**HOMEWORK 8 – ANSWERS**

1. b) Using $t(d) = t_0 + sd = t_0 + \frac{d}{v}$ we find using equations from Lecture 16:

   $t_0 = -1.067 \text{ m.y.}$  
   $s = 0.0113 \text{ m.km}^{-1}$  
   $v = 8.85 \text{ cm yr}^{-1}$

c) Taking the weights into account we get (using equations from Lecture 24)

   $t_0 = -0.678 \text{ m.y.}$  
   $s = 0.0101 \text{ m.km}^{-1}$  
   $v = 9.88 \text{ cm yr}^{-1}$

e) The slope is the inverse of the velocity of the Pacific plate over the Hawaiian hotspot. We find the values to be in general agreement with plate tectonic theory. The intercept represents the age of the volcanism at the hotspot. It is not zero because the ages sampled along the chain are taken from the uppermost (i.e. youngest) parts of each volcano. Since volcanoes take several 100 kyr to form we anticipate a negative intercept value. Also, there are some suggestions that the plate velocity has increased over the last few m.y.

2. a) Our fracture zone model becomes

   $$d(x) = c_1 + c_2 x + c_3 H(x - x_0)$$

for some fixed choice of $x_0$ (we must fix this value since the model is nonlinear in $x_0$).

b) By plotting the data and eyeballing I guessed $x_0 = 630 \text{ km}$. Our matrix equation $A \cdot c = y$ becomes

   $$\begin{bmatrix}
   1 & x_1 & H(x_1 - x_0) \\
   1 & x_2 & H(x_2 - x_0) \\
   \vdots & \vdots & \vdots \\
   1 & x_n & H(x_n - x_0)
   \end{bmatrix} \begin{bmatrix}
   c_1 \\
   c_2 \\
   \vdots \\
   c_3
   \end{bmatrix} = \begin{bmatrix}
   y_1 \\
   y_2 \\
   \vdots \\
   y_n
   \end{bmatrix}$$

The solution becomes $c = (A^TA)^{-1}A^Ty$; I found

$$\begin{bmatrix}
-3639.7 \\
-0.4268 \\
-619
\end{bmatrix}$$

c) It seems clear that $x_0$ must be in the range $620 \leq x_0 \leq 660$. By trying all $x_0$ in this range (i.e., every 1 km interval) I solved for $c$ and evaluated the misfit $E = e^T e$. A plot of $E$ versus $x_0$ indicated the best value was $x_0 = 638.5 \text{ km}$. The new solution becomes

$$\begin{bmatrix}
-3700.1 \\
-0.2959 \\
-679.7
\end{bmatrix}$$

d) Our step $c_3$ must equal the difference in the predicted depths. We find

$$d(t_{old}) - d(20) = c_3 = \alpha \left( \sqrt{t_{old}} - \sqrt{20} \right) \Rightarrow t_{old} = \left( \frac{t_1}{\alpha} + \sqrt{20} \right) = 41.14 \text{ m.y.}$$