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NOTE

Confusion of the mathematical notation for defining the residence time*

YUAN-HUI LI

Lamont-Doherty Geological Observatory of Columbia University, Palisades, New York 10964, U.S.A.

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Abstract—The inverse of the mean residence time of an element in the sea is, by the first approximation, equal to the removal rate constant of that element in the sea.

THE CONCEPT of the residence time (or the passage time) of an element in the sea, τ, was first introduced by BARTH (1952) and is defined as:

$$\tau = \frac{\text{total mass of an element in the sea}}{\text{mass supplied per year}}. \tag{1}$$

Later, GOLDBERG and ARRHENIUS (1958) gave the residence time a mathematical expression as:

$$\tau = \frac{A}{\mathrm{d}A/\mathrm{d}t}, \tag{2}$$

where they defined A = total mass of an element in the sea and $\mathrm{d}A/\mathrm{d}t$ = mass supplied per year.

Ever since, many standard text books (e.g. *The Sea*, edited by HILL, 1962; *Chemical Oceanography*, edited by RILEY and SKIRROW, 1975; *The Evolution of Sedimentary Rocks*, by GARRELS and MACKENZIE, 1972; *Aquatic Chemistry*, by STUMM and MORGAN, 1970, etc.) adopted equation (2) for defining the residence time.

Unfortunately, the $\mathrm{d}A/\mathrm{d}t$ term in equation (2) does not mathematically correspond to the mass of an element introduced to the sea per year as GOLDBERG and ARRHENIUS (1958) meant it to, but instead $\mathrm{d}A/\mathrm{d}t$ means only the rate of increase of an element in the sea. Therefore at steady state, $\mathrm{d}A/\mathrm{d}t = 0$, as noted earlier by CARRITT (1971).

In order to avoid confusion in the future, it is desirable to replace $\mathrm{d}A/\mathrm{d}t$ in equation (2) with any other notation (e.g. Q or R) to represent the mass of an element supplied per year.

In a single box model system of the sea, let Q and R represent the input rate and output rate of an element in the sea, respectively, and M, the total mass of an element in the sea (the author prefers M to A), then:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = Q - R, \tag{3}$$

if one assumes that for the first approximation the output (or removal) rate of an element in the sea is proportional to the total mass of that element in the sea, i.e. $R = kM$ where k is the rate constant, then equation (3) becomes:

$$\frac{\mathrm{d}M}{\mathrm{d}t} = Q - kM, \tag{4}$$

and at steady state:

$$0 = Q - kM$$

or

$$\frac{1}{k} = M/Q = \tau.$$

Therefore, the inverse of the residence time is equal to the removal rate constant of an element in the sea at steady state.

According to equation (4), the effective way to change M is to vary the input function Q. For example, if one assumes that:

at $t \leqslant t_0$, the sea is in a steady state, i.e. $Q_0 = kM_0$ and

at $t_0 \le t \le t_1$, Q starts to increase exponentially, e.g. $Q = Q_0 \exp[m(t - t_0)]$, due for example, to exponential growth of human activities, then, solving equation (4) one obtains:

$$M = M_0\left\{\frac{m}{m+k}\exp[-k(t-t_0)] + \frac{k}{m+k} \times \exp[m(t-t_0)]\right\} \quad \text{for} \quad t_0 \le t \le t_1. \tag{5}$$

Let $m = 0.05$ (i.e. annual growth rate of 5%) and $t_1 - t_0 = 100$ yr, then at t_1, $M_1/M_0 = 3.8$ for $\tau = 10^3$ yr and $M_1/M_0 = 1.003$ for $\tau = 10^6$ yr. It is clear that the shorter the residence time of the element, the faster the M/M_0 ratio increases. If Q changes back to the original Q_0 at $t \ge t_1$ (collapse of human society or complete control of pollution), then the solution of equation (4) is

$$M = M_0 + (M_1 - M_0)\exp[-k(t - t_1)] \quad \text{for} \quad t \ge t_1. \tag{6}$$

where $M_1 = M$ obtained from equation (5) at $t = t_1$.

It is apparent from equation (6) that the time span for $(M - M_0)$ to reduce to $(M_1 - M_0)/2$, i.e. the mass of element in the sea is half way back to the original steady-state value, is $\ln 2/k = 0.693 \times \tau$. Therefore, the shorter the residence time the faster the system adjusts back to the steady state. But even with $\tau = 10^3$ yr the system will still take about 2.8×10^3 yr (~4 half time) to return to the near-normal state.

In short, the mean residence time of elements not only provides a relative measure of reactivity of elements in the sea but also, by the first approximation, can be inversely related to the removal rate constant of elements in the sea.

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Once the removal rate constant of an element is known, one can semi-quantitatively predict the change of the mass of an element in the sea as caused by the perturbation of the input function.

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