Supporting Information

Zeebe 10.1073/pnas.1222843110

SI Text

Equilibrium and Transient Response

The feedback analysis (starting with Eq. 1) and the formulation of time-dependent climate sensitivity is based on a linearization around an initial equilibrium state (say global mean surface temperature \(T_0\)) and assumes that, under a sustained forcing, a new equilibrium will be established at \(T_0 + \Delta T\). However, the transient response to establish this new equilibrium may take a long time, during which the system is not in equilibrium (e.g., Fig. 2B). Note that this type of behavior is not limited to the present case (e.g., ref. 1). The system’s response time is governed by the sum of the feedbacks and the effective thermal inertia of the system (here, mostly ocean heat uptake; included in the projections; see below). Thus, the feedback analysis and the formulation of time-dependent climate sensitivity as presented here is not limited to equilibrium states. Rather, the transient response can be explicitly modeled, provided that the system’s thermal inertia can be determined, as is the case here.

Long-Term Ocean–Atmosphere–Sediment Carbon Cycle Reservoir Model

The Long-term Ocean-atmosphere-Sediment CARbon cycle Reservoir (LOSCAR) model computes the partitioning of carbon between ocean, atmosphere, and sediments on timescales ranging from centuries to millions of years (2, 3). LOSCAR couples ocean–atmosphere routines to a computationally efficient sediment module. This allows, for instance, adequate computation of CaCO\(_3\) dissolution, calcite compensation, and long-term carbon cycle fluxes, including weathering of carbonate and silicate rocks. The ocean component includes biogeochemical tracers such as total carbon, alkalinity, phosphate, oxygen, and stable carbon isotopes. LOSCAR’s configuration of ocean geometry is flexible and allows for easy switching between modern and paleo versions. We have previously published various tests and applications of the model tackling future projections of ocean chemistry and weathering, \(p\text{CO}_2\) sensitivity to carbon cycle perturbations throughout the Cenozoic, and carbon/calcium cycling during the Paleocene–Eocene Thermal Maximum (2–9). The model’s architecture, its components, tuning, and examples of input and output are described in detail in ref. 3.

Uncertainties in Parameterizations. Several parameterizations were included in the LOSCAR model for the present study, which are subject to uncertainties (see below). For the most part, however, the uncertainties are not critical because they apply equally to projections that calculate and exclude slow feedbacks. The main point of this study is the difference between these two types of simulations (Fig. 2). Uncertainties in the slow feedbacks themselves are examined by varying the slow-feedback parameters (Fig. 3).

Greenhouse Gas Forcing. The global mean surface warming was calculated as:

\[ \Delta T = S(t) (\Delta R_{\text{CO}_2} + \Delta R_{\text{CH}_4} + \Delta R_{N_2O}), \]

where \(\Delta R_n\) is the change in radiative forcing of greenhouse gas (GHG) \(n\) at atmospheric concentration \(C_n\) (10):

\[ R_{\text{CO}_2} = 5.35 \ln([\text{CO}_2]/[\text{CO}_2]_0) \]
\[ = 3.7 \ln([\text{CO}_2]/[\text{CO}_2]_0)/\ln(2) \]
\[ R_{\text{CH}_4} = 0.036 (\sqrt{[\text{CH}_4] - \sqrt{[\text{CH}_4]_0}) \]
\[ R_{N_2O} = 0.12 (\sqrt{[N_2O] - \sqrt{[N_2O]_0}), \]

where the subscript “0” refers to preindustrial concentrations. Note that the greenhouse effect due to changes in water vapor is part of the fast feedbacks and is hence not included here. For a given emission scenario (4), \(C_{\text{CO}_2}\) was calculated using LOSCAR. \(C_{\text{CH}_4}\) and \(C_{N_2O}\) were determined from:

\[ \frac{dC_n}{dt} = (C_n^0 - C_n)/\tau_n + F_n, \]

where \(C_n\), \(\tau_n\), and \(F_n\) are preindustrial concentrations, mean lifetimes (\(\tau_{\text{CH}_4} = 12\) y, \(\tau_{N_2O} = 114\) y; refs. 11, 12), and emissions, respectively. \(F_{\text{CH}_4}\) and \(F_{N_2O}\) were taken proportional to \(CO_2\) emissions and scaled based on published GHG emission ratios (13). Anthropogenic emissions of non-CO\(_2\) GHGs are not to be confused with slow non-CO\(_2\) GHG feedbacks in response to warming (Table 1).

Permafrost. Carbon release from permafrost represents a feedback to the warming. However, it can be included explicitly here as a CO\(_2\) source enhancing radiative forcing, rather than implicitly affecting \(\lambda\) values in a less specific fashion (Feedback Analysis). The permafrost carbon inventory, \(C_p\), was determined from:

\[ \frac{dC_p}{dt} = \left( C_p^0(T) - C_p \right)/\tau_p, \]

where \(\tau_p = 100\) y is the permafrost response time. The equilibrium inventory is \(C_p^0(T) = C_p^0 - C_p(T - T^0)\), where \(C_p^0 = 1,700\) Pg C (ref. 14) is the carbon mass at \(T^0\) and \(C_p^0(T) = 50\) to 150 Pg C·K\(^{-1}\) represents the sensitivity of the reservoir size to warming (14–16). The present standard simulation assumes a small sensitivity of \(C_p^0(T) = 150\) Pg C·K\(^{-1}\).

Oceanic Hydrates. Carbon release from oceanic methane hydrates can also be included explicitly as a carbon source. The change in carbon mass of the oceanic hydrate reservoir was calculated from:

\[ \frac{dM}{dt} = -\left( M_f - M_f^q \right)/\tau_f, \]

with \(M_f = M_f + \text{C}_f \cdot e^{-0.55 \Delta T}\) (ref. 17), where \(\chi_f = 0.4\) is the fraction of the reservoir available for dissociation on timescales \(\leq 10^4\) y (subscript f). The factor “0.55” in the exponential means that the reservoir size drops to 1/\(e\) for a temperature increase of \(\Delta T = 1/0.55 \sim 1.8\) K, where \(\Delta T\) is the warming above preindustrial levels (17). Furthermore, \(\tau_f = 5,000\) y is the response time and \(C_f = 2,000\) Pg C is the initial carbon mass. The fraction of the reservoir that is stable for \(t \leq 10^4\) y is \(M_f = 1 - (1-\chi_f)C_f\) and \(M = M_f + M_f\). Note that \(C_f = 2,000\) Pg C (18) used here is less than one-half of the initial inventory of 5,000 Pg C assumed in ref. 17, resulting in a moderate future warming contribution from methane hydrate dissociation.
Ocean Heat Uptake Efficiency. Ocean heat uptake efficiency is known to delay surface warming over a few centuries (19–21) and was explicitly included here. During heat storage, we may write \( \Delta T = (|R_T - N|)/S(t) \), where \( N = C\cdot d(\Delta T)/d t \) and \( C \) is an effective heat capacity (1, 20, 22). Hence \( d(\Delta T)/dt = (|R_T - \Delta T|/S(t)) = S(t)/\Delta T/T \). \( \tau \) is a surface response time. This expression is readily integrated if a value for \( C \) is assumed. \( C \) and \( \tau \) are not constant but increase substantially over time due to ocean mixing processes (21, 22). Thus, a time-dependent \( C(t) \) was used here (\( \Delta T \neq 0 \)):

\[
\tau_s = S(t)/C(t) = S(t) \left[ C^0 \tau_s^0 + \left( C^\infty - C^0 \right) \Delta T/\Delta T \right],
\]

where \( C^0 \) and \( C^\infty \) were chosen so that \( \tau_s^0 = 20 \) y and \( \tau_s^\infty = 1,000 \) y for \( S = \text{const.} \) as \( t \to 0 \) and \( t \to \infty \), respectively. \( \Delta T \) was determined using (see Main Text):

\[
\Delta T_i = (2\tau_i)^{-1} \int_{t_0}^{t} \Delta T(t) \ dt
\]

with an integration time of 300 y. These parameter choices resulted in \( \sim 85\% \) and \( \sim 97\% \) surface response to instant CO\(_2\) doubling at 100 y and 1,000 y, respectively, slightly faster than the intermediate climate response function of Hansen et al. (21). With this setup, the present LOSCAR model runs also showed reasonable agreement with observed and predicted 20th- and 21st-century warming.

Deep-Sea Sediments and Chemical Weathering. On timescales \( >10^4 \) y, fossil fuel CO\(_2\) neutralization involves reaction with CaCO\(_3\) in deep-sea sediments and enhanced weathering (3, 23, 24). These processes represent slow, negative carbon cycle feedbacks that tend to reduce atmospheric CO\(_2\) levels and are included explicitly in the carbon cycle model LOSCAR (3) (Table 1).

Illustration of Ice Sheet Feedback Parameter

An illustration of the ice sheet feedback parameter used in the future projections (see Main Text) is given below. It is emphasized that the following is merely an illustration—a rigorous analysis of the ice–albedo feedback is substantially more complex.

Consider the short-wave radiation term \( Q(1 - \alpha) \) in a planetary energy balance, where \( \alpha \) is the albedo and \( Q = S_0/4 \), with \( S_0 \) being the solar constant. If only the effect of land ice sheets on albedo is considered (index \( L \)), the change of this term with temperature represents the change in radiative flux due to albedo changes (\( \Delta R_L/\Delta T \), Eq. 1). Hence \( dQ(1 - \alpha)/dT \) may be interpreted as the land ice feedback parameter (25):

\[
\frac{\Delta R_L}{\Delta T} = \frac{dQ(1 - \alpha)}{dT} = -Q \frac{d\alpha}{dT} = \lambda_L.
\]

Assuming a simple representation of albedo:

\[
\alpha = \alpha_L \alpha_R + (1 - \alpha_L) \alpha_R,
\]

where \( \alpha_L \) is the area fraction covered by land ice, and \( \alpha_L \) and \( \alpha_R \) are the albedo of land ice and the remaining area, respectively (both independent of \( T \)), we have:

\[
\frac{d\alpha}{dT} = \frac{d\alpha_L}{dT} (\alpha_L - \alpha_R).
\]

Using Eq. 10 yields:

\[
\frac{d\alpha}{dT} = \frac{d\alpha_L}{dT} (\alpha_L - \alpha_R).
\]

Mean Warming Index

The mean warming index for projection \( i \) defined in the Main Text is:

\[
W_i = (t_2 - t_1)^{-1} \int_{t_1}^{t_2} \Delta T_i(t) \ dt
\]

where \( \Delta T_i(t) \) is the global surface warming above preindustrial levels; \( [t_1, t_2] = [Y_{2013}, Y_{2013}] \). Thus, \( W_i \) represents the calculated average future warming from the present until year 10,000. For the present simulations, \( W_i \) predominantly measures the longevity of the warming because the projections show a long warming tail, rather than intense peak warming followed by rapid cooling (Fig. S1). If the latter was true, intense/short-lived warming could produce the same average warming as less intense/long-lived warming. However, the present simulations do not show intense/short-lived warming (Fig. S1).

Regarding the warming threshold for the irreversible deglaciation of the Greenland ice sheet, the timescale of melt depends strongly on the magnitude and duration of the temperature overshoot above the threshold (27, 28). This feature is better represented here by \( W_i \) (Eq. 14) than by the maximum warming. For example, all but one simulation with \( W_i > 3.1 \) K also include sustained warming \( \Delta T > 3.1 \) K for more than 4,000 y (Fig. S1). The temperature anomaly for the projection with 2,000 Pg C and \( \sum_{i=0}^{12} \alpha_i = 0.12 \) W m\(^{-2}\) K\(^{-1}\) (green line) shows a slightly shorter duration above 3.1 K. However, this is partly compensated for by a larger overshoot above the threshold of 3.1 K (maximum \( \Delta T = 4.5 \) K).

Silicate Weathering and Final Equilibrium

On timescales of \( 10^3 \) to \( 10^6 \) years following the onset of a perturbation (e.g., GHG release), equilibrium will finally be restored in the coupled carbon cycle–climate system. This assumes absence of any other external perturbation, no changes in boundary conditions, orbital forcing, etc. Under a continued, sustained forcing, the final equilibrium would be different from the initial equilibrium before the perturbation. If the forcing disappears over time, then the final and initial equilibrium would be equal. Within the current framework, these two situations may be illustrated by varying the strength of the silicate weathering feedback in the model (Fig. S2).
Enhanced silicate weathering and subsequent carbonate burial ultimately removes excess carbon from the ocean–atmosphere system. The silicate weathering feedback in LOSCAR is parameterized based on (3, 6, 29–31):

\[ F_{\text{si}} = F_{\text{si}}^0 \left( \frac{p_{\text{CO}_2}/p_{\text{CO}_2}^0}{n_{\text{si}}} \right)^{n_{\text{si}}}, \]  

where the superscript “0” refers to the initial (steady-state) value of the weathering flux and \( p_{\text{CO}_2} \), respectively. The parameter \( n_{\text{si}} \) controls the strength of the weathering feedback. For example, for \( n_{\text{si}} = 0 \) there is no silicate weathering feedback; \( F_{\text{si}} \) is constant, only the carbonate weathering feedback is active (3). The carbon that has been added to the exogenic carbon cycle would accumulate in the surface reservoirs, part of which would remain in the atmosphere. This causes a sustained forcing that remains constant after several 10 thousand years (Fig. S2; \( n_{\text{si}} = 0 \)). This example illustrates the system’s approach to the final equilibrium under a sustained forcing. The final equilibrium surface temperature is elevated relative to the initial value.

For \( n_{\text{si}} > 0 \), the enhanced silicate weathering flux and subsequent carbonate burial removes carbon from the ocean–atmosphere system as long as the partial pressure of atmospheric \( \text{CO}_2 \) remains higher than the initial \( p_{\text{CO}_2}^0 \). The timescale of restoring the final equilibrium via silicate weathering is probably of the order of 500 ky to 1 My but is uncertain because of uncertainties in the weathering parameterization. Typical values assumed for \( n_{\text{si}} \) may range from 0 to 0.6 (6, 32) (Fig. S2); the standard value used in the present LOSCAR simulations is 0.2 (Main Text). The case for \( n_{\text{si}} > 0 \) illustrates the system’s approach to the final equilibrium when the forcing disappears over time. The initial and final equilibrium surface temperatures are equal. Again, this scenario only applies in the absence of any other external perturbation/forcing, no changes in boundary conditions, orbital parameters, etc.
Fig. S1. Selected future warming projections with $W_i > 3.1$ K (Fig. 3). The numbers in the legend indicate total carbon input (in petagrams of carbon); $\Sigma J^n_i$ (in watts per square meter per kelvin). The horizontal dashed line indicates the warming threshold of 3.1 K given in ref. 1 for the irreversible deglaciation of the Greenland ice sheet.


Fig. S2. Illustration of the system’s approach to final equilibrium. (A) Global surface temperature anomaly and (B) atmospheric CO$_2$ concentration. The graphs indicate three different strengths of the silicate weathering feedback (see text). Moderate slow-feedback parameters and carbon emissions of 2,500 Pg C over 500 y were used in all three scenarios (Fig. 2).