EFFECTS OF VARIABLE VERTICAL DIFFUSIVITY IN THE CFDL MODEL

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ABSTRACT

The sensitivity of the CFDL ocean model to stratification-dependent diapycnal diffusivity $K_d \propto N^{-1}$ has been examined. Provided that the value of $K_d$ in the upper ocean is small (of order 0.1 cm$^2$s$^{-1}$), model diagnostics such as meridional heat flux, meridional stream function, and direction of deep interior flow are relatively insensitive to changes in the deep diffusivity. However, use of $K_d \propto N^{-1}$ significantly changes the model's deep ocean density stratification, suggesting that the specific parameterization chosen for $K_d$ may be important in problems which are sensitive to deep ocean characteristics.

INTRODUCTION

The question of the sensitivity of ocean models to the particular form and magnitude of diffusion coefficients is of considerable interest to microscale observers, who naturally wish to know the broader significance of the scales they measure. Moreover it is central to the question of how much faith we should place in present numerical ocean models, particularly those used in the predictive sense required by climate concerns. However, while the perceived need for sensitivity studies is great, it is not clear how best to proceed with them. Models with sufficient horizontal resolution to resolve mesoscale eddies (hence eliminating the need to parameterize the associated stirring and mixing effects) are presently being run but are so computationally intensive that the thermohaline circulation is not calculated (Semtner and Chervin, 1988) and/or does not achieve steady-state (F. Bryan, this volume). Thus it is not yet possible to carry out an exploration of the effects of different parameterizations of $K_d$, the diapycnal eddy diffusivity for mass, in eddy-resolving ocean models. If we back off to non-eddy-resolving models which parameterize the effects of eddies on the mass field by a horizontal eddy diffusivity $K_h$, assumed to be much larger than $K_d$, it becomes possible to carry out extensive investigation of the $(K_d, K_h)$ parameter space. Unfortunately, there is no guarantee that the observed sensitivity to $K_d$ in such models will necessarily carry over to eddy-resolving models. In this Catch-22 situation, we are left with the reality that the models presently being coupled with atmospheric models for climate studies are of the non-eddy-resolving variety. Thus it seems necessary to investigate the parameter-space sensitivity of such coarse resolution models.
This paper reports preliminary results of an investigation of the effects of a non-constant \( K_d \) on the widely distributed GFDL model (Cox, 1984). In particular, we examine the stratification-dependent diffusivity proposed by Gargett (1984): \( K_d = a_o N^{-1} \), with \( a_o = 10^{-3} \text{ cm}^2\text{s}^{-2} \). To facilitate comparison with Bryan's (1987) investigation of the effects of different constant values of \( K_d \), we chose a model configuration which is roughly similar in both physical configuration and forcing.

The standard GFDL code was implemented in a flat-bottomed ocean basin of dimension 60° in both latitude and longitude. Resolution was 1.5° in both horizontal directions, while the 5 km depth range was divided into 15 levels, weighted to provide increased resolution in the upper ocean. The model is forced by a steady double-gyre zonal wind stress, resulting in the standard double-gyre form for the barotropic stream function (Fig. 1). The surface layer \( T \) (temperature) and \( S \) (salinity) are required to relax to the zonally constant meridional profiles shown in Figure 1, with relaxation time constant of 25 days. Convection is incorporated by setting \( K_d \) to a very large value \( (10^4 \text{ cm}^2\text{s}^{-1}) \) where unstable density gradients were detected (in place of the standard 'convective adjustment' algorithm). The diffusion tensor was rotated to parallel the local isopycnal slope (Redi, 1982), with a view to reducing implicit mixing across isopycnals due to horizontal diffusion \( K_h \) acting in regions of sloping isopycnals: the isopycnal diffusivity \( K_p = 1 \times 10^7 \text{ cm}^2\text{s}^{-1} \) in all experiments. Unfortunately, a non-zero value of \( K_h \) must be retained in order to maintain numerical stability, so the experiments are not free

Fig. 1. Two-gyre barotropic stream function resulting from the GFDL model forced by the zonally constant wind stress shown at the left. Thermohaline forcing is by relaxation of surface layer \( T \) and \( S \) to zonally constant values shown at the right. Heavy lines mark standard sections used to display N-S and E-W variation of model results.
from effects of $K_n$. Table 1 gives the values of $K_d$, $K_p$ and $K_h$ used in the three experiments which are reported here.

Experiment 1 was started from rest and spun up using the accelerated convergence mechanism of Bryan (1984). After ~1000 years of equivalent real time (ERT), the system reached a quasi-steady-state, in which the time trends of area-averaged deep ocean temperatures were less than $(5 \times 10^{-3})^\circ$C/century. Subsequent experiments, initiated from the end state of Exp. 1, took a few hundred years ERT to regain quasi-steady-state after change of $K_d$.

Table 1. Values of diapycnal diffusivity $K_d$, isopycnal diffusivity $K_p$, and horizontal diffusivity $K_h$ used in model runs. $a_o = 1 \times 10^{-3}$ cm$^2$s$^{-2}$.

<table>
<thead>
<tr>
<th>EXP</th>
<th>$K_d$ (cm$^2$s$^{-1}$)</th>
<th>$K_p$ (cm$^2$s$^{-1}$)</th>
<th>$K_h$ (cm$^2$s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2</td>
<td>$a_o N^{-1}$</td>
<td>$1 \times 10^{-7}$</td>
</tr>
<tr>
<td>2</td>
<td>$a_o N^{-1}$</td>
<td>$1 \times 10^{-7}$</td>
<td>$1 \times 10^{-7}$</td>
</tr>
<tr>
<td>3</td>
<td>$a_o N^{-1}$</td>
<td>$1 \times 10^{-7}$</td>
<td>$0.33 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

EFFECTS OF $K_d = a_o N^{-1}$

Implementation of the stratification-dependent diapycnal diffusivity $K_d = a_o N^{-1}$ was straightforward and produced no problems. The resulting distribution of $K_d$, shown in Figure 2, has low values in near-surface ($z < 500$m) mid-ocean regions; with increasing depth (decreasing stabilities), values rise, reaching $0(1$ cm$^2$s$^{-1}$) near 1 km and 5–7 cm$^2$s$^{-1}$ in the bottom kilometer of the basin. Even higher values near the northern wall are associated with low stabilities downstream of the region (in the northeast part of the domain) where convection drives the thermohaline cell. These low stabilities are advected south in a western boundary undercurrent, where they are visible in the section at 20$^\circ$N (not shown).

Although the field of $K_d$ varies by nearly two orders of magnitude from the once-canonical value of 1 cm$^2$s$^{-1}$, the effect of such variation on the advective heat flux carried by the ocean (a quantity of primary importance to global climate models) is relatively slight. As seen in Figure 3, using $K_d = a_o N^{-1}$ (Exp. 2) instead of the constant 0.2 cm$^2$s$^{-1}$ (Exp. 1) decreases the poleward heat flux by at most ~30% over the mid-latitude band of interest. The poleward heat flux is calculated by averaging the product $vT$, where $v$ is meridional velocity and $T$ is temperature, over both the vertical and longitudinal directions. The insensitivity of $< vT >$ occurs because changes in the T and v fields tend to partially compensate. Experiment 2 results in an ocean which is slightly colder above the main thermocline ($z \approx 750$m) and substantially warmer (by ~0.5$^\circ$C) below. By
itself this change would lead to considerably lower poleward heat flux; however it is accompanied by an increase of ~20% in the strength of the main meridional overturning cell (Fig. 4) which nearly compensates for the T field changes.

Fig. 2. Field of diapycnal diffusivity $K_d = a_0 N^{-1}$ associated with the steady state of Experiment 2 (Table 1): units are (cm$^2$s$^{-1}$). (a) Expansion of upper 1000m. (b) Entire water column.

Fig. 3. Over mid-latitudes, implementation of $K_d = a_0 N^{-1}$ (Exp. 2) yields a poleward heat flux approximately 30% lower than that associated with constant $K_d = 0.2$ cm$^2$s$^{-1}$ (Exp. 1).
Fig. 4. The zonally averaged meridional overturning stream functions for (a) Exp. 1 and (b) Exp. 2. Dashed contours indicate clockwise flow, solid contours counterclockwise flow; contour interval is $2.5 \times 10^6$ m$^3$s$^{-1}$.

Another point of interest is whether the meridional flow in mid-gyre remains poleward (Stommel and Arons, 1960) or whether a diapycnal diffusivity proportional to $N^4$ reverses the direction of this flow, a possibility suggested in Gargett (1984). The answer is that the meridional flow of Exp. 2 not only remains poleward in direction, but increases in magnitude relative to Exp. 1, as seen in Figure 5. While the strong surface flows of Exp. 2 (Fig. 5(b)) are comparable to those in Exp. 1 (Fig. 5(a)), the region of significant poleward flow in the deep ocean moves out from the western boundary and extends throughout the deep interior of the gyre. The vertical boundary between the northward-flowing surface western boundary current and the western boundary undercurrent (WBUC) deepens by ~500m and the region of equatorward flow associated with the WBUC extends much further into the interior (more than halfway across the domain in the near bottom layers). Why does the equatorward interior flow derived by Gargett (1984) not emerge in the model? Recall that the conditions which led to equatorward flow were geostrophy of the velocity field and a vertical advection/diffusion balance for the density field. It is the latter assumption which fails. While the computed model velocity fields obey geostrophy (within 20%) nearly everywhere in the domain, the density field does not achieve steady-state by balancing $\omega \bar{\rho}_z$ with $(K_{ij}\bar{\rho}_z)_z$ in the deep interior.
Fig. 5. E–W sections of meridional velocity within the subtropical gyre (20°N) for (a) Exp. 1 and (b) Exp. 2: contour interval is 0.003 cm s⁻¹. In both experiments, deep interior flow is poleward.

The preceding comparisons between Experiments 1 and 2 lead to the conclusion that a stratification-dependent diffusivity of the form suggested by Gargett (1984) doesn’t matter much to the poleward heat flux carried by the model ocean, to the meridional overturning cell, or to the meridional sense of the deep circulation. ("Much" is of course a relative term: to some applications, 30% change in, say, poleward heat flux, might be important.) Are we thus able to conclude that the possible depth or stratification dependence of $K_d$ need not concern basin-scale modellers, provided only that the constant value used is chosen to produce an appropriate thermocline depth? Insofar as we are interested in the deep ocean (of major importance to the problem of the ocean as CO₂ sink, for example), the answer to this question is no. One of the major shortcomings of the GFDL model with constant $K_d$ is the vertical uniformity of the deep water density field. Figure 6(a) shows the E-W section across the "subtropical gyre" at 20°N for Exp. 1. Below ~2 km depth, the vertical density gradient has a nearly uniform low value. A N-S section at 30° longitude (Fig. 6(b) shows that weak uniform deep gradients occur throughout the domain. In contrast, Exp. 2 yields a significantly more strongly stratified deep ocean (Fig. 6(c) and 6(d)). Both this stronger stratification and higher deep water temperatures (previously mentioned in discussing the heat flux) move Exp. 2 in the direction of the real ocean, suggesting that an inappropriate representation of diapycnal processes is at least partially responsible for the incorrect deep ocean stratification produced by the GFDL model.
THE EFFECTS OF $K_h$: IMPLICIT DIFFUSION

An unexpected benefit of implementing $K_d = a_o N^{-1}$ was a dramatic reduction in unwanted implicit diapycnal diffusion associated with $K_h \neq 0$. As mentioned previously, a non-zero value of $K_h$ yields diapycnal mixing in the presence of sloping isopycnals. A measure of the relative importance of this implicit diffusivity to that explicitly included in the model is given by a normalized diffusivity diagnostic defined as

$$\text{NDD} = \frac{K_h m^2}{K_d} = \frac{K_h}{K_d} \frac{\nabla_p \cdot \nabla_h \rho}{\frac{\partial^2 \rho}{\partial z^2}},$$

where $K_h$, $K_d$ are the explicit horizontal and diapycnal diffusivities and $m$ is the isopycnal slope (referenced to local pressure). N/S sections of the logarithm of NDD (Fig. 7(a) and (b)) show enormous changes between Exps. 1 and 2, particularly in the 1-4 km depth range where the explicit diapycnal fluxes of Exp. 1 are dominated by implicit diffusion.

In Exp. 1, only the upper kilometer of the subtropical gyre has diapycnal fluxes determined by the explicit diffusivity ($\log \text{NDD} < 0$, NDD < 1), while in Exp. 2 much of the intermediate and deep water is effectively free from implicit diffusion. While some of the observed change in NDD results from increased $K_d$ at depth in Exp. 2, this can account for only about one order of magnitude. The rest of the change (approximately three
orders of magnitude at some depths) results from decreases in $m^2$, the mean square isopycnal slope.

Encouraged by the decrease in implicit diffusion in Exp. 2, we attempted a further reduction of implicit diffusion effects by decreasing $K_h$ (to $0.33 \times 10^7 \text{ cm}^2\text{s}^{-1}$) in Exp. 3. The model retained stability at this lower value of $K_h$, but the result was not a straightforward reduction in implicit diffusion. Instead (Fig. 7(c)), while Exp. 3 shows lower NDD

![Diagram](image)

Fig. 7. Contours of $\log(\text{NDD}) = \log \left(\frac{K_h}{m^2/K_d}\right)$ where $K_h$, $m^2$ and $K_d$ are respectively the implicit and explicit diapycnal diffusivities present in (a) Exp. 1, (b) Exp. 2 and (c) Exp. 3 (see Table). Values of $\log(\text{NDD}) > 0$ indicate regions where implicit diffusion is the dominant diapycnal transfer mechanism.
near the northern boundary, in much of the mid-depth interior, particularly near the equator, NDD has actually increased. What apparently happens is that reduction of $K_n$ reduces the rate at which $V_h \rho$ is removed, thus increasing $m^2$; the increase in $m^2$ is larger than the decrease in explicit $K_n$, hence NDD increases. While readily "understood" in this way, such a result is counterintuitive to those (most?) of us whose intuition has developed around linear systems, and illustrates the complexity of "the" sensitivity problem in nonlinear systems. Determining "the" sensitivity of any ocean model to sub-grid-scale processes is immensely complicated by the degree of interdependence between horizontal and vertical (or isopycnal and diapycnal) sub-grid-scale parameterizations.

Moreover, considering the range of possible parameterizations of just diapycnal fluxes, it seems unlikely that it will be possible to determine appropriate forms of these fluxes by using observables to constrain the models. The problem is that the same model may give very much the same result using very different values and/or forms for the various diffusivities involved. As an example, Figure 8 is a comparison of the meridional overturning stream function resulting from our Exp. 3 (with $K_d = a_2 N^{-1}$, $K_p = K_\rho = 0.33 \times 10^7 \text{ cm}^2\text{s}^{-1}$) and one of Bryan's (1987) runs (with $K_v = 0.1 \text{ cm}^2\text{s}^{-1}$, $K_n = 1 \times 10^7 \text{ cm}^2\text{s}^{-1}$ and $K_p = 0$): allowing for the different meridional extent of the basins, the two studies give results

![Figure 8](image)

Fig. 8. Zonally-averaged meridional overturning stream function for (a) Exp. 3 of this work and (b) Bryan's (1987) Exp. 2. The contour interval is $2.5 \times 10^6 \text{ m}^2\text{s}^{-1}$: dashed line is clockwise, solid line is counter-clockwise flow. The similarity of the two results (apart from the difference in meridional extent of the domain) illustrates that the GFDL model can produce similar results with very different choices for the diffusivities used to parameterize mixing.
which are remarkably similar in both form and magnitude. If we are to learn anything about appropriate parameterizations from models, it is clearly necessary to look for diagnostic fields which are more sensitive to these parameterizations than, for example, the meridional overturning stream function (or the meridional heat flux) and/or to carry various passive tracers as well as the active tracers T and S. It also seems clear that any contribution which the observational community can make as to appropriate forms of diffusivities/fluxes will provide welcome restrictions for model sensitivity studies.

REFERENCES