HORIZONTAL SCALES OF WIND FORCED INERTIAL MOTIONS

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ABSTRACT

Wind forced upper ocean inertial motions are simulated using a theory appropriate for regions of low oceanic mesoscale activity with the goal of identifying the causes of horizontal variability in inertial currents. Mixed layer inertial currents are modeled as a uniform slab forced by the wind stress. The subsequent evolution of these currents and their propagation below the mixed layer is analyzed using a perturbation analysis of the linearized equations. The resulting equations allow only near-inertial frequency dynamics and suppress other classes of motion.

Realistic wind fields are generated from Seasat scatterometer data by horizontally advecting the measured fields over the simulated ocean at a constant speed. The scatterometer data appear to have sufficient resolution to make such simulations, although there is considerable uncertainty about their absolute precision. The models predict scales similar to those observed. The horizontal scales of the simulated inertial currents are set by the advection speed and the horizontal scales of the applied wind stress field. Typically, both scales are important, indicating that the complexities of realistic wind fields are needed for realistic models of wind forced inertial motions — even for wind features such as fronts which appear to be two-dimensional on the synoptic scale.

INTRODUCTION

Near-inertial oscillations are an important velocity component in the upper ocean, commonly contributing half or more of the kinetic energy and a somewhat smaller fraction of the 10 m shear (D’Asaro, 1985a). They play a key role in theories of mixed layer deepening (Niiler and Kraus, 1977; Price, 1981) and have been observationally linked with patches of enhanced mixing (Kunze and Lueck, 1986; Gregg et. al., 1986). Under some circumstances they may be sufficiently nonlinear to generate wave-forced and Stokes flows of several centimeters per second (Price, 1983; White, 1986).

Recent years have seen considerable work on the theory of near-inertial motions, particularly those forced by storms (Price, 1983; Gill, 1984; Greatbatch, 1984; Kundu, 1986). One result of these studies is that, although some of the characteristics of these motions are determined by the stratification, the evolution rate, amplitude, and horizontal structure depend on
the wind stress field. In particular, the horizontal scale of the inertial motions is crucial in
determining the rate at which inertial motions transfer energy from the directly forced
motions in the mixed layer into the deeper ocean. Because these scales are set by the wind,
realistic models of wind forced inertial motions will require realistic wind stress fields. The
above theoretical work, however, considered only highly idealized wind fields such as circu-
lar hurricanes (Price, 1983; Greatbatch, 1984), two-dimensional fronts (D’Asaro, 1985a;
Kundu, 1986), and sinusoidal storms (Gill, 1984). This is because measurements of two-
dimensional, time-dependent wind stress fields with a spatial and temporal resolution
sufficient to drive these models are not available. A possible exception are wind field mea-
surements by the Seasat microwave scatterometer, which have a 50 km resolution. In this
paper upper ocean inertial motions will be simulated using a linear model driven by the
Seasat wind fields. The goals are to determine the resulting horizontal scales and compare
these with oceanic observations and to assess the suitability of scatterometer winds for use in
upper ocean modeling.

A LINEAR MODEL OF INERTIAL FREQUENCY DYNAMICS

Consider the linear, Boussinesq, f-plane, hydrostatic equations for a flat bottomed ocean
with a buoyancy frequency $N(z)$ and forced by a wind stress $\tau$ which is modeled by a body
force with a depth distribution $Z(z)$:

\begin{align}
    u_t - fv &= -P_x + \frac{\tau_x}{\rho_0 H} Z(z) \\
    v_t + fu &= -P_y + \frac{\tau_y}{\rho_0 H} Z(z) \\
    N^2 w &= -P_{zt} \\
    u_x + v_y + w_z &= 0
\end{align}

(1) (2) (3) (4)

where the reference density $\rho_0$ has been absorbed into the pressure. Previous studies have
shown distinctly different responses for the barotropic and baroclinic modes (Gill, 1984).
We are interested in the baroclinic response in this study and will therefore use rigid lid
boundary conditions

\begin{equation}
    w(0) = w(-B) = 0.
\end{equation}

(5)

Equations (3)-(5) can be combined to form

\begin{equation}
    P_t = I \nabla_H \cdot \mathbf{u}
\end{equation}

(6)

where $\nabla_H \cdot \mathbf{u} = u_x + v_y$. 
\[ I = (1-M) \int_{-B}^{z} dz' N^2(z') \int_{-B}^{z} dz'' \] 

is an integral operator that combines eqs. (3) and (4), and

\[ M = \frac{1}{B} \int_{-B}^{z} dz \] 

is an integral operator required by (5).

Following Gill (1984), the wind stress will be distributed uniformly over a mixed layer of depth \( H \) so that

\[ Z(z) = -\frac{H}{B} + \begin{cases} 
1 & z > -H \\
0 & z < -H 
\end{cases} \]

where the first term ensures that \( MZ = 0 \) so that there is no projection upon the barotropic mode.

Assuming velocity, vertical, horizontal, and time scales \( V, D, L, \) and \( f^{-1} \) respectively, the nondimensional forms of eqs. (1), (2), and (6) are

\[ u' = v' - \varepsilon P_x + T_x Z(z) \]

\[ v' = -u' - \varepsilon P_y + T_y Z(z) \]

\[ P' = \nabla_H \cdot u' \]

where

\[ \varepsilon = \frac{g \Delta \rho D}{\rho_0 f^2 L^2} \]

\[ \frac{g \Delta \rho}{\rho_0} = \int_{-B}^{0} N^2 dz \]

and \( T = \tau/(\rho_0 HVf) \). The prime denotes dimensionless variables. \( I \) and \( P \) are nondimensionalized by \( g \Delta \rho Z/\rho_0 \) and \( fLU \varepsilon \), respectively. Combining (10) and (11)

\[ U' + iU' = -\varepsilon (P'_x + iP'_y) + FZ'(z') \]
\[ P'_{t'} = \frac{1}{2} l' \left[ U'_{x'} + U'_{x'}^* - i(U'_{y'} - U'_{y'}^*) \right] \]

where \( U = u' + iv' \), \( F = T_x + iT_y \), and \(*\) denotes complex conjugation. The use of this complex number technique can, at times, result in wrong answers. It does not do so in any of the results presented here and is used for the sake of clarifying the exposition.

For \( \varepsilon = 0 \), eq. (15) describes the behavior of a slab mixed layer forced by a horizontally uniform wind stress. Many investigators (see Pollard, 1980; D'Asaro, 1985a) have found this to be a useful model of the generation of mixed layer inertial currents. For inertial motions of finite, but large, horizontal scale, a small \( \varepsilon \) approximation may be appropriate. We will derive an asymptotic approximation to (15) and (16) that is valid for small \( \varepsilon \) using the method of multiple scales (Kervorkian and Cole, 1981). This is inspired by Hasselman (1970) and also expands upon some results in D'Asaro (1985a). We expand \( U \) and \( P \) in a perturbation expansion

\[ U = U_0 (t'', \tau'') + \varepsilon U_1 (t'', \tau'') + \cdots, \]

where \( t'' = t' (1 + O(\varepsilon^2)) \) and \( \tau'' = \varepsilon \tau' \) are "fast" and "slow" time variables. The equations are expanded in orders of \( \varepsilon \) with the \( \tau'' \) equation chosen so as to eliminate resonance. The resulting solution to order \( \varepsilon \) is

\[ U = \left[ \tilde{U}_F (t'', z') + \tilde{U} (\tau'', z') \right] e^{-i\tau''} + \varepsilon U_1 (z, \tau'') e^{i\tau''} + \varepsilon U_G (z') . \]

The complex inertial amplitude (in brackets) is the sum of a directly forced component \( \tilde{U}_F \) and a component \( \tilde{U} \) due to the inviscid evolution of previously forced motions.

The directly forced motions, described by (15) with \( \varepsilon = 0 \), can be divided into Ekman-type motions

\[ U_E = -iF Z'(z') \]

and a residual \( \tilde{U}_F \) described by

\[ \frac{\partial' \tilde{U}_F}{\partial t''} = i \frac{\partial' F}{\partial t''} e^{i\tau''} . \]

\( \tilde{U}_F \) describes the time dependent part of the solution and is thus of interest here. The dynamics of the free part of the solution \( \tilde{U} \) are described by

\[ \frac{\partial \tilde{U}}{\partial t''} = -\frac{1}{2} i l' \nabla r^2 \left[ \tilde{U} + \tilde{U}_F (z') \right] . \]

In addition, there are several order \( \varepsilon \) terms. Evolving inertial motions are slightly
anisotropic. This is described by a component $U_1$ which rotates in a direction opposite to pure inertial currents and is given by

$$U_1 = \frac{1}{4} \frac{d}{dr} \left[ \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + 2i \frac{\partial^2}{\partial xy} \right] (\tilde{U}^* + \tilde{U}_F^*) \, Z'(z') \right) \, . \quad (22)$$

The wind forcing also generates order $\epsilon$ motions $U_G$ described by

$$\frac{\partial U_G}{\partial t''} + i U_G = i \left[ \frac{\partial' \cdot \mathbf{T}}{\partial x} + i \frac{\partial' \cdot \mathbf{T}}{\partial x} \right] \left\{ \nabla' \cdot \mathbf{T} + \int_0^{t''} \frac{d}{dt} \mathbf{T} \, dt \right\} \, , \quad (23)$$

which represent the noninertial and geostrophic forced motions. These are not of interest here.

The dimensional forms of (18), (20), (21), and (22) are

$$U = \left[ \tilde{U}_F (t) Z(z) + \tilde{U} (t, z) \right] e^{-i t} + U_1 e^{i t} + U_G \quad (24)$$

$$\frac{\partial \tilde{U}_F}{\partial t} = i e^{i t} \frac{\partial}{\partial t} \left( \frac{\tau_x + i \tau_y}{\rho_0 H} \right) \quad (25)$$

$$\tilde{U}_t = -\frac{i}{2f} \mathbf{I} \nabla^2 (\tilde{U} + \tilde{U}_F \, Z) \right) \, . \quad (26)$$

$$U_1 = \frac{1}{4f} \left[ \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + 2i \frac{\partial^2}{\partial xy} \right] (\tilde{U}^* + \tilde{U}_F^* \, Z) \right) \, . \quad (27)$$

respectively.

Equations (24)-(27) describe the evolution of the velocity field for $\epsilon \ll 1$. They are valid as long as $\tilde{U}$ varies on a time scale that is long compared with $f^{-1}$. These equations allow only mean and inertial motions, as defined by (24). Higher frequency internal wave motions have been filtered from the equations of motions by the perturbation analysis. The evolution equation (26) is only first order in time, as opposed to the full equations, (1)-(4), which are second order in time. They are thus simpler to analyze and can be numerically integrated with larger time steps.

**APPLICATION TO STORM RESPONSE**

D'Asaro (1985b) computed $\tilde{U}_F$ from a large number of long term wind records. He found that inertial motions are forced (i.e., $\tilde{U}_F$ changes) only intermittently, so that the concept of
energetic "storms," each of which produces a large response in the ocean, separated by long periods of "calm," is a useful idealization for upper ocean inertial motions. The goal of this study is to simulate the inertial motions initially generated by one such "storm" in the mixed layer and thermocline. The subsequent evolution of these motions, although it can be modeled using these equations, is a more ambitious project which is currently under way.

The ocean is initially assumed at rest, $\bar{U} = 0$, and $\bar{U}_F$ is computed from the wind stress field $\tau(x, y, t)$. The evolution of the wave field for small times is computed from (26),

$$
\bar{U} = \bar{U}_F(t)Z(z) - \left[ \frac{i}{2f} \int_0^t \nabla_H^2 \bar{U}_F(t) dt \right] IZ(z).
$$

(28)

Assuming that $\bar{U}_F$ is nearly constant except for a brief period of forcing at $t = 0$, this simplifies to

$$
\bar{U} = \bar{U}_F(t)Z(z) - ft \left[ \frac{i}{2f} \nabla_H^2 \bar{U}_F(0) \right] IZ(z).
$$

(29)

The two terms in (29) have a different depth dependence. The depth structure is illustrated in Fig. 1 assuming

$$
N^2(z) = \begin{cases} 
0 & z > -H \\
N_0^2 e^{(z+H)b} & z < -H 
\end{cases}
$$

(30)

with $H = 50$ m, $b = 1000$ m, $N_0 = 5 \times 10^{-3}$ s$^{-1}$ (3 cph) and $B = 5000$ m. $Z(z)$ (Fig. 1b, solid) is large only in the mixed layer and is very small ($O(H/B)$) beneath. $IZ(z)$ (Fig. 1b, dashed) has a maximum value in the mixed layer and is continuous across the mixed layer base. Since this is a baroclinic flow, $IZ$ is large and of opposite sign in the deep ocean, with a zero at mid-depth.

At $t = 0$ the inertial motions are described by $Z(z)$ and are thus concentrated in the mixed layer. A characteristic profile of inertial currents at a finite time is shown in Fig. 1c by assuming the bracketed term in (29) is equal to $e^{0.53t} \bar{U}_F$. Inertial motions are now present both in the mixed layer and in the deeper ocean. Equation (29) therefore describes the "propagation" of inertial motions from the mixed layer into the thermocline.

Several characteristics of the velocity profiles (Fig. 1c) are notable. The inertial velocity is uniform across the mixed layer, jumps sharply across the mixed layer base, and then decays with depth in the thermocline, resulting in a velocity maximum beneath the mixed layer base. The thickness of this maximum is set by the stratification. For stratifications that include a thin seasonal thermocline, the velocity maximum is thinner and thus more distinct than that in Fig. 1c. Examples of this pattern for various stratifications can be seen in Rubenstein (1983), Gill (1984), D'Asaro (1985a), and Kundu (1986). It should be noted that since
Fig. 1. Vertical structure of simulated wind forced inertial motions assuming $N^2$ profile shown in a). b) Vertical structure of motions directly forced by the wind ($Z$, solid) and of their initial evolution (IZ, dashed). c) A sample velocity profile with components $Z + \sin(0.53)IZ$ (solid) and $\cos(0.53)IZ$ (dashed).
$IZ$ has no discontinuity across the mixed layer base, the shear at the mixed layer base is unchanged by the dynamics and in the time scales considered here.

The rate of energy transfer can be estimated from (29) by computing the total energy below the mixed layer

$$E_{therm} = \int_{-B}^{-H} \frac{1}{2} |\tilde{U}|^2 = \frac{1}{4} b |\tilde{U}_0|^2 ,$$

where

$$\tilde{U}_0 = i f t L_t^2 \nabla H \tilde{U}_F$$

is the value of $\tilde{U}$ immediately below the mixed layer base. $L_t$ is a horizontal length scale set by the stratification

$$L_t^2 = \frac{H b N_0^2}{2 f^2}$$

with a value of about 7 km for the above parameters. Equations (31)-(33) assume $H \ll b \ll B$ for simplicity. A more general expression, valid under this same approximation, is

$$L_t^2 = \frac{H g \Delta \rho}{2 \rho_0 f^2} ,$$

where $\Delta \rho$ is the change in density across the thermocline. Notice that the energy in the thermocline increases quadratically with time, as has been found by other investigators (Price, 1983; Gill, 1984).

One measure of the rate of energy transfer is the time $t_{trans}$ for the thermocline energy (31) to equal the energy $\frac{1}{2} H |\tilde{U}_F|^2$ initially in the mixed layer:

$$f t_{trans} = \left( \frac{2 H}{b} \right)^{1/2} \frac{1}{L_t^2 k_F^2}$$

where

$$k_F^2 = \frac{\nabla H \tilde{U}_F}{|\tilde{U}_F|}$$
is a characteristic horizontal wavenumber. Note that $t_{\text{trans}}$ depends strongly on the horizontal scale of the flow, specifically the scale associated with $\nabla^2 U$.

AN ADVECTIVE HYPOTHESIS

The above model requires a two-dimensional, time varying wind stress field. The required temporal resolution is clearly of order $f^{-1}$. The required spatial resolution can be estimated from observational estimates of the time for mixed layer inertial currents to decay, which are in the range of 2-20 days (Pollard and Millard, 1970; Pollard, 1980). Equating this time to $t_{\text{trans}}$ in (36) yields $k_F^{-1}$ in the range of 50-175 km, which is comparable to the few existing observations (D'Asaro, 1985a; Kundu and Thompson, 1985). Time varying wind fields with this resolution have not been measured over the ocean.

The problem can be simplified by assuming that the dominant cause of temporal variability is the advection of spatial structure — or equivalently, that on the forcing time scale of a few $f^{-1}$, storms move unchanged across the ocean. Similar assumptions are commonly made in the analysis of meteorological data on these spatial scales (Bond and Fleagle, 1985), which may provide a partial justification. The utility of this assumption is that simulations can now be made using a single, high resolution spatial wind field, such as is available from the Seasat scatterometer. Assuming an advection speed $C$ in the +x direction, the wind stress field can be written

$$\tau(x, y, t) = \tau(x - Ct, y, 0) \quad (37)$$

Away from the regions of forcing and to first order in $\epsilon$, $U = (\tilde{U} + \tilde{U}_F) e^{-i(f-k_A x)}$ so that

$$\nabla^2 U = -k_A^2 U + \frac{\partial^2}{\partial y^2} U \quad (38)$$

$$= -(k_A^2 + k_y^2) U = k_F^2 U \quad (39)$$

where $k_A = f/C$ is an advective wavenumber and $k_y$ is a characteristic wavenumber perpendicular to the advection direction. Relations (38) and (39) also apply to $\tilde{U}$. For the theory to be valid, $k_A^2 L_i^2 << 1$. Equivalently, storms must move fast enough that $C >> L_i f \approx 0.7 \text{ m s}^{-1}$, a condition that is easily met by most mid-latitude storms.

SCALES OF INERTIAL MOTIONS

In the above model, wind stress variations in the advection (x) and cross-advection (y) directions play fundamentally different roles. The x variations act as time variations and thus generate inertial motions as described by (25). The resulting inertial motions have an x wavenumber spectrum that is narrow band with a peak wavenumber near $k_A$. Accordingly,
the amplitude of the inertial motions depends on the variance of the wind stress in this band. The y variations in wind stress act to generate y variations in the inertial currents. The resulting y wavenumber spectrum of inertial currents reflects all the scales of the wind field and is thus broad band.

The wavenumber spectrum of the wind field over the ocean is typically red with an approximate $k^{-2}$ shape (Freilich and Chelton, 1986) and a correlation scale of many hundred kilometers. The spectrum of the wind stress field should be similar. The equation (25) relating the directly forced inertial currents, $\bar{U}_F$ to the wind stress contains no spatial derivatives. The y wavenumber spectrum of $\bar{U}_F$ should therefore be roughly the same as that of the wind stress (i.e., $k^{-2}$) as should their y coherence lengths.

The thermocline inertial currents $\bar{U}_0$ depend on $\nabla^2 \bar{U}_E$ as given by (32). The y wavenumber spectrum of $\bar{U}_0$ will thus be the sum of $-k_y^2 U_F$, with a spectrum of approximately $k^{-2}$, and $\partial^2 \bar{U}_0/\partial y^2$ with a spectrum of approximately $k^{-2}$. Unless the wind stress is redder than $k^{-5}$, which seems unlikely given the numerous observations of small-scale atmospheric variability (Houze and Hobbs, 1982), this argument predicts a blue y wavenumber spectrum for $\bar{U}_0$ and thus indeterminately large values for $k_y$, $k_F$, $\bar{U}_0$, and $t_{\text{trans}}^{-1}$. Such a result violates the assumptions of the theory, which requires slow evolution of the inertial motions and therefore is valid only for wavenumbers such that $L_{F}^{-2} k_y^2 \ll 1$. Thus the wind stress fields that drive this model must not contain wavenumbers higher than this limit. Practically, this is not a problem, since measurements of wind stress fields with 10 km resolution are currently nonexistent. Conceptually, however, it means that all the predictions of this theory apply to spatially averaged variables and that the evolution rates predicted here are a function of the averaging scale. Small horizontal scales will evolve faster than large ones, and the coherence scales of the field will be a function of time. A full study of this problem requires a time stepping model, which is beyond the scope of this paper. Within the above limitations, it is still clear that the inertial motions can be diagnosed using a wind field with a known finite resolution. This is the approach taken here.

The above considerations indicate that the relative magnitude of $k_y$ and $k_A$ is an important parameter of inertial current models. If $k_A \gg k_y$, the dynamics becomes two-dimensional, and only one wavenumber, $k_A$, is important. In this case, the details of the wind field are not important in determining the structure and evolution rates of wind forced inertial motions. $U_F$ and $\bar{U}_0$, for example, always 90° out of phase. If $k_y \geq k_A$, the details of the wind stress become important and a broad band of spatial scales enters the dynamics. The relative phase of $U_F$ and $\bar{U}_0$, for example, now depends on the local value of $\nabla^2 U_F$.

One goal of the following simulations is to determine the importance of $k_y$ for realistic wind fields. This will require a wind stress field with a resolution sufficient to resolve cross-advection scales smaller than $k_A^{-1}$. The Seasat scatterometer winds have a resolution of 50 km, which compares favorably with a value of $k_A \approx (100 \text{ km})^{-1}$ for a typical advection speed of 10 m s$^{-1}$. 
SIMULATION TECHNIQUES

Wind stress fields were computed using data from the Seasat scatterometer (Brown, 1986) supplied by the Jet Propulsion Laboratory. Estimates of the 10 m wind were edited for attenuation, and one of the four possible directions was chosen as described by Levy and Brown (1986). These were converted to estimates of surface wind stress using the formula of Large and Pond (1981) with no correction for boundary layer stratification. This results in a slightly irregular, gappy array of surface stress estimates with a spacing of about 50 km, such as that shown in Fig. 2a.

The above model requires a smooth estimate of \( \tau(x, y) \) so that (25) can be evaluated. In addition, some smoothing is required to reduce noise in the wind estimates. Two-dimensional smoothing splines (Wahba, 1984) were used to accomplish both tasks. These fit a smooth function \( f(x, y) \) with continuous first derivatives to \( n \) data points \( f_i(x_i, y_i) \) to minimize

\[
\frac{1}{n} \sum_{i=1}^{n} (f_i - f(x_i, y_i))^2 + \lambda^2 J(f),
\]

(40)

where \( J(f) \) is an isotropic sum of second derivatives of \( f \) that measures its smoothness. This criterion is isotropic, unlike that for bicubic splines, and therefore imposes no symmetries on the fitted function. The relative balance between the smoothness of the fit and the closeness of the surface to the data is controlled by the parameter \( \lambda \). A value of \( \lambda = 1000 \) m is used here. This value yields an rms difference between the raw and smoothed stress fields somewhat larger than the official error estimates for the scatterometer (1.6 m s\(^{-1}\) in wind speed and 16\(^\circ\) in direction (Born et al., 1982)). Typically about 20-30% of the stress field variance was removed by this method. The two components of \( \tau \) were independently smoothed using this technique. An example of the resulting wind stress field is shown in Fig. 2b.

The wind fields from two Seasat passes over the eastern North Pacific on September 11, 1978, are used here. These were chosen because they cover the low and cold front of a strong cyclone and because the same data have been previously analyzed for their meteorological content (McMurdie and Katsaros, 1985). Each section of data used was small enough so that the advection speed was approximately constant and large enough to encompass a well-defined portion of the storm.

For each wind stress field and advection vector \( \mathbf{C} \), \( \mathbf{U_F} \) was computed using (25) and (37) along paths through the data parallel to \( \mathbf{C} \) and ending at \( t = 0 \). The value of \( f \) corresponding to the final point was used throughout. The value of \( \nabla^2 \mathbf{U_F} \) was estimated by evaluating \( \mathbf{U_F} \) on a 25 km grid and fitting a quadratic surface to all points within a 50 km radius of each grid point. Estimates of \( U_0 \), \( k_F \), and \( t_{\text{trans}} \) were computed at \( t = 0 \) using \( L_f = 7 \) km.
Fig. 2. Simulation using Seasat scatterometer winds from 900Z, Sept. 11, 1978.
  a) Dealiased surface stress vectors. Front and low locations are indicated.
  b) Smoothed wind stress field sampled on a 25 km grid with superimposed contours of wind stress magnitude. c) Complex inertial amplitude of wind forced motions, $\bar{U}_F$. Advection direction and corresponding time axis are indicated. d) Inertial currents corresponding to c).
RESULTS

Response to a Low

Figures 2 and 3 show the simulated inertial response to a Seasat wind field measured at 900Z, September 11, 1978. The stress field (Fig. 2a, 2b) shows a well-defined cyclone with the strongest winds in the northeast and southeast quadrants. Warm and cold fronts from McMurrie (1983) have been drawn in Fig. 2a. As expected, the wind stress varies on a scale of several hundred kilometers.

Mixed layer inertial currents are generated by advecting this stress pattern in the direction indicated in Fig. 2c. The resulting field of data can be interpreted either as a space-time map, with the time scale given on the right-hand side of the data or as a true spatial pattern, bearing in mind that \( f \) is constant in the advection direction. The variation of the complex inertial amplitude \( U_F \) is shown in Fig. 2c; the corresponding currents \( U_F e^{-it} \) are shown in Fig. 2d.

In Fig. 2c,d, inertial currents are generated by the sharp changes in wind stress associated with the two wind stress maxima. The strongest inertial amplitudes occur on the right-hand side of the cyclone, because the winds are strongest in this region and because the turning of the winds corresponds to the turning of inertial currents and thus evokes a resonant response (Price, 1981). As expected, the inertial currents show two spatial scales, rotation with a wavenumber \( k_A \), here about 75 km, and a variation of \( U_F \) in the cross-advection direction with a scale of several hundred kilometers, approximately the same scale as that of the wind stress.

The cross-advection scales are shown more clearly in Fig. 3, which shows a number of variables at \( t = 0 \). \( U_F \) (Fig. 3a) varies on a scale of many hundred kilometers, as discussed above. In contrast, the evolving part of the inertial motions, \( U_0 \) (Fig. 3b), shows two scales, as expected from (39). On the scale of several hundred kilometers, \( iU_0 \) mimics the behavior of \( U_F \). For example at 300 km, the imaginary part of \( U_F \) (dashed) is minimum, as is the real part of \( U_0 \) (solid). This is because of the \( k_A^2 \) term in (39). In addition, \( U_0 \) varies with a wavelength of about 150 km. This is caused by the wind stress variations in the cross-advection direction associated with the cyclone. As expected, the most energetic wavenumber is near the Nyquist wavenumber for the scatterometer data. Figure 3c plots both the simulated energy transfer time \( t_{\text{trans}} \) and a horizontal line representing its value assuming \( k_y = 0 \). The energy transfer rate varies spatially owing to the variations of \( k_y \) and is typically 25% below the two-dimensional estimate. Wind stress fluctuations in the cross-advection direction are an important, but not dominant, factor in this case; the two-dimensional approximation will be useful, but not accurate.

Effect of Advection Speed

Figures 4 and 5 show the effect of decreasing the advection speed from the observed 8.4 m s\(^{-1}\) to 4.0 m s\(^{-1}\), corresponding to a decrease in \( k_A \) from about 85 km to about 40 km. Since there is, in general, less variance in the wind stress at smaller scales, we would expect the inertial amplitudes to be decreased. A comparison of Figs. 2 and 4 shows this to be the
Fig. 3. Simulated quantities at \( t = 0 \) for Fig. 2. a) Real (solid) and imaginary (dashed) components of \( U_F \). b) Same but for \( U_0 \), the thermocline inertial currents. c) Time for energy to be transferred from mixed layer to thermocline. Solid line indicates two-dimensional approximation.
case. The biggest change, however, is a concentration of the inertial motions into a small region. At this location the two wind maxima are separated by about one wavelength, $2\pi k_A^{-1}$, resulting in a coherent forcing of the inertial motions. In general, we would expect that a change in advection speed would increase $k_A$ while $k_y$ would remain roughly the same, thus decreasing $t_{trans}$ by about a factor of 4 and increasing the accuracy of the two-dimensional approximation. Comparing Figs. 3c and 5c, we note that $t_{trans}$ decreases as expected but its deviation from the two-dimensional value is about the same. The concentration of inertial motions into a small region has increased the small-scale variability of the inertial motions more than simple scaling argument would suggest. Mid-latitude storms, it would appear, are difficult to characterize with simple spectral arguments.

![Graph](image)

Fig. 4. Same as 2d) but with slower advection speed.

Response to a Cold Front

Figures 6 and 7 show the simulated response to the cold front associated with the same storm. The Seasat wind stresses (Fig. 6a) shift sharply from northeast to northwest at the front, which is drawn to coincide with the location of the wind shift. The strongest winds (Fig. 6b) occur near the front and on the edge of the associated low. Inertial currents are generated primarily by the frontal wind shift (Fig. 6c). This storm advects quickly (17.8 m s$^{-1}$) so that the advective wavenumber is small, $k_A \approx (180 \text{ km})^{-1}$. The decay time due to $k_A$ alone is long, approximately 200f$^{-1}$ (Fig. 7c). Accordingly, $k_y$ is mostly much larger than $k_A$, and $t_{trans}$ is commonly a factor of 3 less than the two-dimensional value. The two-dimensional approximation would not be appropriate in this case.
Fig. 5. Simulated quantities at $t = 0$ for Fig. 4.
Fig. 6. Simulation of response to a cold front using Seasat scatterometer winds from 1840Z, Sept. 11, 1978. a)-c) same as Fig. 2.
Fig. 7. Simulated quantities at \( t = 0 \) for Fig. 6.
On a typical weather map a cold front appears to be two-dimensional. Such a map has a resolution of a few hundred kilometers. These data suggest that significant deviations from two-dimensionality exist on the smaller scales relevant to the generation of inertial motions. If the advective speed of the frontal case is decreased to match that of the low case, so that $k_A$ is the same, the decay rate also becomes the same. Thus the intensity of the cross-advection variations is about the same for the two cases. The frontal case appears to be no more two-dimensional than the low case.

**Scatterometer Errors**

The above analysis uses smoothed versions of the scatterometer data. The amplitude of the small scale wind variations that determine $k_y$ depends on the degree of smoothing applied. Here the amount of variance removed is comparable to the official estimates of scatterometer accuracy. The scatterometer, however, is known to be less accurate in regions of precipitation and unsteady winds (Brown, 1986), exactly the conditions under which it is being used here. The errors may therefore be higher than average, although a quantification of the amount of extra error and its nature is not available. A second source of error can be seen in Fig. 6a, in which the frontal line zigzags as it passes obliquely through the lines of data. This is an aliasing effect due to a mismatch of the scatterometer footprint and its spatial resolution. Finally, the ocean is driven by wind stress not wind. Currently the ability of the scatterometer to measure wind stress is poorly known and the ad hoc scheme used here for computing stress from scatterometer wind is subject to an unknown error. Because of these errors, the work done here must be regarded as qualitative. Until the errors are better understood, it will be difficult to do quantitative work on the spatial scales of interest here using scatterometer data.

**CONCLUSIONS**

Wind forced near-inertial motions necessarily reflect the characteristics of the wind field with which they are forced. The limited simulations done here are encouraging in that they produce inertial currents with spatial scales and evolution rates comparable to those observed. They indicate that, in general, the two-dimensional approximation in which the advective scale $C/f$ dominates (Kundu, 1986) is not appropriate, since the wind field itself usually contributes scales smaller than this. Realistic models of mid-latitude near-inertial motions will probably have to use realistic wind stress fields with a spatial resolution of at least $C/f$. Satellite scatterometry appears to be the only method of obtaining data on these scales. Considerable work on the interpretation of scatterometer measurements in terms of surface wind stress will be needed, however, before quantitative simulations of inertial motions are possible.

**ACKNOWLEDGMENTS**

Scatterometer data for this work was dealiased and edited by Gad Levy. Brad Bell provided the two-dimensional spline algorithms and educated me as to their meaning. Kraig Winters helped keep the perturbation analysis of the equations of motion mathematically correct. This work was supported by the Office of Naval Research under Contract N00014-84-C-0111.
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