SMALL-SCALE MIXING: A FIRST-ORDER PROCESS?

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ABSTRACT
To within a factor of 2, observations from diverse sites in the mid-latitude thermocline demonstrate that \( \langle \varepsilon_I W \rangle = 7 \times 10^{-10} \langle N^2/N_0^2 \rangle \langle S_{10}/S_{CM}^4 \rangle \) W kg\(^{-1} \), where \( \langle \varepsilon_I W \rangle \) is the average rate of turbulent dissipation due to internal waves, \( N_0 = 0.0052 \), and \( S_{CM} \) is the root mean square shear larger than 10 m calculated with the Garrett and Munk model. When the observed shear matches the Garrett and Munk level, this scaling yields a vertical eddy diffusivity of \( 5 \times 10^{-6} \) m\(^2\) s\(^{-1} \), independent of depth. Levels this low suggest that small-scale mixing is not a major process forming the thermocline and can be omitted from dynamical models, although it must be important for water mass modification occurring over decades. In some places, however, intense shear produces much higher mixing. Owing to the strong sensitivity to shear, the occurrence of significant mixing can be parameterized in models only by including the climatology of internal waves.

INTRODUCTION
Since Munk’s (1966) Abyssal Recipes, small-scale mixing has been considered a major factor in the creation and formation of the thermocline. Subsequent modelers have strengthened this belief by stressing the need for turbulent transport similar to Munk’s vertical eddy diffusivity of \( K_v = 1 \times 10^{-4} \) m\(^2\) s\(^{-1} \). A recent example is Frank Bryan’s (1986) sensitivity study using primitive equations. These modelers assume that one eddy coefficient represents the fluxes generated by small-scale turbulence, at least when the fluxes are averaged over several years and the 100 km grid scale used for models of the general circulation. Recently, however, Ann Garlett (1984) and Garlett and Holloway (1984) argue that \( K_v \) varies inversely with the buoyancy frequency \( N \), producing a vertical gradient in diffusivity that profoundly affects the interior circulation of ocean basins.

Within this general context, our observational strategy has been to ask

- When and where is turbulence important?
- How can turbulent transport be parameterized?
- What is the energy flux through the internal wave spectrum?
The first question is being answered gradually as data from diverse places and seasons are reported. Below are some observations that span the full range of probable importance.

Except for the preliminary $u_3 T'$ measurements of Jim Moun, the second question is being addressed indirectly. $K_T$ is calculated using simple models and measurements of $\chi_T \equiv 6 \kappa_T (\partial T' / \partial x_3)^2$ and $\varepsilon \equiv 7.5 \nu (\partial u'_1 / \partial x_3)^2$. The vertical eddy coefficient for heat, $K_T$, is inferred by assuming a simple balance between local production and diffusive smoothing of temperature fluctuations (Osborn and Cox, 1972), which yields

$$K_T = \frac{\chi_T}{2 \kappa_T (\partial T' / \partial x_3)^2} \quad [\text{m}^2 \text{ s}^{-1}]$$  \hspace{1cm} (1)

A similar approach (Osborn, 1980) gives the vertical buoyancy flux in terms of

$$K_P \leq \frac{0.2 \varepsilon}{N^2} \quad [\text{m}^2 \text{ s}^{-1}]$$ \hspace{1cm} (2)

The assumptions leading to these expressions appear reasonable, and the two approaches seem to give consistent values. No one, however, has verified either formula by comparison with changes in average properties. Furthermore, at this meeting Holloway continued to assert that inferring diffusivities from $\varepsilon$ may be quite wrong. Although his arguments seem implausible to me, they cannot be dismissed. In any event, the lack of any confirmation of these inferential techniques is disquieting.

Because we think that breaking internal waves produce most of the small-scale mixing, the first two questions lead to the third. Things are somewhat better with this question, since it can be addressed more directly—by relating $\varepsilon$ observations to the intensity of internal waves. The major observational questions reduce to assessing isotropy and the statistical adequacy of the sampling. Working for the past nine years, we have compiled six sets of simultaneous observations of shear and $\varepsilon$, allowing us to address the last question with new data. The results, however, apply as well to the first two questions.

INTERNAL WAVE SCALING OF TURBULENT DISSIPATION

By comparing simultaneous observations of $\varepsilon$, $N$, and shear calculated over 10 m intervals, we find that

$$\langle \varepsilon_{IW} \rangle = 7 \times 10^{-10} \langle N^2 / N_0^2 \rangle \langle S_{10}^4 / S_{GM}^4 \rangle \quad [\text{W kg}^{-1}]$$ \hspace{1cm} (3)

where $N_0 = 5.2 \times 10^{-3} \text{ s}^{-1}$, $S_{10}$ is the observed 10 m shear, and $S_{GM}$ is the corresponding shear in the Garrett and Munk spectrum. Thus, $\varepsilon_{IW}$ varies as the fourth power of the shear and the square of the buoyancy frequency.
Taking $u_1$ and $u_2$ as the eastward and northward velocity components, the shear components are computed as

$$S_{x_1} = \sqrt{2.11 \frac{\Delta u_1}{\Delta x_3}}, \quad S_{x_2} = \sqrt{2.11 \frac{\Delta u_2}{\Delta x_3}}$$

(4)

where $\Delta x_3 = 10$ m. The factor of $\sqrt{2.11}$ corrects for the attenuation of the first-difference filter and makes the shear variance comparable to that obtained by integrating a spectrum to 0.6 rad m$^{-1}$. Then,

$$S_{10}^4 = (S_{x_1}^2 + S_{x_2}^2)^2$$

(5)

For comparison, the variance of the 10 m shear in the Garrett and Munk model is

$$S_{GM}^2 = \frac{3\pi}{2} j^* E_{GM} b N_0^2 k_3^G (N/N_0)^2 = 1.91 \times 10^{-5} (N/N_0)^2 \quad [s^{-2}]$$

(6)

Garrett and Munk did not estimate the fourth moment, but they did assume that $S_{x_1}$ and $S_{x_2}$ are statistically independent and normally distributed. We examined two of our data sets and found that both conditions are satisfied. Applying normal probability statistics, H. Seim (personal communication, 1988) showed that the average of the fourth moment is twice the square of the variance. We also verified this by comparing $2 \langle S_{10}^2 \rangle$ with $\langle S_{10}^4 \rangle$. Therefore, for the reference shear we use

$$\langle S_{GM}^4 \rangle = 2 \langle S_{GM}^2 \rangle^2$$

(7)

The functional form of $\langle \varepsilon_{HW} \rangle$ is the same as that predicted by McComas and Müller (1981) and by Henyey et al. (1986), provided the shear varies as $S_{GM}^2 \propto E_{GM}$, as it does in Eq. (6).

McComas and Müller assumed that nonlinear interactions among internal waves are weak and calculated the flux of energy through the spectrum due to two resonant mechanisms: parametric subharmonic instability and induced diffusion. Supposing that all of this flux is dissipated, they estimated

$$\varepsilon_{MM} = \left( \frac{27\pi}{32\sqrt{10}} + 1 \right) \pi^2 j^*_s b^2 f N^2 E_{GM}^2 \quad [W \ kg^{-1}]$$

(8)

where $j^*_s = 3$ and $b = 1300$ m are parameters in the Garrett and Munk model.

Henyey et al. recognized that interactions within the internal wave spectrum are not necessarily weak, but argued that the primary interactions occur between widely different wavenumbers. Since most of the energy in the internal wave spectrum is at low
wavenumber, they followed the evolution of energy at high wavenumbers by using the ray-tracing equations. After several further assumptions, they obtained

$$\varepsilon_{HF} = (1.67/\pi)j^2 b^2 f \cosh^{-1}(N/f) N^2 E_{GM}^2 \quad [W \text{ kg}^{-1}]$$

(9)

Thus, the two predictions have the same $fN^2E_{GM}^2$ dependence. Neglecting the $\cosh^{-1}(N/f)$ factor, which varies little, Eq. (3) is twice $\varepsilon_{HF}$ and about one third $\varepsilon_{MM}$.

**COMPARISON WITH OBSERVATIONS**

The need to scale with $S_{10}$ as well as with $N$ is demonstrated in Fig. 1. Three cruise averages of $\langle \varepsilon \rangle / \nu \langle N^2 \rangle$ are compared in the left panel. We made the PATCHEX observations in the outer reaches of the California Current when the internal wave shear was nearly indentical with the Garrett and Munk levels (Gregg and Sanford, 1987).

Fig.1. Scaling comparisons for three mid-latitudes averages. Including shear-scaling greatly reduces the large differences remaining when $\varepsilon$ is scaled only with $N$. 
Becoming bored with Garrett and Munk's energy level, we left PATCHEX a few days early and headed for a mature coastal jet off Crescent City. Although neither the shear nor $\varepsilon$ showed signatures in the coastal jet, both were much higher than at PATCHEX. We collected the RING82I data south of New England while waiting for an outbreak of cold air (Larson, 1988). Even before the arrival of the storm trailing the cold air behind it, both shear and dissipation were very high: averaged vertically, $\langle \varepsilon \rangle / \nu \langle N^2 \rangle$ for RING82I was 58 times the similar average for PATCHEX, compared with 9 times higher for PATCHEX north.

Applying the full scaling greatly reduces the variability in dissipation rates (Fig. 1, right panel). Averaged vertically, $\langle \varepsilon \rangle / \langle \varepsilon_{IW} \rangle$ is 1.52 for RING82I, 1.11 for PATCHEX north, and 1.00 for PATCHEX (Table 1). The very high dissipation rates for RING82I resulted from internal energy levels of $4.3E_{GM}$, compared with $3.2E_{GM}$ for PATCHEX north. We also have a fourth data set from mid-latitudes, DRIFTER (not shown in the figures, but listed in Table 1). Its vertical average is $\langle \varepsilon \rangle / \langle \varepsilon_{IW} \rangle = 0.5$, on the low side, but within the factor of 2 claimed for the scaling.

Since these four sets of profiles were taken at 31°–42°N, the variation in $f$ is much smaller than the statistical scatter of the data. Consequently, we used 34°N, the latitude of PATCHEX, in computing $\varepsilon_{IW}$ for them. Three of these data sets were taken.

Table 1. Vertical averages. Excluding CSALT, $\langle \varepsilon \rangle / \nu N^2$ varies from 0.21–58.3 times the PATCHEX value, compared with 0.5–1.5 for $\langle \varepsilon \rangle / \langle \varepsilon_{IW} \rangle$, strongly demonstrating the need to scale with shear as well as with buoyancy frequency. For CSALT, $\langle \varepsilon \rangle / \langle \varepsilon_{IW} \rangle$ is larger than predicted, consistent with salt fingers producing turbulence in addition to that generated by internal waves. For TROPIC HEAT 2, we use a conditional average calculated by setting to zero all raw $\varepsilon$'s less than $10^{-10}$ W kg$^{-1}$. Averaging all the data, including values dominated by noise, gives $\langle \varepsilon \rangle / \nu N^2 = 5.1$ and $\langle \varepsilon \rangle / \langle \varepsilon_{IW} \rangle = 1.99$ and 2.16, with and without $f$-scaling, respectively.

<table>
<thead>
<tr>
<th>Cruise</th>
<th>$\langle \varepsilon \rangle / \nu N^2$</th>
<th>$\langle S_{in}^2 \rangle / E_{GM}$</th>
<th>$\langle S_{in}^4 \rangle / S_{GM}^4$</th>
<th>$\langle \varepsilon \rangle / \langle \varepsilon_{IW} \rangle$</th>
<th>$\langle \varepsilon \rangle / \langle \varepsilon_{IW} \rangle$</th>
<th>$f$-scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>PATCHEX (1.0–9.5)</td>
<td>13.1</td>
<td>0.98</td>
<td>1.02</td>
<td>1.00</td>
<td>1.00</td>
<td>1.08</td>
</tr>
<tr>
<td>RING82I (0.5–2.5)</td>
<td>763.4</td>
<td>4.31</td>
<td>43.66</td>
<td>1.52</td>
<td>1.52</td>
<td>1.35</td>
</tr>
<tr>
<td>PATCHEXn (1.0–9.5)</td>
<td>123.4</td>
<td>3.17</td>
<td>9.89</td>
<td>1.11</td>
<td>1.11</td>
<td>1.08</td>
</tr>
<tr>
<td>DRIFTER (0.5–2.0)</td>
<td>14.8</td>
<td>1.54</td>
<td>2.19</td>
<td>0.50</td>
<td>0.50</td>
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<tr>
<td>TROPIC HEAT 2 (1.0–9.5)</td>
<td>2.7</td>
<td>0.86</td>
<td>0.80</td>
<td>0.96</td>
<td>0.96</td>
<td>2.33</td>
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<tr>
<td>CSALT (1.0–7.0)</td>
<td>17.2</td>
<td>0.70</td>
<td>0.64</td>
<td>3.52</td>
<td>3.52</td>
<td>8.44</td>
</tr>
</tbody>
</table>
in diffusively stable profiles below the shallow salinity minimum in the eastern Pacific. RING821, however, sampled a profile that was diffusively unstable, to the diffusive regime above 1.5 MPa and to salt fingering below 1.5 MPa. In addition, many strong thermohaline intrusions were found. Nevertheless, Larson (1988) found no relationship between the intrusions and the dissipation. Furthermore, he determined that the dissipation rates are much larger than can be attributed to double diffusion. Consequently, we attribute the primary dissipation in all of these profiles to internal waves.

Two additional sets of observations were made, both near 12°N: CSALT in the thermohaline staircase east of Barbados (Gregg and Sanford, 1987), and TROPIC HEAT 2 in the central Pacific. Vertical averages give $\langle \varepsilon \rangle/\langle \varepsilon_{IW} \rangle = 3.5$ for CSALT (Table 1), more than twice any of the others. Multiplying $\langle \varepsilon_{IW} \rangle$ by $f/f_{34^\circ}$ to include the $f$-scaling (Fig. 2) raises this to 8.4. By contrast, although TROPIC HEAT 2 was taken

Fig. 2. Comparisons including the $f$-scaling for two sites at 12°N and PATCHEX, at 34°N. Although both low-latitude sites are higher than predicted by the scaling, the discrepancy is much larger for CSALT than for TROPIC HEAT 2, presumably due to the salt fingering that forms the thermohaline staircase in the CSALT profiles.
at the same latitude, its vertical average is $\langle \varepsilon \rangle / \langle \varepsilon_{IW} \rangle = 0.96$ and 2.3, without and with the $f$-scaling. Therefore, both are higher than expected for that latitude, but CSALT is much higher.

Although these data do not confirm a decrease in dissipation toward the equator, we cannot conclude that the prediction is wrong, because both profiles are diffusively unstable to salt fingering. The existence of the staircase and other evidence observed during CSALT is consistent with weak salt fingers in the interfaces. Although we found no staircases during TROPIC HEAT 2, the profile is diffusively unstable to salt fingers between 2 and 6 MPa. The major difference between the two profiles is the density ratio, $R_\rho \equiv (\alpha \partial \tilde{T}/\partial x_3)/((\beta \partial \tilde{s}/\partial x_3)$, which is 4 for TROPIC HEAT 2 and 1.6 for CSALT. Schmitt (1979) argues that fingers become more intermittent as $R_\rho$ increases, and are therefore less likely to form staircases. Whether this also implies lower buoyancy fluxes is uncertain, as Kunze (1987) argues that the fluxes increase with $R_\rho$. Therefore, it is possible that 1) $\varepsilon_{IW}$ decreases toward the equator as predicted, 2) salt fingering in the tropical halocline obscures the decrease, and 3) dissipation due to the salt fingering varies with $R_\rho$.

CONCLUSIONS AND DISCUSSION

From the summaries in Table 1 and Fig. 3, we conclude that

- In the mid-latitude thermocline, to within a factor of 2, the average dissipation rate scales as $\langle \varepsilon_{IW} \rangle = 7 \times 10^{-10} \langle N^2/N_0^2 \rangle \langle \bar{S}_{10}/S_{GM} \rangle$ W kg$^{-1}$. The functional form matches predictions by McComas and Müller (1981) and Henyey et al. (1986), but the magnitude is one third of McComas and Müller's estimate and twice that of Henyey et al.

- Two sets of observations at $12^\circ$N had higher dissipation rates than expected when the $f$-scaling in the predictions is included. One of the sets was taken through the thermohaline staircase east of Barbados, making salt fingers a likely source of the additional dissipation. Although the second set did not have a staircase, much of the profile was diffusively unstable to salt fingers, and we cannot exclude them as an extra source of dissipation.

By virtue of its success with data as disparate as PATCHEX and RINGS21, it seems likely that the scaling is at least approximately correct for mid-latitudes. What are the implications?

When the internal wave shear matches the Garrett and Munk value, Eq. (2) gives $K_\rho \leq 5 \times 10^{-6}$ m$^2$ s$^{-1}$, independent of $N$. This diffusivity is close to $K_\rho \approx 10^{-5}$, increasing the evidence in support of Garrett’s (1984) ‘zeroth-order view’ of internal waves and mixing. Although this may well demonstrate that we have a first-order understanding of the important dynamics linking internal waves and mixing, we should not ignore the possibility that we may have the right answer for the wrong reasons. Had the scaling not worked, plausible excuses are close at hand, beginning with the linear interactions used by McComas and Müller and the ad hoc assumptions made by
Fig. 3. Summary of vertical averages of the scalings.

Henyey et al., including the discrepancies between the Garrett and Munk spectrum and Pinkel’s (1985) observations, and ending with Holloway’s questioning of the entire basis for inferring diffusivity from dissipation rates. Thus, in spite of the success of the $\langle \varepsilon_{IW} \rangle$ scaling, too many issues remain unresolved for us to be fully confident that we now have a first-order understanding of the dynamics.

The scaling gives a constant and small diffusivity when internal waves are at the background level modeled by Garrett and Munk. How well must we know that constant? Is $K_p \leq 5 \times 10^{-6}$ so low that it can be set to zero in dynamical models? If so, small-scale mixing is not a first-order dynamical process in these places, although, over decades, it may modify water masses. If mixing in most of the thermocline is of secondary importance, then working out the details is not a high priority problem for oceanography, even though it is interesting fluid mechanics. To sort this out, we need numerical sensitivity studies. However, present models cannot run with such low diffusivities; owing to their large grid spacings, the models require higher diffusivities for numerical stability. While waiting for the next generation of computers, we should focus on confirming the dissipation scaling, Eq. (3), and testing our formulas, Eqs. (1) and (2).

Ledwell’s (this volume) tracer release offers the only prospect for testing Eqs. (1) and (2) in the open ocean, but the dissipation scaling indicates that we will have severe sampling problems. The preliminary plans are to release the tracer along parallel
tracks tens of kilometers long and to sample it at intervals no shorter than 6 months. After a year, the tracer streaks will extend at least 500 km. If internal wave breaking is the major process mixing the tracer vertically, then the variability of the internal wave field establishes the sampling requirements for microstructure, or for $S_{10}$. Horizontal coherence lengths for internal waves are about 10 km (D'Asaro and Perkins, 1984). Although the energy levels do not necessarily change over short distances, they may—as we learned to our dismay when we found a very energetic inertial feature 50 km away after making the DRIFTER microstructure observations. Since we may need to profile continuously for 1–2 days to estimate $\varepsilon$ and $\chi_T$ to within a factor of 2 at one place, no conceivable amount of sampling can give adequate averages over a 500 km box until we know much more about the spatial variability of internal waves. To compound the difficulties, Briscoe and Weller (1984) report an annual cycle in the energy of high-frequency waves and episodic bursts of low-frequency waves. Thus we must conclude that ship-based microstructure observations will be inadequate for a decent statistical comparison with the tracer. Therefore, the primary data must come from floats or moorings. As one approach, I am working with Russ Davis to add temperature microstructure to the bobbing floats (ALICE) that he is building with Doug Webb.

Although we have doubts about the importance of small-scale mixing accompanying background internal wave levels, there is little question about its significance when internal waves rise above the background. For instance, in PATCHFEX north $K_p = 5 \times 10^{-5}$ m$^2$ s$^{-2}$ throughout the upper kilometer; and in RING82I, $K_p \approx 5 \times 10^{-4}$ from 0.5–1.5 MPa and $\approx 8 \times 10^{-5}$ from 1.5–2.5 MPa. Based on observed changes over the winter in another warm-core ring, Terry Joyce (personal communication, 1989) inferred diffusivities similar to those we measured in 82I. Since diffusivities this large produce important water mass changes, they need to be incorporated into numerical models by including the variability of internal wave shear. Until now, the hope was that the vertical fluxes of all processes between mesoscale and microscale could be parameterized by simple eddy coefficients that were either constant or, more recently, varied only with $N$. It now seems that we cannot get off so simply and that we must deal with the intermediate processes, at least where small-scale mixing is large.

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