Interpolation and Gridding

GG691 Lab 4
March 22, 2007
Agenda

• Demonstrate 1-D spline
  – Cubic splines and splines in tension
• Grid a 2-D topography data set
  – Use Matlab’s `griddata` function
    • Linear and cubic Delaunay triangulation
    • Nearest neighbor
    • Sandwell’s [1987] biharmonic Green’s function
  – General Green’s function for Cartesian splines with or without tension
**Cartesian Green’s functions**

**Minimum Curvature**

\[ \nabla^4 \phi = \delta \]

<table>
<thead>
<tr>
<th>Dimension</th>
<th>( \phi(r) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D</td>
<td>( r^3 )</td>
</tr>
<tr>
<td>2-D</td>
<td>( r^2 \log r )</td>
</tr>
<tr>
<td>3-D</td>
<td>( r )</td>
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Cartesian Green’s functions
Splines in tension

$$\nabla^4 \phi - p^2 \nabla^2 \phi = \delta$$

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Formula</th>
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<tbody>
<tr>
<td>1-D</td>
<td>$$\phi(r) = e^{-pr} + pr$$</td>
</tr>
<tr>
<td>2-D</td>
<td>$$\phi(r) = K_0(pr) + \log(pr)$$</td>
</tr>
<tr>
<td>3-D</td>
<td>$$\phi(r) = \frac{1}{pr} \left[ e^{-pr} - 1 \right] + 1$$</td>
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Spherical Gridding

• When area is too large to ignore Earth’s curvature we must interpolate on the sphere.

• Parker [1993] found solution for minimum curvature ($\theta$ is angular distance)

$$\phi(\theta) = \text{dilog}\left(\sin^2 \frac{\theta}{2}\right)$$

• Janet B and I are including tension and find

$$\phi(\theta) = \frac{\pi P_v (-\cos \theta)}{\sin v \pi} - \log(1 - \cos \theta)$$

$$v = -\frac{1}{2} + \sqrt{\frac{1}{4} - p^2}$$
Process of Solution

• Calculate matrix \( R \) with radial distances between all pairs of the \( n \) observation points.

• Let observations \( z = z_0 + \Delta z \)
  – \( \Delta z \) are the deviations from the mean \( z_0 \).

• Evaluate the Green’s function for all these distances, yielding square \( n \) by \( n \) matrix \( G \).

• Solve for the \( n \) weights \( w = G^{-1}\Delta z \).

• For each evaluation point \( x_k \)
  – Get \( r \), the distance vector to all \( n \) data points.
  – \( z_k = z_0 + r^T w \).