Seabed sediment attenuation profiles from a movable sub-bottom acoustic vertical array

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The acoustic lance [S. S. Fu, M.Sc. Thesis, University of Hawaii at Manoa (1994); Fu et al., J. Acoust. Soc. Am. 99, 234–241 (1996)] consists of a linear array of acoustic receivers below an acoustic source, all mounted on the outside of a core barrel or independent probe which is embedded in the seafloor. In earlier studies lance travel time data were processed to give in situ compressional wave sound speed as a function of depth. In this study lance waveforms are processed to extract compressional wave attenuation \( A = Q^{-1} \) as a function of depth. The processing technique is unusual because the \( L_1 \) norm is used instead of the usual \( L_2 \) norm, the model space of attenuation profiles is exhaustively searched within the limits of discretization, and the marginal posterior probability density function of attenuation is computed explicitly at each depth. The technique is described in terms of Bayesian inverse theory using the notation of Tarantola. © 1999 Acoustical Society of America. [S0001-4966(99)05306-0]

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INTRODUCTION

In this study we use a new processing technique to generate in situ compressional wave attenuation profiles in three different types of surficial marine sediment. The measurements are in the band 5–20 kHz, and the profiles are in the interval 0–4 mbsf (meters below sea floor). The profiles show that attenuation can vary rapidly with depth, with sediment type, and with location in the same sedimentary environment.

The acoustic lance (Fu, 1994; Fu et al., 1996a) is an instrument developed to obtain in situ compressional sound speed and attenuation profiles for the upper few meters of seabed sediment. As shown in Fig. 1, the lance consists of a linear array of receivers mounted on a core barrel or probe. A broadband source and a solid state recording system are mounted on the weight-stand of the probe, above the receivers. In use, lance is lowered by cable from a surface vessel; when the tip of the probe touches bottom, a trigger turns on the electronics. The lance signal spectrum is in the range of 5 kHz to 20 kHz, as shown in Fig. 2. A typical set of recorded waveforms is shown in Fig. 3.

Earlier studies using lance waveforms have concentrated on the extraction of sound speed profiles, whereas this study is concerned mainly with the extraction of attenuation profiles. Methods of extracting attenuation from seismic data have been reviewed by Jannsen et al. (1985) and Bromirski et al. (1992, 1995). Attenuation is much more difficult to extract than sound speed, as whole waveforms must be used, and if multiple receivers are used then corrections must be made to the data so that the spectrum of the source–receiver wavelet is effectively the same at each receiver. For this reason many attenuation studies use data containing multiple seismic phases recorded on the same instrument. Our method of extracting attenuation from the waveforms is an application of geophysical inverse theory to the well known spectral ratio method.

In seismic studies, attenuation results are often presented in terms of attenuation \( A = Q^{-1} \) where \( Q \) is a dimensionless quantity called the acoustic quality factor. A review of the mathematics of \( Q \), including a rigorous definition suitable for both low and high frequencies, was given by O’Connell and Budiansky (1978). For the high frequency waves of this study a good physical interpretation of \( A \) is provided by regarding \( A \) as \( 1/\pi \) times the fractional loss in amplitude per cycle. The plane wave \( \exp[i2\pi fx/c] \), where \( f \) is temporal frequency, \( x \) is distance and \( c \) is sound speed, decays like \( \exp(-\alpha x) \), where \( \alpha \) is given by \( \alpha = \pi f A/c \). It follows that \( A \) is twice the ratio of the imaginary and real parts of \( 1/c \). Attenuation is used in some papers as a synonym for \( \alpha \), but here the word attenuation is always used to mean \( A \). Note that when \( A \) is constant, \( \alpha \) is proportional to the first power of frequency. The attenuation measured from lance data is effective attenuation, as it contains the effects of both intrinsic attenuation and scattering attenuation. The well known relation between these three quantities is

\[
A_{eff} = A_{int} + A_{sc}.
\]

(1)

Although the dependence of \( A \) on frequency is much debated (e.g., Stoll, 1985; Kibblewhite, 1989), here we accept the important assumption that \( A \) is independent of frequency. This assumption is not essential for our method, but it is
appropriate for our data set which spans less than 3 octaves in frequency.

Attenuation profiles from four lance data sets will be obtained below. Two of the data sets are from deployments at sites 624 and 627, located in Kiel Bay, Baltic Sea (Fu et al., 1996b; Wilkens and Richardson, 1996). The seabed of Kiel Bay is composed of high-porosity clays that contain gas bubbles in some areas, and show evidence of significant biological activity (Richardson, 1994; Orsi et al., 1996). However, sediment velocities do not suggest the presence of gas bubbles at sites 624 and 627 (Fu et al., 1996b).

The third lance data set we analyze is site A2 in the Halekulani sand channel off the island of Oahu, Hawaii. The sediments there are carbonate sands. As the gravity corer was unable to penetrate this hard bottom, a jet probe was used to penetrate the sand approximately 2 m in depth. The lance was left in the sand for two months prior to taking the acoustic measurements.

The fourth lance data set analyzed is from site S90 on the Eel River delta in northern California. The silty sediments of this area are strongly influenced by biological reworking and periodic flood deposition from the Eel and Mad rivers (Nittouer and Kravitz, 1995).

**SPECTRAL RATIOS**

In processing the lance data for attenuation we assume that a lance signal at depth \( z \) below the receiver has a spectrum of the form

\[
S(\omega, z) = H(\omega) \frac{1}{z} \exp \left[ i \omega \int_0^z \frac{dz}{c(z)} \right].
\]

In this equation \( \omega = 2\pi f \) is radian frequency; \( H \) is the spectral transfer function of the system consisting of acoustic source, receiver and recording system; \( 1/z \) is the geometrical spreading factor for spherical spreading; and \( c(z) \) is the sound speed profile. Sound speed is taken to have the simple form

\[
\frac{1}{c} = \frac{1}{c_R} \left( 1 + \frac{iA}{2} \right),
\]

in which \( 1/c_R \) is the real part of \( 1/c \). Parametrizations of sound speed that include the frequency dependence of \( c_R \) and \( A \) were extensively tested, but they gave no improvement in results over the frequency-independent formula just given. From the expression for \( S \) it follows that the ratio of signal spectra \( S_1 \) and \( S_2 \), from receivers at respective depths \( z_1 \) and \( z_2 \), is given by

\[
\frac{S_2}{S_1} = \frac{H_2}{H_1} \frac{z_1}{z_2} \exp \left( i \omega T_{12} - \frac{\omega}{2} T_{12} A_{12} \right),
\]

in which the traveltimes from receiver 1 to receiver 2 is
Taking the logarithm of each side, and defining the data quantity \( \phi_{12}(f) \) by

\[
\phi_{12} = \ln \left( \frac{S_2}{S_1} \right) \frac{W_2}{W_1}.
\]

(9)

gives the spectral ratio formula

\[
\phi_{12}(f) = -\pi f T_{12} A_{12}.
\]

(10)

It follows that on a plot of \( \phi(f) \) vs \( f \) the plotted points should cluster about the straight line through the origin with slope \(-\pi T_{12} A_{12}\); this is the basis of the spectral ratio method.

As will be seen below, in our data set the line \( \phi(f) \) does not go through the origin. We attribute this to a change in receiver transfer function \( H \) from sediment to water. Examination of the derivation above shows that if going from sediment to water changes the gain on each receiver by a slightly different factor then the equation for \( \phi(f) \) will have a non-zero intercept. To allow for this, we use the following more general spectral ratio formula:

\[
\phi_{12}(f) = B - \pi f T_{12} A_{12}.
\]

(11)

in which \( B \) is unknown, but independent of \( f \). Other types of error are also present; for example, some energy travels through the probe itself. The effects of these other types of errors are neglected in the theory, but they are accounted for to some extent in our error analysis.

We wish to give more weight to values of \( f \) at frequencies where the signal-to-noise ratio is high. From the definition of \( \phi_{12} \) as a spectral ratio of the data we estimate the standard error in \( \phi_{12} \) by

\[
|d\phi_{12}| = \left| \frac{dS_1}{S_1} \right| + \left| \frac{dS_2}{S_2} \right| + \left| \frac{dW_1}{W_1} \right| + \left| \frac{dW_2}{W_2} \right|.
\]

(12)
at each frequency. To estimate the error $|dS_1|$ we Fourier analyze the noisy time series in a time window immediately preceding arrival of the signal $S_1$, assigning the amplitude spectrum value of the noise to $|dS_1|$, and similarly for the other signals in water and sediment. The windows used for Fourier analysis of signals and noise are shown in Fig. 4.

As spectral estimation is an important component of our procedure, we tested windows of different length and shape. Following Bromirski et al. (1995), tests were conducted by taking an actual recorded lance signal, and propagating it synthetically to the next receiver (1 m distant) using a dispersive propagation operator with given attenuation. The original seismogram and the dispensively propagated seismogram were then analyzed (by the inverse theory procedure given later in the paper) using various time windows for spectral estimation. Equation (11) of Bromirski et al. (1995) gives the propagation operator as

$$
\varphi(f) = \exp \left\{ i2\pi T \frac{f_j}{f} \left[ 1 + i \tan \left( \frac{\pi\gamma}{2} \right) \right] \right\},
$$

in which $T$ is the traveltime difference between the receivers, $f_j$ is a reference frequency, and $\gamma = \pi^{-1} \tan^{-1} A$. This propagation operator is based on the Kjartansson (1979) constant-$Q$ formula for dispersion.

Figure 5 shows plots of $\varphi(f)$ for various window lengths, using various actual signals. The value of $A$ used for propagation and the value of $A$ obtained from $\varphi(f)$ are also shown on each panel. Panels (a) and (b) of Fig. 5 show that a 640 µs window is satisfactory for analysis of the water signals, and that when too short a window is used $\varphi$ is badly behaved. Panels (c) and (d) show that a 640 µs window is also adequate for analysis of the Eel River delta signals. Based on these results we used a 640 µs window for analysis of the Eel River data. For the Kiel Bay analyses a 640 µs window was also used. Although the signals in the carbonate sand were not longer than those from Eel River, panels (e)–(f) show that a 1280 µs window works well for the carbonate sand. A 2560 µs window (not shown) works almost as well, however, it was not used on the data analysis because a window that long would include surface reflections.

**INVERSE THEORY**

Consider the forward equation

$$
d_s = G(m) + e,
$$

in which $d_s$ is a vector of measured data values, $m$ is a vector of parameters whose values are sought, $G$ is a model of the physical process, and $e$ is a vector of data errors. Notice that we distinguish between the measured data $d_s$ and the true data $d$ whose value is unknown. For example, the pressure values recorded by our hydrophones are not the true pressure values because the hydrophone is imperfect. This is an example of instrument error. Errors caused by noise processes in the apparatus used to record the data are also called instrument errors. On the other hand some types of error are due the inadequacy of our physical model, and these types of errors are referred to as theory errors. In our case an example

![FIG. 6. A two-stage model for geophysical inversion. Noise process $n_2$ includes geophysical parameters whose values we are not attempting to obtain from the experiment, but which may affect the data; the statistics of $n_2$ are assumed to be known. Noise process $n_1$ affects the sensors and recording system; its statistics are also assumed known. In general, the dependence of $d$ on $n_2$ is nonlinear, and the dependence of $d_s$ on $n_1$ is nonlinear.](image)

![FIG. 7. Inversion for the interval between receiver 2 and receiver 4 at site 624. (a) The posterior distribution $\sigma(A,B)$. (b) The marginal posterior distribution of $A$ obtained from $\sigma(A,B)$ by integration over $B$.](image)
\[ \sigma(m,d) \propto \nu(d_0|d) \theta(d|m) \rho(m), \]  

(15)

where \( \sigma \) is the joint posterior distribution of model and data, \( \nu \) is the instrument distribution, \( \theta \) is the theory distribution and \( \rho \) is the prior distribution of \( m \). [This equation becomes very familiar as soon as one inserts typical forms of the distributions on the right hand side; for example: \( \nu = \exp(-[d-d_0]^2) \), \( \theta = \exp(-[d-G(m)]^2) \), and \( \rho = \exp(-[m-m_0]^2) \), where \( \| \cdot \|_v \) and \( \| \cdot \|_\theta \) are norms in the data space, and \( \| \cdot \|_\rho \) is a norm in the model space.]

Duijndam (1988) has noted that relation Eq. (15) can be derived using the concept of conditional probability and its consequence, Bayes rule. As usual we define \( p(a|b) \), the conditional probability of event \( a \) given event \( b \), by the relation \( p(a|b) = p(a,b)/p(b) \) where \( p(a,b) \) is the joint probability of \( a \) and \( b \), and \( p(b) \) is the probability of \( b \). Accepting the notion of conditional probability, one sees that \( p(a,b) \) can be written as \( p(a|b)p(b) \) or as \( p(b|a)p(a) \), and equating these two expressions gives Bayes rule in the form \( p(a|b) = p(b|a)p(a)/p(b) \). Here we will extend Duijndam’s remarks by using Bayes rule to derive Eq. (15) in a way that reveals how \( \theta \) and \( \nu \) are related to the noise processes. As shown in Fig. 6, suppose that the instrument is subject to a noise process with unknown parameters \( n_1 \) and that the propagation (i.e., the theory) is contaminated by a different noise process with unknown parameters \( n_2 \). Beginning with Bayes rule we write

\[ p(m,d,n_1,n_2|d_0) = \frac{p(d_0|m,d,n_1,n_2)p(m,d,n_1,n_2)}{p(d_0)}. \]  

(16)

Notice that \( p(d_0) \) must be nonzero because it is a value of \( d \) that has actually been observed. Henceforth we omit the denominator, and use the proportional sign ‘\( \propto \)’ in place of the equal sign. Next notice that \( d_0 \) depends only on \( d \) and on the instrument noise process \( n_1 \), hence \( p(d_0|m,d,n_1,n_2) \) may be written \( p(d_0|d,n_1) \). Our relation has now simplified to

\[ p(m,d,n_1,n_2|d_0) \propto p(d_0|d,n_1)p(m,d,n_1,n_2). \]  

(17)

Using conditional probabilities we substitute the product \( p(d|m,n_1,n_2)p(m,n_1,n_2) \) for \( p(m,d,n_1,n_2) \), giving

\[ p(m,d,n_1,n_2|d_0) \propto p(d_0|d,n_1)p(d|m,n_1,n_2)p(m,n_1,n_2). \]  

(18)

As the true data \( d \) do not depend on the instrument noise, \( p(d|m,n_1,n_2) \) simplifies to \( p(d|m,n_2) \). Also, as \( m, n_1, \) and \( n_2 \) are independent variables, the joint distribution \( p(m,n_1,n_2) \) reduces to the product \( p(m)p(n_1)p(n_2) \). Our relation becomes

\[ p(m,d,n_1,n_2|d_0) \propto p(d_0|d,n_1)p(d|m,n_2)p(m)p(n_1)p(n_2). \]  

(19)

Finally, integration over the unknown noise parameters \( n_1 \) and \( n_2 \) yields

\[ \int dn_1 \int dn_2 p(m,d,n_1,n_2|d_0) \]

\[ \propto \left[ \int dn_1 p(d_0|d,n_1)p(n_1) \right] \times \left[ \int dn_2 p(d|m,n_2)p(n_2) \right] p(m). \]  

(20)

Comparing this relation with Tarantola’s relation we see that the left hand side is \( \sigma(m,d) \), the term \( \{ \cdot \} \) is the instrument distribution \( \nu(d_0|d) \), the term \( \{ \cdot \} \) is the theory distribution \( \theta(d|m) \), and \( p(m) \) is just the prior distribution of the data, \( \rho(m) \).

The derivation just given shows that Tarantola’s equation can be quickly derived by Bayes rule; it also shows that knowledge of the way in which errors arise in particular cases can be used to infer the forms of \( \nu \) and \( \theta \). For example, if one takes \( \nu \propto \exp(-[d-d_0]^2) \), then the noise process with parameters \( n_1 \) can be used to obtain the (covariance) matrix in the norm \( \| \cdot \|_v \). This fact will not be of much help here, as our noise is poorly understood, but it is helpful in other problems. A reader who has trouble with the derivation just given should try repeating it without \( n_1 \) and \( n_2 \). The Tarantola relation Eq. (15) is then obtained in two quick steps but without insight into \( \nu \) and \( \theta \).

We now take the usual next step with Tarantola’s equation, eliminating the true data \( d \) from the left hand side by integrating over the data space, thereby obtaining the posterior distribution of the model itself,

\[ \sigma_M(m) \propto \rho(m) \int dd \nu(d_0|d) \theta(d|m). \]  

(21)

When the model space has dimension two then one can display \( \sigma_M \) by a contour plot or perspective view, but when the model space has dimension greater than two it is often helpful to display the marginal posterior distributions of the components of \( m \) (e.g., Basu and Frazer, 1990). The marginal posterior distribution (MPD) of the \( k \)th component of \( m \) is obtained by integrating over all the \( N_M - 1 \) other components:

\[ \sigma_k(m_k) \propto \int dm_1 \cdots \int dm_{k-1} \int dm_{k+1} \cdots \int dm_{N_M} \sigma_M(m). \]  

(22)

This last relation is used below to display the attenuation profiles.

**DATA ANALYSIS**

It is possible to invert for the attenuation profile in a single step. However, as a form of quality control, we choose a two-step inversion procedure. In the first step we invert separately for the interval attenuations \( A_{ij} \) for each pair of receivers \( i < j \). (Notice that this includes nonadjacent receivers; thus many intervals overlap.) The interval attenuations found in this way are not necessarily consistent with each other. In the second step we use the interval attenuations as
data to invert for the attenuation profile. The second step can be regarded as a means of forcing the attenuations of overlapping intervals to be mutually consistent.

In the first inversion step we seek the attenuation of an interval between two receivers. The data vector $d_i$ is the vector $[\phi(f_1), \phi(f_2), \cdots, \phi(f_{N_f})]$ where $N_f$ is the number of frequencies, and the model vector $m$ is $[A, B]$. The instrument error due to the noise in the hydrophone and recording system is thought to be negligible compared to the theory error, so we chose an instrument function $\nu(d_a | d)$.

Our theory errors are significant, as indicated by the need to include a nonzero intercept $B$, but they are poorly understood and we cannot relate them to any specific aspect of the propagation. Thus we take

$$\theta(d|m) \propto \exp\left(-\sum f |\phi(f) - B + \pi f TA| \right),$$

where $|d\phi|$ is obtained from the noise and signal amplitude spectra by formula Eq. (12). The prior distribution $\rho(m)$ is imposed in the form of a search window, and as the manifold of $m$ is roughly Cartesian, so we take $\rho(m)$ to be constant within the search window and zero outside of it. Notice that the manifold of $m$ would not be Cartesian if the model vector were $[Q, B]$ instead of $[A, B]$. In consequence of the delta function for $\nu$, the integration in going from $\sigma(m, d)$ to $\sigma_M(m) = \sigma(A, B)$ is trivial, and $\sigma_M(m)$ is given by the right hand side of the last equation with $d_a$ substituted for $d$.

Figure 7 shows the results of a first step inversion for the interval between receivers 2 and 4 at site 624. Figure 7(a) shows the contours of the posterior distribution $\sigma(A, B)$. The contour plot was generated by explicitly computing $\sigma$ over the portion of model space indicated by the axes of the figures. Figure 7(b) shows the marginal posterior distribution (MPD) of the attenuation, $\sigma_A(A) = \int dB \sigma(A, B)$. In this example the peak of $\sigma(A, B)$, i.e., the MAP model, has the same attenuation value as the peak of the MPD. This is not always the case, and the theory indicates that the value of $A$ from the MAP model is preferable to the value of $A$ at which the MPD $\sigma_A(A)$ takes its peak.

For the second inversion step we need a theoretical relation between the attenuation of a large interval and the

**TABLE I. Single interval attenuations at sites 624, 627, SA1 and S90.**

<table>
<thead>
<tr>
<th>Receiver pair</th>
<th>Site 624 Kiel Bay</th>
<th>Site 627 Kiel Bay</th>
<th>Site A2 Carb. sand</th>
<th>Site S90 Eel River</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ij}$</td>
<td>$dA_{ij}$</td>
<td>$A_{ij}$</td>
<td>$dA_{ij}$</td>
<td>$A_{ij}$</td>
</tr>
<tr>
<td>$A_{12}$</td>
<td>0.11 ± 0.009</td>
<td>0.0068 ± 0.008</td>
<td>0.0348 ± 0.011</td>
<td>0.0627 ± 0.41</td>
</tr>
<tr>
<td>$A_{13}$</td>
<td>0.0333 ± 0.003</td>
<td>0.0217 ± 0.005</td>
<td>0.0535 ± 0.010</td>
<td>0.0637 ± 0.036</td>
</tr>
<tr>
<td>$A_{14}$</td>
<td>0.0127 ± 0.002</td>
<td>0 + 0.002</td>
<td>0.0441 ± 0.008</td>
<td>0.0505 ± 0.013</td>
</tr>
<tr>
<td>$A_{15}$</td>
<td>0.0286 ± 0.004</td>
<td>0.0036 ± 0.004</td>
<td>0.0441 ± 0.005</td>
<td>0.0208 ± 0.007</td>
</tr>
<tr>
<td>$A_{16}$</td>
<td>0.0213 ± 0.007</td>
<td>0.0042 ± 0.002</td>
<td>nd nd</td>
<td>0.0049 ± 0.007</td>
</tr>
<tr>
<td>$A_{17}$</td>
<td>0.0244 ± 0.002</td>
<td>0.0088 ± 0.003</td>
<td>nd nd</td>
<td>nd nd</td>
</tr>
<tr>
<td>$A_{23}$</td>
<td>0 ± 0.004</td>
<td>0.034 ± 0.005</td>
<td>0.1403 ± 0.025</td>
<td>0.0648 ± 0.04</td>
</tr>
<tr>
<td>$A_{24}$</td>
<td>0 ± 0.001</td>
<td>0 ± 0.001</td>
<td>0.0743 ± 0.013</td>
<td>0.0389 ± 0.019</td>
</tr>
<tr>
<td>$A_{25}$</td>
<td>0.0164 ± 0.002</td>
<td>0 ± 0.001</td>
<td>0.0512 ± 0.006</td>
<td>0.0356 ± 0.028</td>
</tr>
<tr>
<td>$A_{26}$</td>
<td>0.0019 ± 0.002</td>
<td>0.0036 ± 0.003</td>
<td>nd nd</td>
<td>0.0014 ± 0.017</td>
</tr>
<tr>
<td>$A_{27}$</td>
<td>0.0172 ± 0.002</td>
<td>0.0036 ± 0.003</td>
<td>nd nd</td>
<td>nd nd</td>
</tr>
<tr>
<td>$A_{34}$</td>
<td>0 ± 0.002</td>
<td>0 ± 0.001</td>
<td>0.0125 ± 0.032</td>
<td>0.0565 ± 0.032</td>
</tr>
<tr>
<td>$A_{35}$</td>
<td>0.0257 ± 0.004</td>
<td>0 ± 0.001</td>
<td>0.0268 ± 0.001</td>
<td>0 ± 0.024</td>
</tr>
<tr>
<td>$A_{36}$</td>
<td>0.0132 ± 0.003</td>
<td>0.0011 ± 0.004</td>
<td>nd nd</td>
<td>0 ± 0.008</td>
</tr>
<tr>
<td>$A_{37}$</td>
<td>0.0151 ± 0.002</td>
<td>0.0025 ± 0.003</td>
<td>nd nd</td>
<td>nd nd</td>
</tr>
<tr>
<td>$A_{45}$</td>
<td>0.0526 ± 0.005</td>
<td>0.0196 ± 0.009</td>
<td>0.0187 ± 0.026</td>
<td>0.0108 ± 0.07</td>
</tr>
<tr>
<td>$A_{46}$</td>
<td>0.0263 ± 0.004</td>
<td>0.0026 ± 0.003</td>
<td>nd nd</td>
<td>0 ± 0.011</td>
</tr>
<tr>
<td>$A_{47}$</td>
<td>0.0244 ± 0.006</td>
<td>0.0069 ± 0.003</td>
<td>nd nd</td>
<td>nd nd</td>
</tr>
<tr>
<td>$A_{56}$</td>
<td>0 ± 0.001</td>
<td>0 ± 0.002</td>
<td>nd nd</td>
<td>nd nd</td>
</tr>
<tr>
<td>$A_{57}$</td>
<td>0.0133 ± 0.005</td>
<td>0 ± 0.002</td>
<td>nd nd</td>
<td>nd nd</td>
</tr>
<tr>
<td>$A_{67}$</td>
<td>0.0435 ± 0.005</td>
<td>0.014 ± 0.013</td>
<td>nd nd</td>
<td>nd nd</td>
</tr>
</tbody>
</table>
attenuations of its subintervals. Recalling Eq. (6) it can be seen that for the interval between receiver 1 and receiver 4 the relation is

\[ A_{14} = \frac{1}{T_{14}} (T_{12}A_{12} + T_{23}A_{23} + T_{34}A_{34}), \]  

(24)
in which \( T_{ij} \) is the traveltime difference between receivers \( i \) and \( j \). For a set of four receivers the data vector is given by \( d = [A_{12}, A_{13}, A_{14}, A_{23}, A_{24}, A_{34}] \) and the model vector is \( m = [A_{12}, A_{23}, A_{34}] \). This notation is not confusing if it is remembered that the appearance of a single interval attenuation, \( A_{ij} \), for example, in \( d \) represents an observation, while its appearance in \( m \) represents a quantity to be determined. (Note that an inversion constrained to preserve single interval attenuations would be trivial.) For an experiment with only four receivers the theory is the linear matrix equation,

\[
\begin{bmatrix}
A_{12} \\
A_{13} \\
A_{14} \\
A_{23} \\
A_{24} \\
A_{34}
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 & 0 \\
T_{12}/T_{13} & T_{23}/T_{13} & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & T_{23}/T_{24} & T_{34}/T_{24} & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\begin{bmatrix}
e_{12} \\
e_{13} \\
e_{14} \\
e_{23} \\
e_{24} \\
e_{34}
\end{bmatrix}
\]  

(25)
in which \( e_{ij} \) is the error in \( A_{ij} \). In the general case of \( N \) receivers the number of rows in the data vector and system matrix is \( N(N-1)/2 \) where \( N \) is the number of receivers. If a particular receiver is omitted then the rows and columns corresponding to that receiver are suppressed. This affects the condition of the matrix, an important item in a conventional \( L_2 \) inversion for the MAP model. Here we generate \( \sigma_M \) explicitly, so the condition of the matrix does not affect the computation.

In the second step of inversion, our data consist of the interval attenuations obtained in step 1. The errors in the theory are negligible compared to the errors in the data, so we take the theory distribution to be \( \nu(m|d) = \delta(d - G \cdot m) \). In the four-receiver example of Eq. (25) \( G \) is the \( 3 \times 4 \) matrix on the right hand side. Each data error \( |dA_{ij}| \) is estimated as the half-width of the peak of the marginal distribution \( \sigma_M(A_{ij}) \). For the instrument distribution we therefore choose

\[ \nu(d_o|d) \propto \exp \left[ -\sum_{i < j} \left[ A_{ij} - A_{ij}^o \right]^2 \right] / |dA_{ij}| \]  

(26)
in which the \( A_{ij}^o \) are the interval attenuations obtained in step 1. The delta nature of \( \theta \) makes the integration in Eq. (20) trivial and the posterior distribution for the model becomes

\[ \sigma_M(m) \propto \exp \left[ -\sum_{i < j} \left[ (G \cdot m)_{ij} - A_{ij}^o \right]^2 / |dA_{ij}| \right] , \]  

(27)
in which \((G \cdot m)_{ij}\) is the row of \( G \cdot m \) corresponding to interval \( ij \). The distribution \( \sigma_M(m) \) is explicitly generated by exploration of the model space, but for display purposes we generate the marginal distributions. An example of a marginal distribution with five receivers is

\[ \sigma_{23}(A_{23}) \propto \int dA_{12} \int dA_{34} \int dA_{45} \sigma_M(m) . \]  

(28)

**ATTENUATION RESULTS**

As shown in Fig. 8, in the lance deployments at Kiel Bay, eight receivers penetrated the sediments at both sites. The signals from the first receiver (R0) were clipped and were therefore not used for the attenuation calculations, thus the shallowest interval at which attenuation can be measured is approximately 0.5–1.0 mbsf. The remaining 7 receivers (6 intervals) give 21 different receiver pairs, and thus 21 different depth intervals. For each pair of receivers we construct the posterior distribution \( \sigma(A,B) \) as shown in Fig. 7(a) and the MPD \( \sigma_A(A) \), shown in Fig. 7(b). Not all contour plots were as definitive as this one. As shown in Table I, 16 intervals were analyzed at site 624, and 13 intervals were analyzed at site 627.

At the Halekulani sand channel only five receivers penetrated this relatively hard sediment. At the Eel River site, five receivers penetrated the deltaic sediments, with one receiver remaining above the sediment water interface. Lance signals from Eel River were noisy in the band 15–20 kHz, because source placement on the core head resulted in significant energy traveling down the body of the corer. (This placement was for protection of the source in case the corer overturned.) Almost every spectral ratio from Eel River had

**FIG. 9.** The marginal posterior distribution (MPD) of attenuation at each depth, site 624 (silty clay), Kiel Bay. Row 1 shows \( \sigma_{12}(A_{12}) \), row 2 shows \( \sigma_{23}(A_{23}) \), and so forth.
outliers in the 15–20 kHz band, so this band was automatically suppressed in the attenuation calculation by the signal-to-noise weighting in Eq. (23).

As an example of the second stage inversion, we use site 624; the matrix equation analogous to Eq. (24) for Kiel Bay, site 624 is

\[
\begin{bmatrix}
A_{12}^{(o)} & 0.11 \\
A_{13}^{(o)} & 0.0333 \\
A_{14}^{(o)} & 0.0127 \\
A_{15}^{(o)} & 0.0286 \\
A_{16}^{(o)} & 0.0213 \\
A_{17}^{(o)} & 0.0244 \\
A_{18}^{(o)} & 0.0164 \\
A_{19}^{(o)} & 0.0172 \\
A_{20}^{(o)} & 0.0127 \\
A_{21}^{(o)} & 0.0132 \\
A_{22}^{(o)} & 0.0323 \\
A_{23}^{(o)} & 0.0526 \\
A_{24}^{(o)} & 0.0263 \\
A_{25}^{(o)} & 0.0244 \\
A_{26}^{(o)} & 0.0133 \\
A_{27}^{(o)} & 0.0435 \\
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0.502 & 0.498 & 0 & 0 & 0 & 0 \\
0.33 & 0.328 & 0.342 & 0 & 0 & 0 \\
0.247 & 0.246 & 0.256 & 0.25 & 0 & 0 \\
0.198 & 0.197 & 0.205 & 0.201 & 0.198 & 0 \\
0.166 & 0.165 & 0.171 & 0.167 & 0.166 & 0.166 \\
0 & 0.327 & 0.341 & 0.333 & 0 & 0 \\
0 & 0.197 & 0.205 & 0.201 & 0.198 & 0.198 \\
0 & 0 & 0.506 & 0.494 & 0 & 0 \\
0 & 0 & 0.342 & 0.334 & 0.33 & 0 \\
0 & 0 & 0.256 & 0.25 & 0.247 & 0.247 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0.503 & 0.497 & 0 \\
0 & 0 & 0 & 0.336 & 0.332 & 0.332 \\
0 & 0 & 0 & 0.5 & 0.5 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

in which the column vector of errors on the right hand side is understood. To generate the posterior distribution \( \sigma_M(m) \) for this equation we use Eq. (27) with eight possible values of \( A \) at each depth. This gave \( 6^8 \approx 262 \, 000 \) different attenuation profiles. The MPDs \( \sigma_{ij}(A_{ij}) \), generated by Eq. (28) with the integration replaced by a sum, are shown in Fig. 9. Figures 10–12 show the MPDs for the other sites. The second stage inversion was carried out twice for the Eel River data (Fig. 13) because at that site the first receiver was slightly above the water bottom. Similar results are obtained whether the first receiver is included or excluded, although when the first receiver is excluded the MPDs are broader.

Results for all four sites are summarized as attenuation profiles in Fig. 13, where dashed lines show the MAP pro-

![FIG. 10. MPD of attenuation at each depth, site 627 (silty clay), Kiel Bay.](image)

![FIG. 11. MPD of attenuation with depth, site A2, Halekulani sand channel (carbonate sand), Oahu, Hawaii.](image)
files and solid lines show the profiles generated by picking the peak of $\sigma_d(A)$ at each depth, i.e., picking the peaks in Figs. 9–12. Table II includes the compressional sound speed distribution in depth in the interval 0–4.1 mbsf. At sites 624 and 627, it can be seen that sound speed is fairly constant at both sites, varying between 1430 m/s and 1460 m/s (Fu et al., 1996b). Thus the differences in attenuation at the two sites do not correlate with sound speed. The attenuation profile at Kiel Bay, site 624, shows high attenuation zones at 0.5–1.1 mbsf and 2.3–2.9 mbsf, and moderate attenuation in the interval 2.9–3.5 mbsf. These higher attenuation zones are thought to result from the presence of very small amounts of gas bubbles, although the sound speed and signal character do not suggest gas bubbles at this site (Fu et al., 1996b). Site 627, also from Kiel Bay, shows uniformly low attenuation with low variability, suggesting that gas bubbles are not present there even in small amounts. In contrast with the soft silty clays of Kiel Bay, the results from the Halekulani sand channel show a high attenuation, decreasing slowly with depth, possibly due to compaction. The data from the Eel River delta show the highest attenuation values, decreasing relatively rapidly with depth. The high attenuations at shallow depths in the Eel River delta and the Halekulani sand may both be due to biological reworking; however, the high attenuation in the Eel river sediments may also be the result of periodic flood deposition, as microbedding in sediments is known to increase attenuation (e.g., Frazer, 1994).

**DISCUSSION**

The sediments at sites 624 and 627 in Kiel Bay are soft silty clays with high water content. Waves and bottom currents induced by storms can impact the seafloor to all water depths in this area (Orsi et al., 1996). The attenuation profile at Kiel Bay, site 624, shows high attenuation zones at 0.5–1.1 mbsf and 2.3–2.9 mbsf, and moderate attenuation in the interval 2.9–3.5 mbsf. Although acoustic turbidity is common in the sediments of Kiel Bay, resulting from the methane gas bubbles beginning about 1 m below the seafloor (Richardson and Briggs, 1996; Orsi et al., 1996), sound speed and signal character do not suggest gas bubbles at these two sites (Fu et al., 1996b). Sound speed does not vary much with depth here, being confined to the interval 1428–1460 m/s (Table II). The zones of higher attenuation may be sand layers formed by storm impact or layers of laminated coarser silt. Sand size particles are known to increase attenuation (Hamilton, 1972) as is micro-bedding (e.g., Frazer, 1994). Site 627, also from Kiel Bay, shows uniformly low attenuation with low variability, suggesting much more homogeneous fine grained sediments.

In contrast with the soft silty clays of Kiel Bay, the results from the Halekulani sand channel, off Oahu island, show a high attenuation decreasing slowly with depth. The sediments covering Halekulani sand channel are carbonate sands. Sand deposits in the Hawaiian Islands are varied in response to microclimates mainly created by island topogra-
Table II. Compressional sound speeds and attenuations at sites 624, 627, SA1 and S90.

<table>
<thead>
<tr>
<th>Depth (mbsf)</th>
<th>Soundspeed (m/s)</th>
<th>Attenuation $A = Q^{-1}(Q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0–0.5</td>
<td>1430</td>
<td>nd</td>
</tr>
<tr>
<td>0.5–1.1</td>
<td>1439</td>
<td>0.05 (20)</td>
</tr>
<tr>
<td>1.1–1.7</td>
<td>1428</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>1.7–2.3</td>
<td>1446</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>2.3–2.9</td>
<td>1429</td>
<td>0.043 (23)</td>
</tr>
<tr>
<td>2.9–3.5</td>
<td>1444</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>3.5–4.1</td>
<td>1442</td>
<td>0.033 (30)</td>
</tr>
<tr>
<td>Hekulani Carb. sand, Site A2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0–0.69</td>
<td>1617</td>
<td>$&gt;0.05$ ($&lt;20$)</td>
</tr>
<tr>
<td>0.69–1.03</td>
<td>1653</td>
<td>$&gt;0.05$ ($&lt;20$)</td>
</tr>
<tr>
<td>1.03–1.37</td>
<td>1660</td>
<td>0.045 (22)</td>
</tr>
<tr>
<td>1.37–2.07</td>
<td>1626</td>
<td>0.022 (45)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Depth (mbsf)</th>
<th>Soundspeed (m/s)</th>
<th>Attenuation $A = Q^{-1}(Q)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0–0.5</td>
<td>1430</td>
<td>nd</td>
</tr>
<tr>
<td>0.5–1.1</td>
<td>1447</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>1.1–1.7</td>
<td>1428</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>1.7–2.3</td>
<td>1446</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>2.3–2.9</td>
<td>1447</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>2.9–3.5</td>
<td>1453</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>3.5–4.1</td>
<td>1460</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>Eel River Delta, Site S90</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0–0.5</td>
<td>1477</td>
<td>nd</td>
</tr>
<tr>
<td>0.1–0.5</td>
<td>1480</td>
<td>$&gt;0.05$ ($&lt;20$)</td>
</tr>
<tr>
<td>0.5–0.9</td>
<td>1493</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>0.9–1.3</td>
<td>1492</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
<tr>
<td>1.3–1.7</td>
<td>1524</td>
<td>$\leq 0.005$ ($&gt;200$)</td>
</tr>
</tbody>
</table>

*Sound speeds at 0 mbsf are averaged bottom water sound speed.

phy, waves, tidal currents, stream discharge and catastrophic events (Coulbourn et al., 1988); however, there is no apparent relation between grain size and the spatial location of a sample or the vertical location of a sample within the deposit in the Hekulani sand channel (Ericson et al., 1995). Although decreasing attenuation with depth in sands may result from reduced porosity by compaction (Hamilton, 1972), our experiment was in a carbonate sand layer of only 2 m thickness, and compaction should not contribute significantly to the attenuation decrease with depth. More likely, sediment reworking by bottom water currents or waves reduces the strength of grain contacts, resulting in increased attenuation as well as lowered rigidity.

The data from the Eel River delta, northern California, show high attenuation values at shallow depths of 0.1–0.5 mbsf, decreasing relatively rapidly with depth. Site S90 is located on the downslope of the delta in water of depth 90 m. The sediments at the site are a mixture of silts and clays, and the study area is strongly influenced by both biological reworking and periodic flood deposition from the Eel and Mad rivers (Nittrouer and Kravitz, 1996). Initial laboratory measurements on core data collected at 1 cm intervals show higher variability in sound speed (from 1450–1520 m/s at 400 kHz) within the interval of 0.1–0.5 mbsf, than outside this interval, which suggests that microbeds due to periodic floods may be present in this shallow depth zone. Gorgas et al. (1996) also saw a strong correlation between attenuation and sound speed in the core data. They suggest that the faster layers may have a higher sand content, and thus a higher attenuation.

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