Wave-induced deep equatorial ocean circulation

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introduction
Observations - Mean U at 159°W

Equatorial Deep Jets (EDJs)

Tall Equatorial Jets (TEJJs)

Firing 87
Observations - Mean U at 159°W

Tall Equatorial Jets (TEJs)

Firing 87
A modelling challenge

Observations

JAMSTEC
1/4°, 54 levels

POP
1/10°, 106 levels
A theoretical challenge

JAMSTEC – Mean U over 500-900 m

Source of PV?
High-frequency equatorial waves generated by the instabilities of the swift surface currents

JAMSTEC – Snapshot of equatorial V

New Guinea

Galápagos Islands
Spectrum of equatorial $V$ between 400 and 1000 m

Central Pacific  
Eastern Pacific  
Eastern Atlantic

Eriksen and Richman 88, Tang et al. 88, Weisberg and Horigan 81
High-frequency equatorial waves generated by the instabilities of the swift surface currents rectifies into deep mean equatorial currents
test of the working hypothesis
in idealized numerical simulations
Configuration of the model

primitive equations / POP model
rectangular basin: 20°S-20°N and 8000 km long
1/4° x 1/4° and 100 levels
weak explicit dissipation, constant N

monthly-periodic meridional surface stress
1) weak amplitude, 2) moderate amplitude
"Moderate" – Snapshot of equatorial V
Beam amplitude

“weak” $\Rightarrow$ no dissipation

“moderate” $\Rightarrow$ dissipation

U: zonal velocity
c: zonal phase speed
Snapshot of equatorial V within the beam

in “moderate”
  ↓
multiple nonlinear interactions
  ↓
small scales
  ↓
dissipation
Mean U within the beam

“weak”

“moderate”
“Moderate” – Snapshot of equatorial V
Mean U averaged along the beam

"weak"

"moderate"
Mean U west of the beam in “moderate” versus Mean U at 159°W in observations and JAMSTEC
Mean U averaged along the beam

“moderate” with modulated forcing

“moderate” with $1/8^\circ \times 1/8^\circ$ 400 levels
understanding the numerical simulations using analytical solutions
Two processes at play

\[ U = \]

1) a component opposing the *Stokes drift*  
   kinematic  
   localized to the beam  
   no need of dissipation or b.c

2) a *Lagrangian component*  
   dynamic  
   remote effect  
   needs dissipation and b.c.
case without dissipation
Stokes drift of a surface gravity wave

Wave field trajectory

Net drift

Off-shore to beach

Z

X
Stokes drift of a Yanai wave

\[
\begin{align*}
\text{Eq.} & \\
5^\circ \text{N} & \\
0 \text{ days} & \\
5^\circ \text{S} & \\
\end{align*}
\]

- wave field
- trajectory
- net drift
Stokes drift of a Yanai wave
Stokes drift without dissipation

\[ V_S = 0 \]
Potential vorticity

\[ \frac{f + \zeta}{h} \]
Lagrangian component without dissipation

\[ V_L = 0, \quad \beta V_L = 0 \]

- Mass conservation
- Eastern boundary

\[ U_L = 0 \]
Stokes trajectory *versus* actual trajectory
Stokes trajectory versus actual trajectory

\[ x, y \]

\[ 0, \pi \]

\[ 3\pi/4, \pi/4 \]
Stokes trajectory versus actual trajectory
Stokes: $U_S$

$U = -U_S$
"weak" numerical solution

inviscid analytical solution
case with dissipation
Dissipation = - D x (U, V)

Beam amplitude

U: zonal velocity

C: zonal phase speed
Stokes drift with dissipation

$20 \times V_S$

$U_S$
mean Lagrangian $\zeta = \zeta_L$
Mean Lagrangian $\zeta_L$

$\beta V_L \approx -D \zeta_L$

Andrews and McIntyre 78
Lagrangian component with dissipation

$20 \times V_L$

$U_L$
"moderate" numerical solution

viscid analytical solution
conclusions
numerical and analytical solutions

\[ \Downarrow \]

TEJs can result from the rectification of a beam of Yanai waves dissipated in the vertical
Mean flow is composed of two components:

1) a component opposing the *Stokes drift* kinematic localized to the beam from inviscid wave solution

2) a *Lagrangian* component dynamic remote effect from inviscid wave solution

\[ \beta V_L \approx - D \xi_L \]

dissipation is key to obtain TEJs all across the basin
mechanism to produce the TEJs

energy transfer from high to low frequency basin scale effect
robust
physically meaningful
consistent with JAMSTEC

other models?
test in nature?
other deep currents?

mechanism for the EDJs

partially similar mechanism but between Yanai and inertia-gravity waves?
Spectrum of equatorial V between 400 and 1000 m

Central Pacific

Eastern Pacific

Eastern Atlantic

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mechanism to produce the TEJs

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Thank you...

Eric,
Jay, Dennis, Doug, Fei-Fei and Bo,
Ryo and Andrei,
Dailin,
IPRC computer facility,
Jules and Derek,
Friends, Chèvre, Cam, Jérôme,
Captain Flam,
Pierre,
Nancy,
My parents, brothers and sisters
Emano
Thank you
Merci!
Mahalo
شكراً
extras
Moderate – Smoothed equatorial U averaged along the beam
Moderate – Smoothed U at x = 5000 km and along the equator
Amplitude of the TEJs

- Westward within beam
- Eastward within beam
- Westward west of beam
- Eastward west of beam
There will be blood – Eulerian solution (1)

First-order Yanai beam

\[
\begin{align*}
\partial_t u - fv + \partial_x p & = -ru \\
\partial_t v + fv + \partial_y p & = -rv \\
-1/N^2 \partial_{zzt}p + \partial_x u + \partial_y v & = 0
\end{align*}
\]

\[(u, v, p) \propto e^{i(kx + mz - \omega t)} \text{ avec } (k, \omega) \in \mathbb{R}, m \in \mathbb{R}\]
There will be blood – Eulerian solution (2)

Equations for the second-order mean flow

\[-fV + \partial_x P = F_x\]
\[+fV + \partial_y P = F_y\]
\[\partial_x U + \partial_y V = g/N^2 \partial_z F_\rho\]

\[
\begin{bmatrix}
F_x \\
F_y \\
F_\rho
\end{bmatrix} = - (\vec{u} \cdot \nabla) \begin{bmatrix}
u \\
v \\
\rho
\end{bmatrix}
\]
There will be blood – Eulerian solution (3)

Second-order mean flow

\[
W = 0
\]

\[
V = \frac{1}{\beta} \nabla \times (F_x, F_y)
\]

\[
U = \frac{1}{\beta} \int_{xE}^{x} \partial_y \left\{ \nabla \times (F_x, F_y) \right\} \, dx
\]

\[
W = -\frac{g}{N^2} F_\rho
\]

\[
V = -\frac{g}{N^2 y \cdot \partial_z F_\rho}
\]

\[
U = -\int_{x}^{xE} \frac{g}{N^2} \left\{ 2 \partial_z F_\rho + y \cdot \partial_{yz} F_\rho \right\} \, dx
\]
There will be blood – Lagrangian solution (1)

PV equation in isopycnal coordinates

\[
\begin{align*}
\frac{D}{Dt} \left[ \frac{f + \zeta}{h + \bar{h}} \right] &= -r \frac{\zeta}{h + \bar{h}} \\
\frac{D}{Dt} &= \partial_t + u \partial_x + v \partial_y
\end{align*}
\]
There will be blood – Lagrangian solution (2)

Mean Lagrangian PV equation

\[ \frac{D^L}{Dt} \left[ f + \zeta \right]^L = -r \left[ \frac{\zeta}{h+h} \right]^L \]

\[ \frac{D^L}{Dt} = \partial_t + U_L \partial_x + V_L \partial_y \]

Andrews and McIntyre 78
There will be blood – Lagrangian solution (3)

Time-mean Lagrangian PV equation

\[ V_L = -\frac{r}{\beta} \left[ \dot{\zeta}_L - \frac{\zeta h^L}{h} \right] \]