the turbulent boundary layer is downwards, implying that above this depth, the seamount is (on average) acting as a sink for the lateral flow field. It is interesting to consider some simple geometrical shapes to see how the ocean may choose to achieve the local balances described by Eqs (22) and (23).

Consider first a conical seamount. The lateral width of the boundary layer and the vertical diffusivity, $D$, are assumed to be independent of depth and so the area, $A$, the circumference, $C$, and the radius of the seamount are all proportional to $(z_0 - z)$, where $z_0$ is the height of the top of the seamount. $A_z$ is negative and constant along the cone so that the downwelling volume flux, $Ae$, is depth-independent and the horizontal velocity outside of the boundary layer, $u$, is zero. From where does the downwelling volume flux in the turbulent boundary layer originate? The answer lies in a singularity at the apex of the cone, as is indicated in Fig. 13 (a). The real ocean does not of course exhibit such singular behaviour, but it is not obvious how things will adjust. Perhaps the density gradient near the apex will change sufficiently in the vicinity of the seamount to ensure a smoother velocity structure near the apex.
Fig. 10. Sketch of an ocean in steady state with no interior mixing and with boundary mixing occurring at only one side-wall. All the upwelling of Bottom Water occurs in the turbulent boundary layer, and there is no net exchange of volume between the boundary region and the ocean interior.

Fig. 11. Two turbulent boundary regions can share the task of upwelling the Bottom Water. The volume flux divergence in one boundary layer is matched by the convergence at the other. There is still no divergence in the interior, although there must now be a mean flow there to effect the exchange of mass between the different boundary layers.
Fig. 12. These graphs indicate how increased vertical mixing near one boundary leads to outflow from that boundary. The area of the boundary mixing is assumed constant with height so that the dianeutral velocity is given by $eN^2 = \left[DN_z^2\right]_z$. The dashed lines are for a depth-independent upwelling velocity, $e$, that would apply if the vertical diffusivity, $D$, were not locally enhanced at the depth indicated.

Two other seamount geometries are considered in Fig. 13 (b) and (c): the seamount radius proportional to $(z_0 - z)^2$ and $(z_0 - z)^{0.5}$. The pinnacle shape of Fig. 13 (b) does not require any singularities, while the more realistic geometry of Fig. 13 (c) has an infinitely large downwelling flow at the apex and a corresponding singularity in the horizontal inflowing mass flux to match (F. Henyey, private communication, 1989). Of the three geometries considered here, all have downwelling in the boundary layer, but the lateral volume flux, $Cu$, is quite different in the three cases. The pinnacle (Fig. 13b) acts as a sink at all depths whereas the cone is a strong sink (a delta function) at the apex and thereafter does not exchange fluid with the ocean interior. The most realistic shape for the top of a seamount, Fig. 13c, has a strong double singularity at its highest point with alternating inflow and outflow. The volume flux of the inflow must be larger than that of the outflow because the turbulent boundary layer carries fluid downward at all depths.

One can imagine constructing a model of a realistic seamount by having the geometry of Fig. 13 (c) at the summit, a straight-sided seamount like Fig. 13(a) near the mid-depth, and a flared section like Fig. 13(b) at the base. The downwelling volume flux in the boundary layer would then be large near the summit, decrease to a constant value on the conical section and then increase on the flared section at the base of the seamount. Meanwhile, the very top of the seamount would act like a sink to the exterior fluid, followed by a source of fluid which decreased to zero on the conical section and then became a sink of constant strength on the flared section.
Fig. 13. Sketches of the seamount cross-section, the area of the boundary mixing, $A$, as a function of height, $z$, the dianeutral volume flux, $Ae$, and the lateral outflowing volume flux, $C u$, for three different seamount geometries. Fig (a) is a cone, Fig (b) has the radius of the seamount proportional to $(z_0 - z)^2$ and Fig (c) has the radius proportional to $(z_0 - z)^{0.5}$. 
It is clear from these simple geometrical examples that the implications of mixing near the top of seamounts are far from solved. Nevertheless, one can reach some useful conclusions about seamounts without a detailed knowledge of the mixing-induced flows near the summit. This is because Eqs. (22) and (23) apply at each depth, $z$, independent of our ignorance of the near-summit region where extra physical processes seem to be needed. From Eq. (22) it is concluded that because $A_z$ is always negative at a seamount, the fluid in the turbulent boundary moves downward so that, on average, seamounts are sinks for the deep ocean. From Eq. (23) it is concluded that the lateral flow into or out of the boundary is governed by the curvature of the seamount profile. A convex region with $A_{zz} < 0$ is a source for the deep ocean, while a concave region where $A_{zz} > 0$ is a sink. Since on average seamounts are sinks for the lateral flow field of the deep ocean, the fluid that is downwelled in the seamount boundary layers must make its way to continental boundaries or to mid-ocean ridges, where, on average, it is upwelled.

The picture that emerges for the deep circulation driven by boundary mixing is quite different to that of Stommel and Arons (1960) where the spatially uniform upwelling must be driven by interior mixing. Instead of broad smooth regions of poleward flow, we have found that the interior deep ocean is quiescent except for the need to transfer fluid from one turbulent boundary layer to another, and for the tendency for seamounts to act as sinks for the horizontal circulation. The contrasting deep circulation patterns are shown schematically in Fig. 14. It is concluded that while it may be satisfactory to regard boundary mixing in terms of an equivalent interior diffusivity for the purpose of understanding the mean distributions of passive tracers, the two types of mixing processes have very different dynamical implications for the mean circulation in the deep ocean.

![Fig. 14. Sketches of the Stommel-Arons (1960) circulation (Fig a), and a circulation caused by boundary mixing at continental margins and seamounts. The boundary layers adjacent to continents must upwell not only the Bottom Water flux, but also the water that is pumped down at the boundaries of seamounts.](image-url)
CONCLUSIONS

The importance of the vertical velocity to the general circulation of the ocean provides much of the incentive for measuring the strength of ocean microstructure and thereby deducing the vertical advection through neutral surfaces (i.e. dianeutral advection). The separate roles of dianeutral advection and dianeutral mixing in causing water-mass conversion have been illustrated in this paper by maps of the relevant vertical derivatives of hydrographic variables on some neutral surfaces from each of the world's oceans. For example, maps of the upwelling (dianeutral advection) caused by small-scale mixing are presented, normalized by the vertical diffusivity. These maps show that there is considerable variability in the magnitude and sign of such upwelling at different depths and at different locations on a neutral surface. Vertical diffusion and vertical advection have no strong tendency to be correlated for any given tracer, and so the common practice in inverse models of lumping the effects of dianeutral advection and diffusion into a single interfacial volume flux is unacceptable. This practice often forces the vertical processes to contribute a term of the wrong sign to the conservation equations.

Two processes – thermobaricity and cabbeling – achieve dianeutral advection without relying on vertical mixing processes, and so this part of the total vertical velocity is invisible to microstructure instruments that measure the dissipation rate of mechanical energy, $\varepsilon$. Thermobaricity and cabbeling are quite strong in the North Atlantic and Southern Oceans, causing contributions to the dianeutral downwelling velocity of order $-1 \times 10^{-7}$ m s$^{-1}$. These in turn make a large impact on the conservation equations of scalars in these regions. Of the two processes, thermobaricity is rather smaller than cabbeling except in the Antarctic Circumpolar Current where it is at least as large as cabbeling and also of the same sign.

It is common to use the dissipation rate of mechanical energy, $\varepsilon$, to infer the vertical diffusivity through Osborn's (1980) formula, $D = \Gamma^2 \varepsilon N^{-2}$. The nonlinearity of the equation of state means that in the thermocline, the real dianeutral velocity will often be $10^{-7}$ m s$^{-1}$ less than the estimate, $D\varepsilon / \varepsilon$, that one would infer from the microstructure data alone.

It is shown that vertical mixing processes do not contribute to cabbeling or thermobaricity. The opposite side of this coin is that when a conservation equation is written for potential density, the contribution of the vertical diffusivity acting on the nonlinear terms of the equation of state must be subtracted from the normal diffusive flux divergence of potential density. Failure to do this in inverse models to date has undoubtedly led to significant errors in the diapycnal velocities of such models.

The final section of the paper considers the contrasting dynamical implications of interior mixing and boundary mixing in the deep ocean. Simple arguments, based largely on the conservation of volume in a steady ocean, have shown that it is quite misleading to regard boundary mixing as equivalent to an "effective interior diffusivity." In an ocean with no interior mixing, Bottom Water is upwelled only in the turbulent boundary layers on continental boundaries, with the interior ocean being largely stagnant, and seamounts causing a local sinking of fluid in their vicinity.

Acknowledgments

Dr Frank Henyey is thanked for first pointing out to the author the singularity in the flow field, due to boundary mixing, that occurs at the apex of any smoothly shaped seamount.
REFERENCES


