SIMILARITY THEORIES AND MICROTURBULENCE IN THE ATMOSPHERIC MIXED LAYER

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ABSTRACT

The cloud-free, convective mixed-layer is the simplest and best understood atmospheric boundary layer regime. Many of the mean and turbulent properties can be well represented by similarity models based upon scaling in terms of the basic forcing processes (e.g., the surface and tropospheric interfacial fluxes). The boundary layer can be divided into three scaling height regimes which, historically, are described with different scaling models: the surface layer (Monin-Obukhov similarity), the mixed layer (top-down/bottom-up diffusion), and the inversion layer (Wyngaard-Lemone entrainment similarity). These models are known to be particularly successful when used to describe the small scale turbulence properties (dissipation rates and structure function parameters for velocity and passive scalars). Combined with these simple models, measurements of small scale turbulence can be used to infer surface fluxes, entrainment rate, and flux profiles.

INTRODUCTION

Compared to the ocean, the atmospheric boundary layer (ABL) has been extensively investigated and, in many ways, is considered to be better understood. This understanding has developed from a combination of information sources: (1) laboratory models (e.g., Willis and Deardorff, 1974; Deardorff and Willis, 1982), (2) three-dimensional, primitive equation large eddy simulations (e.g., Deardorff, 1974; Deardorff, 1980; Moeng, 1984), and (3) atmospheric measurements with aircraft and tethered balloons (e.g., Lenschow, 1973; Kaimal et al., 1976; Lenschow et al., 1980; Brost et al., 1982; Nicholls, 1984). Remote sensors are just beginning to make major contributions to knowledge about atmospheric boundary layers. An interesting example from Brost et al. (1982) is shown in Fig. 1, where aircraft measured profiles of the various terms of the budget equations for the turbulent kinetic energy (TKE) and vertical velocity variance are given. All terms are measured except the pressure terms which are inferred as an imbalance.
Fig. 1. Terms of the TKE budget (upper) and one-half the vertical velocity variance (lower) normalized by $\kappa z_i/\bar{u}_x^3$. For the upper panel the terms are: dissipation (solid circles); shear production (crosses); buoyant production (open triangles); turbulent transport (open squares); and pressure transport (open circles). For the lower panel the terms are: $\epsilon/3$ (solid circles); buoyant production (open triangles); turbulent transport (open squares); and pressure scrambling (open circles).
Given this vast store of information, it is of interest to ponder the present 'state of understanding' of the ABL. I admit that the word understanding is rather subjective and that my own opinions may not be universally held. Given the complexity of ABL dynamics, it is natural to classify the conditions under dynamical regimes (see Table 1).

Table 1. An evaluation of the present 'state of understanding' for several typical ABL regimes.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Understanding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cloudfree, convective</td>
<td>good</td>
</tr>
<tr>
<td>Cloudfree, shear driven</td>
<td>not good</td>
</tr>
<tr>
<td>Stratocumulus</td>
<td>not good</td>
</tr>
<tr>
<td>Tradewind cumulus</td>
<td>not good</td>
</tr>
<tr>
<td>Stable</td>
<td>poor</td>
</tr>
<tr>
<td>Broken cloud</td>
<td>poor</td>
</tr>
<tr>
<td>Mesoscale forcing</td>
<td>poor</td>
</tr>
</tbody>
</table>

One index of our understanding is the existence of simplified conceptual models and useful scaling laws or parameterizations. By this measure, the cloudfree, convective ABL is clearly the best understood regime. It is also closely analogous to the convective regime in the oceanic mixed layer; and the simple scaling laws may be applicable to both the ocean and the atmosphere. Some evidence for this viewpoint is given in Fig. 2, where profiles of the dimensionless rate of dissipation of TKE, $\epsilon/Jo$, are remarkably similar in the ocean and the atmosphere (Jo is the surface buoyancy flux). This paper will present a summary of similarity models developed to describe the ABL. Following these introductory comments are sections on general background, surface flux scaling, entrainment scaling, and top-down/bottom-up diffusion.

BACKGROUND

Microturbulence Variables

The primary small scale turbulence variables of interest to meteorologists are the standard dissipation rates: $\epsilon$ (for TKE), $\chi_T$ (temperature variance), and $\chi_q$ (specific humidity variance). These variables represent the viscous/diffusive loss terms in the TKE or variance budget equations. The dissipations are related to the inertial subrange variables known as the structure function parameters,

$$C_X^2 = \langle (X(r) - X(r+d))^2 \rangle / d^{2/3}$$

(1)
Fig. 2. Rate of dissipation of TKE ($\epsilon$) scaled by the surface buoyancy flux ($J_{b^0}$) as a function of height (or depth) scaled by the mixed layer depth (D). The upper panel is from the ocean and the lower from the atmosphere (Shay and Gregg, 1984).
where $X(r)$ represents the value of the variable $X$ ($X = u$ for velocity, $T$ for temperature, and $q$ for specific humidity) at the position $r$ and $X(r+d)$ is the value of $X$ at a position $d$ distance from $r$. The brackets $<>$ signify the standard ensemble average. In the inertial subrange of isotropic turbulence, the 1-dimensional variance spectral density, $S_X$, is given by the Kolmogorov $-5/3$ law:

$$S_X = 0.25 \ C_X^2 k^{-5/3}$$  \(2\)

where $k$ is the wavenumber magnitude. The structure function parameters and dissipations are related by the Corrsin's equation,

$$C_X^2 = 4 \ \beta_X \chi_X \epsilon^{-1/3}$$  \(3\)

where $\beta_X$ is the Kolmogorov constant ($\beta_u = 0.5$ and $\beta_T = \beta_q = 0.4$) for the variable $X$. Note that for velocity, this implies that $C_u^2 = 2 \epsilon^{-1/3}$. Incidentally, the value 0.25 in eq.(1) is a mathematical constant while the $\beta_X$ are empirically determined by measurement (e.g., Champagne et al., 1977).

The microturbulence variables are important for several reasons. The dissipation is important in dynamics and the inertial-dissipation method is useful for estimating surface fluxes over the ocean. In regions where gradient production is approximately balanced by dissipation, the dissipation profile can be used to infer the flux profile. The structure functions are important in the realm of wave propagation (optical, acoustic and radar) and atmospheric dispersion.

**Scaling Regimes**

Scaling theories have their origins in dimensional analysis, where important variables of the problem are selected and other properties are calculated from dimensionally consistent combinations of those variables. The basic concept of similarity theory is that some selected dynamical property (e.g., $\epsilon$), when normalized by the proper combination scaling variables, can be described by 'universal' function of the scaling variables. In other words, the flow of honey around a basketball is indistinguishable (i.e., 'similar') from the flow of helium around a BB when viewed scaled by diameter and when the flow velocity is adjusted so the Reynolds numbers are the same. Modern ABL similarity theories are now based on arguments about the relative variability (or lack of variability) and magnitude of the terms in the mean and turbulent budget equations (see Fairall and Larsen, 1986, for examples). Since this is considered to be a more firm foundation than the dimensional analysis approach, the universal function of the scaling variables described above is often referred to as a semi-empirical function. The 'semi' supposedly imparts some increased credibility. In fact the functions must be determined by measurements but often the limiting forms of the functions can be determined by scaling arguments so that only one or two constant coefficients must be determined by fitting to real data.
In the ABL the similarity regimes are broken down by vertical scale. Historically, this process has proceeded from the ground up. Fig. 3 depicts a schematic of the typical mean structure of the ABL under convective, well-mixed conditions in a synoptic regime with sufficient subsidence to ensure a healthy capping inversion. Compared to the ABL, the overlying free troposphere is essentially nonturbulent. Table 2 contains a summary of scaling regimes that will be discussed.

Table 2. Summary of the hierarchy of ABL scaling models/ regimes beginning at the surface and moving up to the inversion. The basic scaling parameters, the form of the dimensionless height (ξ), the vertical region of applicability, and an example of a parameterization are given. The symbols will be explained in the text. Scaling parameters for humidity (not shown) are similar to those for temperature.

<table>
<thead>
<tr>
<th>Name</th>
<th>Scaling parameters</th>
<th>ξ</th>
<th>Height Region</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monin-Obukhov</td>
<td>u*,T*, z,L</td>
<td>z/L</td>
<td>constant flux</td>
<td>ε = u*/κz f(ξ)</td>
</tr>
<tr>
<td>Free Conv.</td>
<td>&lt;w'T'&gt;o</td>
<td>-</td>
<td>z&gt;&gt;-L</td>
<td>ε = 0.5(g/T)&lt;w'T'&gt;o</td>
</tr>
<tr>
<td>Mixed Layer</td>
<td>W*,θ*, z_i</td>
<td>ξ&gt;0.1</td>
<td></td>
<td>ε = W*/z_1 F(ξ)</td>
</tr>
<tr>
<td>Top-dn</td>
<td>W*,θ*, z_i, R_x</td>
<td>ξ&lt;0.9</td>
<td></td>
<td>F(ξ) = f_b + Rf_t + R^2 f_t</td>
</tr>
<tr>
<td>Bot-up</td>
<td>W*,θ*, z_i, R_x</td>
<td>ξ&lt;0.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Invers.</td>
<td>We, Δθ, Δu, Γ_T</td>
<td>-</td>
<td>z-z_i</td>
<td>ε = ε_c + Δu^2 Γ_T We/Δθ</td>
</tr>
</tbody>
</table>

SURFACE FLUX SCALING

Monin-Obukhov

Surface layer similarity theory is based on scaling parameters obtained from the surface fluxes (Wyngaard, 1973). The theory is considered valid in the region near the surface where various terms (particularly the gradients) in the TKE and scalar variance budget equations are considerably more dependent on height than the surface fluxes. Thus, the assumptions on which the theory is based are generally valid in the lowest 10% of the ABL. The basic scaling parameters are the friction velocity, u*, and the temperature scaling parameter, T*, given by
Fig. 3. Schematic of a typical convectively mixed ABL. The absolute humidity ($Q$) and virtual potential temperature ($\theta_v$) are shown as a function of height where $h$ is the height of the turbulent ABL.

\[ u_* u_* = -<u'w'>_o \]  \hspace{1cm} (4a)

\[ u_* T_* = -<T'w'>_o \]  \hspace{1cm} (4b)

where the primes denote turbulent fluctuations and the $o$ denotes the surface value. The humidity equations will not be given since they can be obtained from the temperature equations by replacing $T$ with $q$. According to M-O theory, the suitably non-dimensionalized variables are functions of the dimensionless height scale, $\xi$,

\[ \xi = z/L = -\kappa(g/T)z<w'T'>_o/u_*^3 \]  \hspace{1cm} (5)

The standard meteorological convention is that heat flux is positive upward so that convective conditions are characterized by $\xi<0$. Consider as an example, the standard deviation of vertical velocity, $\sigma_w$, written

\[ \sigma_w/u_* = f(\xi) \]  \hspace{1cm} (6)

where $f(\xi)$ represents the semi-empirical dimensionless function.

The M-O forms for the microturbulence variables are

\[ \epsilon \kappa z/u_*^3 = \Phi_\epsilon \]  \hspace{1cm} (7a)

\[ \chi_T \kappa z/(u_* T_*^2) = \Phi_\chi \]  \hspace{1cm} (7b)
The dimensionless dissipation functions were initially measured in the famous Kansas experiment (Wyngaard and Cote, 1971; Wyngaard et al., 1971). For example,

\[ \Phi_e = (1 + 0.5 |\xi|^{2/3})^{3/2} \]

under unstable \((\xi < 0)\) or convective conditions. Similarly, the dimensionless structure function parameters are given by

\[ C_x^2 z^{2/3}/x_*^2 = f_x(\xi) \]

and are shown in Fig. 4. Under unstable conditions, these are represented by the empirical formulae (Fairall and Larsen, 1986)

\[ f_u(\xi) = 4.0 (1 + 0.5 |\xi|^{2/3}) \quad (10a) \]
\[ f_T(\xi) = 4.9 (1 - 7.0 \xi)^{-2/3} \quad (10b) \]
\[ f_q(\xi) = 0.7 f_T(\xi) \quad (10c) \]

**Fig. 4.** Dimensionless structure function parameters (eqs.10) as a function of surface layer stability \((\xi)\).
The empirical formulae for the dimensionless scalar variance dissipation functions can be obtained from eqs. (8) and (10) using the Corrsin relation given in eq. (3). Of course it is important to remember that these empirical formulae are only expected to be correct in an average sense and that any particular measurement (say a one hour average at a surface tower) can be expected to be in error by an amount associated with normal atmospheric variability and deviations from the assumptions of the theory (something on the order of 20% for the case sited above). The average accuracy of eqs. (8) and (10b) is illustrated by a series of aircraft profile data (Fairall et al., 1980) where 15 profiles have been normalized and averaged in bins of $\xi$ (Fig. 5). The apparently poorer fit of the temperature structure data is believed to be caused by a failure of the isotropic assumption very near the surface (the temperature probe spacing was not much smaller than the altitude).

Fig. 5. Composites of aircraft measurements of $\epsilon$ and $C_T^2$ (non-dimensionalized) indicating the average fit of eqs. (10).
Free Convection

The mathematical form of the dimensionless functions as $z/(\cdot L)$ becomes large is referred to as the convective limit. This limit may be obtained by $z$ becoming large at fixed $L$ or by $L$ becoming small at fixed $z$. Physically, the convective limit is obtained when the buoyant production of TKE greatly exceeds the surface shear production (the two are equal when $z=L$). Since the shear production due to the surface shear becomes unimportant, the friction velocity, $u_\star$, is not a relevant scaling parameter in the convective limit. This means that $T_\star$ is also no longer relevant but is replaced as a scaling parameter by the surface buoyancy flux. The asymptotic behavior of the dimensionless functions is expressed as $f(\xi) \approx \xi^m$ where $m$ is a constant that is selected $u_\star$ to drop out of the scaling relation. For example, let us consider the relation for vertical velocity variance. As $-\xi$ becomes large, eq.(6) becomes

$$\sigma_w = u_\star \left[ \kappa (g/T) z <w'T'>_o / u_\star^3 \right]^m$$

Clearly if $m=1/3$, then $u_\star$ will drop out of eq.(11) yielding

$$\sigma_w = \left[ \kappa (g/T) <w'T'>_o z \right]^{1/3}$$

This implies that $\sigma_w$ increases as $z^{1/3}$ near the surface, which is consistent with observations (Wyngaard, 1973).

The forms of the dimensionless functions for the dissipation and structure functions (eqs.(8) and (10)) have been constructed to obtain the proper free convection limits. Thus, the structure function parameters have a $z^{2/3}$ dependence very near the surface but approach a $z^{-4/3}$ behavior as $z$ increases (see Fig. 6). Because the TKE has an additional source (buoyancy), the behavior for $\epsilon$ is quite different. Near the surface $\epsilon$ decreases as $z^{-1}$ (which reflects the decrease in the shear production from the surface) but for large $z$, $\epsilon$ becomes independent of height,

$$\epsilon = (0.5)^{3/2} (g/T) <w'T'>_o$$

Note that eq.(13) implies $\epsilon/J_p^0 = \text{constant}$ (recall the discussion concerning Fig. 2), where we used $J_p^0 = (g/T) <w'T'>_o$. These asymptotic behaviors are very consistently observed under convective conditions in the lower ABL (an example is given in Fig. 7).

Mixed-layer Scaling

The dynamics of the convective ABL are greatly influenced by the total depth, $z_1$, which determines the vertical scale of the largest eddies. Clearly, $z_1$ is a natural choice for a length scale for mixed-layer scaling and it is natural to assume $\xi = z/z_1$ as the dimensionless length scale. The mixed layer velocity scale (usually called the convective mixing velocity) is easily obtained by writing eq.(12) in terms of $\xi$,

$$\sigma_w = \left[ (g/T) z_1 <w'T'>_o \right]^{1/3} (z/z_1)^{1/3}$$
Fig. 6. Schematic representation of the altitude dependence of $C_T^2$ at fixed $L$ for the asymptotic limits of surface layer scaling.

Following the philosophy expressed by eq.(6), the dimensionless vertical velocity standard deviation is

$$\sigma_w/W_\ast = f(z/z_i)$$

where the velocity scale is

$$W_\ast^3 = (g/T)<w'T'>_0 z_i$$

Similarly, the microturbulence parameters can be expressed in convective scaling,

$$\epsilon z_i/W_\ast^3 = F_\varepsilon(\xi)$$

$$C_T^2 z_i^{2/3}/\theta_*^2 = F_T(\xi)$$

where $\theta_*=<w'T'>_0 W_*$. 
Failure of Surface Flux Based Scaling

The convective limit forms of eq.(17) are

\[ F_{ef}(\xi) = \text{constant} = 0.4 \quad (18a) \]

\[ F_{Tf}(\xi) = A_T \xi^{-4/3} \quad (18b) \]

where \( A_T = 2.7 \) is a constant obtained from eq.(10b). As described earlier (Fig. 2 and Fig. 7), the convective forms are often well followed in the lower ABL but deviations are usually observed in the upper ABL (particularly for the scalar structure function parameters). Further examples for \( C_T^2 \) are given in Fig. 8. The upper panel is from the classic convective regime of the so-called Minnesota experiment (Kaimal et al., 1976). Note that eq.(18b) is followed until \( \xi > 0.5 \). The fact that the free convection form is not followed is, of itself, no cause for concern. Since \( F(\xi) \) is an empirically determined function, we simply find the function that does fit the data. As long as our normalization collapses all conditions onto a single curve, the similarity hypothesis is valid. However, contrast the Minnesota data with several other field experiments (the lower panel in Fig. 8) where the deviations vary considerably.

![Graph](image)

**Fig. 7.** An example of a profile of \( \varepsilon \) (X's) and \( C_T^2 \) (triangles). Note the deviations of the \( C_T^2 \) profile from the \(-4/3\) altitude dependence in the upper mixed layer, the tendency of \( \varepsilon \) to approach a constant, and the dramatic decrease of both variables above the ABL.
Fig. 8. Average normalized $C_T^2$ profiles from the Minnesota experiment (upper panel) and several other major field experiments (Wyngaard and LeMone, 1980). The lack of similarity for this scaling approach is apparent in the upper ABL.
Clearly there is no 'universal' function of the form $F(\xi)$. In order to construct a universal function we must consider at least one additional scaling parameter. Also, a careful examination of individual profiles shows that there are two separable deviations from the free convective forms: (1) a gradual increase in $C_T^*$ [relative to the value predicted by eq. (18b)] in the upper ABL and (2) a dramatic, spike-like increase confined to the interfacial region of the capping inversion. Both of these features are due to the entrainment process. The spike at the inversion is due to the large temperature variance created by mixing warm tropospheric air with cool ABL air at the interface. This behavior is described by the Wyngaard-LeMone interfacial entrainment model (Wyngaard and LeMone, 1980) which is described in the next section. The steady mixing and dissipation of the temperature variations as the recently entrained parcels are transported downward in the ABL lead to the smoother feature in the upper ABL. This behavior is described by top-down and bottom-up diffusion theory (Fairall, 1987).

THE ENTRAINMENT REGION

Background on Entrainment

Between the turbulent ABL and the nonturbulent free troposphere above is a transition or interfacial region characterized by rapid changes (or jumps) in the mean meteorological variables. Entrainment is the process whereby this boundary is eroded by the turbulent diffusive processes in the ABL. The erosion process creates a vertical flux in the interfacial region referred to as the entrainment flux, $\langle w'x' \rangle_1$, for the quantity $X$. This is illustrated in Fig. 9 where the upper panel shows a typical profile for the potential temperature and buoyancy flux and the lower panel shows mean and flux profiles for some unspecified quantity, $X$. Note that the buoyancy flux is negative in the entrainment region because the entrainment process transports warmer air downward and, therefore, consumes TKE. The entrainment flux is also represented as

$$\langle w'x' \rangle_1 = -\text{We} \Delta X$$

(19)

where We is the entrainment velocity and $\Delta X$ the increase [$X_0-X_1$ in Fig. 9] in $X$ across the inversion. The rate of growth of the height of the inversion is

$$\frac{dz_1}{dt} = \text{We} + W_s$$

(20)

where $W_s$ is the subsidence velocity at $z=z_1$. Under cloud free, convective conditions the buoyancy flux in the entrainment region is typically about 20% of the surface value,

$$\text{We}_F = -0.2 \langle w'T' \rangle_0 / \Delta \theta$$

(21)
Fig. 9. Typical ABL structure for potential temperature (upper panel) and a conservative scalar (lower panel). The mean profile is on the left and the vertical flux profile is on the right.
Wyngaard-LeMone Scaling

Wyngaard and LeMone (1980) developed a scaling model for the scalar dissipations and structure function parameters in the interfacial region by considering the scalar variance budget equation

\[ \partial <x^2>/\partial z = -2\langle w'x' \rangle \partial X/\partial z - \partial /\partial z (\langle w'x'^2 \rangle) - \chi_X \]  

(22)

By considering only the average of the various terms of eq.(22) defined over the interfacial region, \( \Delta z=h_2-h_1 \), by

\[ \bar{\chi}_X = (\Delta z)^{-1} \int x \, dz \]  

(23)

(where the integral is from \( h_1 \) to \( h_2 \)), it can be shown that the scalar variance dissipation is approximately balanced by the gradient production term. Integrating by parts and assuming a cubic shape for the mean potential temperature profile in the interfacial region that matches the mean gradient of the lower free troposphere, \( \Gamma_T \), yields

\[ \bar{\chi}_X = (\Delta X)^2 \Gamma_T \omega_e / \Delta \theta \]  

(24)

In the special case of \( X=\theta \), the dissipation is about a factor of five smaller than indicated by eq.(24) because the temperature flux profile is zero at \( z=h_1 \) by definition (see Wyngaard and LeMone, 1980; Fairall, 1984).

The scalar structure function parameter can be evaluated using eq.(24) and the mean Corrsin relation [eq.(3)] where it is assumed that the average value of the TKE dissipation rate is one half the convective limit (\( \epsilon_f \))

\[ \bar{\epsilon} = 0.5 \epsilon_f = 0.5 \frac{W_\star^3}{z_i} \]  

(25a)

\[ \bar{C}_x^{2/3} \left( \frac{z_i}{\Delta X} \right)^{2/3} = 2.3 \left( \frac{\Gamma_T z_i}{\Delta \theta} \right) \frac{\omega_e}{W_\star} \]  

(25b)

In the presence of inversion region velocity shear, a similar process can be followed (Fairall, 1984) to yield

\[ \bar{\epsilon} = \frac{3}{2} \epsilon_f \left( \frac{\omega_e}{\Gamma_T} \right) \Delta \theta + \frac{1}{2} \left( \Delta U \right)^2 \Gamma_T \omega_e / \Delta \theta \]  

(26)

If the velocity shear is zero and \( \omega_e \) is set to \( \omega_e \), then eq.(26) will reduce to eq.(25a). Notice that if we define the dimensionless inversion layer thickness, \( \alpha = \Delta z/z_i \), then eq.(25b) can be written in the natural interfacial scaling parameters

\[ \bar{C}_x^{2/3} \left( \Delta z \right)^{2/3} / \left( \Delta X \right)^2 = 2.3 \alpha^{-1/3} S_T \omega_e / W_\star \]  

(27)

The quantity \( S_T = \Gamma_T \Delta z/\Delta \theta \) is the ratio of the stability of the free troposphere to the stability of the inversion. A representative atmospheric value is \( \alpha^{1/3} = 0.6 \).
TOP-DOWN AND BOTTOM-UP DIFFUSION

Background on ABL Diffusion

The concept of a mixed-layer is based on the observation that the mixing in the convective ABL is sufficiently strong to minimize the vertical gradients of the mean variables. This idea is often expressed in terms of an gradient diffusion coefficient, $K$,

$$<w'x'> = -K \frac{\partial X}{\partial z} \quad (28)$$

For a given value of the flux, the gradient will become very small if $K$ becomes very large. Thus the vertical gradients in the mixed-layer are considered to be negligible or quite small. An important consequence of this assumption is that the flux becomes linearly dependent on height in the ABL (also true if the gradient is independent of time). Because convection is highly skewed (small, powerful updrafts but broadly diffuse downdrafts), the situation is more complicated than implied by the simple local gradient approach implied by eq.(28). Wyngaard and Brost (1984) have shown that the vertical diffusion of a passive, conservative scalar through the convective ABL can be considered as the superposition of top-down components driven by the scalar fluxes at the mixed-layer top and bottom-up components driven by fluxes at the bottom. This situation is depicted in Fig. 10 where a purely bottom-up (surface flux but no entrainment flux) configuration is shown in the left panel while the top-down configuration is shown in the right panel. An example of purely bottom-up diffusion could occur for evaporation from the ocean (finite surface flux) when the free tropospheric air had the same humidity as the ABL (no jump in $q$ at the inversion). An example of purely top-down diffusion would occur with the dilution by entrainment of a completely nonreactive and insoluble gas (i.e., no deposition to the surface so the surface flux is zero) that was present in lower concentrations above the ABL.

The linear flux profile of the variable, $X$, is represented as the sum of two linear components

$$<w'x'> = <w'x'_e> + <w'x'_b> = <w'x'>_o (1-\xi) + <w'x'>_i \xi \quad (29)$$

In top-down/bottom-up notation, this is written

$$<w'x'> = <w'x'>_o [e_b(\xi) + R_x e_t(\xi)] \quad (30)$$

where $e_b=(1-\xi)$, $e_t=\xi$ and $R_x=<w'x'>_i/<w'x'>_o$. The vertical gradient can be decomposed similarly as

$$\frac{\partial X}{\partial z} = \frac{\partial X_b}{\partial z} + \frac{\partial X_t}{\partial z} \quad (31)$$

where the individual components are given by

$$\frac{\partial X_b}{\partial z} = -\frac{<w'x'>_o}{(W_x z^4_1)} g_b(\xi) \quad (32a)$$
Fig. 10. Schematic representation of purely bottom-up up diffusion of a scalar with a surface source (leftmost grouping of panels) and purely top-down diffusion of a scalar with no surface source or loss (rightmost grouping of panels). The quantities shown are the mean scalar concentration, $C$, the vertical flux of $C$ ($<c'w'>$ in the notation used in this paper but indicated with an overbar in the figure), the dimensionless gradient function, $g_b$ or $g_t$ [eq. (32)], and the dimensionless gradient diffusion coefficient, $k_b$ or $k_t$.

$$\frac{\partial X_t}{\partial z} = -\left[ <c'w'>_i / (N_x z_i) \right] g_t(\xi)$$  (32b)

where $g_b = 0.4\xi^{-3/2}$ and $g_t = 0.7(1-\xi)^{-2}$ represent the empirical bottom-up and top-down dimensionless mixed-layer gradient functions in the notation of Moeng and Wyngaard (1984). The dimensionless total gradient is expressed

$$-(z_i / X_x) \frac{\partial X}{\partial z} = g_b(\xi) + R_x g_t(\xi)$$  (33)

where $X_x = <c'w'>_o / W_x$ as before. These functions were determined from so-called large eddy simulations (LES) where tracers were numerically introduced to simulate purely top-down or bottom-up diffusion.

From eqs. (30) and (33) we can show that the dimensionless gradient diffusion coefficients are the ratio of the flux and gradient functions
[i.e., \( K_b/W_x = e_b/g_b \)], which are shown in Fig. 11. Notice that in most of the ABL the bottom-up diffusion coefficient is about 2.5 times greater than the top-down coefficient (again, a manifestation of the skewed nature of convection). This approach permits the slightly positive potential temperature gradient often observed in the upper half of the ABL without demanding a negative diffusion coefficient.

**Scalar Variance**

Moeng and Wyngaard (1984) applied the top-down/bottom-up formalism to fluctuations of scalars in the ABL by expanding on the standard Reynolds decomposition of variables:

\[
x = X + x' \quad (34a)
\]

\[
x' = x_b' + x_t' \quad (34b)
\]

where \( X \) represents the mean and \( x' \) the total fluctuation which is due to the superposition of top-down and bottom-up components. The total variance of \( X \) is

\[
\langle x'^2 \rangle = \langle x_b'^2 \rangle + \langle x_t'^2 \rangle + 2\langle x_b' x_t' \rangle \quad (35)
\]

which can be represented as

\[
\sigma_X^2 = f_b(\xi) + 2R_x f_{tb}(\xi) + R_x^2 f_t(\xi) \quad (36)
\]

**Structure Function Parameters**

The results for the scalar variance can be applied to the structure function parameters (Fairall, 1987). This yields expressions of the form

\[
C_{z_1}^2 = A_x h_b(\xi) + 2R_x h_{tb}(\xi) + R_x^2 h_t(\xi) \quad (37)
\]

where \( A_x = 2.7 \) and \( A_q = 1.9 \). The present estimates for these empirical functions are

\[
h_b = 3.7 \xi^{-2/3}(z_1/(-L))^{2/3}(1-7z/L)^{-2/3} \quad (38a)
\]

\[
h_{tb} = 6 \xi^{-1/12} \quad (38b)
\]

\[
h_t = 12 \xi^{-1/12}(1-\xi)^{-3/2} \quad (38c)
\]

with the dimensionless entrainment rates being

\[
R_T = -0.2 [1 + 3.2 (-L/z_1)] \quad (39a)
\]

\[
R_q = (\Delta q/\Delta \theta)(\theta_x/Q_x)R_T \quad (39b)
\]

Similarly, the cospectral structure function parameter, \( C_{z_1} \), can be written in top-down/bottom-up form (see Fairall, 1987).
Fig. 11. Dimensionless top-down and bottom-up gradient diffusion coefficients, $K/(w_\star z_\star)$, from the LES studies by Wyngaard and Brost, (1984).

Sample profiles using eq. (37) are shown in Fig. 12 for typical values of $R$. The slightly different shape for the temperature and humidity structure function profiles are due to the difference in sign in $R$ which is typical for the atmosphere. Therefore, the crossterm detracts from $C_T^2$ but adds to $C_Q^2$. Sample comparisons against atmospheric data for the humidity structure function are shown in Fig. 13. The upper panel is an average from the AMTEX experiment (Wyngaard and LeMone, 1980) with $R_q$ values chosen to bracket the typical observed value. The lower panel is from overland with negligible surface moisture flux (i.e., a purely top-down diffusion case). Since $Q_{\star\star}=\langle w'q'\rangle/\tilde{w}^\star$, the normalization is by $Q_{\star\star}=\langle w'q'\rangle/\tilde{w}^\star$. 
Fig. 12. Dimensionless scalar structure function parameters (temperature and humidity) based on a top-down/bottom-up diffusion model [eqs.(37)] for typical atmospheric values of the entrainment parameter, $R_x$. 
Fig. 13. Normalized profiles of humidity structure function parameter. In the upper panel, the crosses represent average atmospheric data from the AMTEX experiment (Wyngaard and LeMone, 1980). The solid lines are the top-down/bottom-up model equations for $R_0=0.5$ and $1.0$, values that are expected to bracket the AMTEX averages. In the lower panel, the open circles represent average aircraft measurements (Druilhet et al., 1983). The solid lines are from the model with $R_0=2$, $5$, $20$, and $50$. In this case, $C_0^2$ is normalized by an inversion flux scaling parameter, $Q_{x}\sim R_0 Q_{x}$, because the surface flux was so small.
TKE Dissipation Rate

The top-down/bottom-up closure for $\varepsilon$ has not been worked out to date, but the general form can be easily outlined. This is actually a bit risky because the momentum is not a passive, conservative scalar (although momentum is usually reasonably well-mixed). Another factor working against us in this regard is the conceptual difficulty involved in defining a tracer for velocity. We also do not, to date, have an analysis of LES information for velocity from the top-down and bottom-up point of view. At any rate, the dimensionless form of the TKE equation gives

$$\frac{\varepsilon z_i}{W^*} = M + B + T + P$$

(40)

where the various terms are

Shear production: $M = -\langle u'w' \rangle \frac{\partial U}{\partial z} \frac{z_i}{W^*} \frac{3}{\varepsilon}$

Buoyant production: $B = (g/T) \langle w'T' \rangle \frac{z_i}{W^*} \frac{3}{\varepsilon}$

Turbulent transport: $T = -\frac{\partial \langle w'e' \rangle}{\partial z} \frac{z_i}{W^*} \frac{3}{\varepsilon}$

Pressure transport: $P = -\frac{\partial \langle w'p' \rangle}{\rho} \frac{z_i}{W^*} \frac{3}{\varepsilon}$

The dimensionless shear and buoyant terms are straightforwardly worked out in terms of existing functions,

$$M = \frac{u^4_*}{(W^*)^4} \left[ e_b e_b + R_u (e_b e_t + e_t e_b) + R_u^2 e_t e_t \right]$$

(40a)

$$B = e_b + R_T e_t = 1 - \xi + R_T \xi$$

(40b)

Notice that the shear term does not scale as the other terms but is multiplied by the factor

$$\left( \frac{u_*}{W^*} \right)^4 = \left( -L \kappa / z_i \right)^{4/3}$$

(41)

The effect of surface shear on the dimensionless dissipation is given by the bottom-up term

$$M_b = 0.8 \left( -L \kappa / z_i \right)^{4/3} (1 - \xi) \xi^{-3/2}$$

(42)

Without a detailed LES analysis, the turbulent and pressure terms can only be estimated from atmospheric measurements (which are of the top-down and bottom-up components combined) and educated guesses about their scaling forms.
CONCLUSION

The behavior of the classic microturbulence variables in the cloudfree, convective ABL are well described by scaling models based on using the interfacial fluxes (surface and inversion) to construct appropriate scaling parameters. This approach appears to be particularly successful for the small scale turbulence properties, which probably implies that the small scale diffusive processes are more locally determined and less subject to mesoscale modulation effects. If we think of Monin-Obukhov similarity as the near surface form of the bottom-up scaling, then the top-down and bottom-up scaling approach can account for the average behavior of the structure functions from the surface to within 10% of the top of the ABL. The implications of this success are not obvious, because of the restricted nature of the theory (cloudfree, convective, etc.). Certainly the scaling approach has been quite useful as a method of estimating surface fluxes (e.g., Fairall and Larsen). With the present trend toward the use of realtime processing combined with surface based remote sensors, the similarity models will be very useful when combined with clear-air radar or acoustic radar doppler systems where the backscatter is proportional to the scalar structure function parameters. In principle, this can result in 'intelligent' ABL profilers capable of producing not only mean velocity profiles but also estimates of surface fluxes, entrainment rates, optical properties, dispersion characteristics and short term forecasts of ABL evolution. With the continued growth in computing power, it may eventually be prudent to utilize much more complicated models (e.g., second order closure or LES) from the beginning. Even if this turns out to be true, the simple scaling models still provide a conceptual framework for thinking about the internal working of boundary layers.

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