Gravitational Force

Geophysical interpretations from gravity surveys are based on the mutual attraction experienced between two masses* as first expressed by Isaac Newton in his classic work *Philosophiae naturalis principa mathematica* (The mathematical principles of natural philosophy).

Newton's law of gravitation states that the mutual attractive force between two point masses**, $m_1$ and $m_2$, is proportional to one over the square of the distance between them.

The constant of proportionality is usually specified as $G$, the gravitational constant. Thus, we usually see the law of gravitation written as shown to the right where $F$ is the force of attraction, $G$ is the gravitational constant, and $r$ is the distance between the two masses, $m_1$ and $m_2$.

$$F = \frac{G m_1 m_2}{r^2}$$
Gravitational Acceleration

When making measurements of the earth's gravity, we usually don't measure the gravitational force, \( F \). Rather, we measure the gravitational acceleration, \( g \). The gravitational acceleration is the time rate of change of a body's speed under the influence of the gravitational force. That is, if you drop a rock off a cliff, it not only falls, but its speed increases as it falls.

\[
F = m_2 g
\]

In addition to defining the law of mutual attraction between masses, Newton also defined the relationship between a force and an acceleration. Newton's second law states that force is proportional to acceleration. The constant of proportionality is the mass of the object.

\[
g = \frac{G m_1}{r^2}
\]

Combining Newton's second law with his law of mutual attraction, the gravitational acceleration on the mass \( m_2 \) can be shown to be equal to the mass of attracting object, \( m_1 \), over the squared distance between the center of the two masses, \( r \).
The gravimeter is simple in concept but elegant in implementation. A mass attached to a spring experiences a larger or smaller pull as gravity varies. The corresponding extensions or compressions of the spring are measured very precisely, so that small changes in gravity can be observed.
Mass and Spring Measurements

The most common type of gravimeter used in exploration surveys is based on a simple mass-spring system. If we hang a mass on a spring, the force of gravity will stretch the spring by an amount that is proportional to the gravitational force. It can be shown that the proportionality between the stretch of the spring and the gravitational acceleration is the magnitude of the mass hung on the spring divided by a constant, $k$, which describes the stiffness of the spring. The larger $k$ is, the stiffer the spring is, and the less the spring will stretch for a given value of gravitational acceleration.

$$x = \frac{mg}{k}$$

Like pendulum measurements, we cannot determine $k$ accurately enough to estimate the absolute value of the gravitational acceleration to 1 part in 40 million. We can, however, estimate variations in the gravitational acceleration from place to place to within this precision. To be able to do this, however, a sophisticated mass-spring system is used that places the mass on a beam and employs a special type of spring known as a zero-length spring.

Instruments of this type are produced by several manufacturers; LaCoste and Romberg, Texas Instruments (Worden Gravity Meter), and Scintrex. Modern gravimeters are capable of measuring changes in the Earth's gravitational acceleration down to 1 part in 100 million. This translates to a precision of about 0.01 mgal. Such a precision can be obtained only under optimal conditions when the recommended field procedures are carefully followed.
How is the Gravitational Acceleration, $g$, Related to Geology?  (and why should we care?)

*Density* is defined as mass per unit volume. For example, if we were to calculate the density of a room filled with people, the density would be given by the average number of people per unit space (e.g., per cubic foot) and would have the units of people per cubic foot. The higher the number, the more closely spaced are the people. Thus, we would say the room is more densely packed with people. The units typically used to describe density of substances are grams per centimeter cubed (gm/cm$^3$); mass per unit volume. In relating our room analogy to substances, we can use the *point mass* described earlier as we did the number of people.

Consider a simple geologic example of an ore body buried in soil. We would expect the density of the ore body, $d_2$, to be greater than the density of the surrounding soil, $d_1$.

Density of the soil = $d_1$

Density of the ore = $d_2$

Such that $d_1 < d_2$
The density of the material can be thought of as a number that quantifies the number of point masses needed to represent the material per unit volume of the material just like the number of people per cubic foot in the example given above described how crowded a particular room was. Thus, to represent a high-density ore body, we need more point masses per unit volume than we would for the lower density soil*.

*In this discussion we assume that all of the point masses have the same mass.
Now, let's qualitatively describe the gravitational acceleration experienced by a ball as it is dropped from a ladder. This acceleration can be calculated by measuring the time rate of change of the speed of the ball as it falls. The size of the acceleration the ball undergoes will be proportional to the number of close point masses that are directly below it. We're concerned with the close point masses because the magnitude of the gravitational acceleration varies as one over the distance between the ball and the point mass squared. The more close point masses there are directly below the ball, the larger its acceleration will be.

We could, therefore, drop the ball from a number of different locations, and, because the number of point masses below the ball varies with the location at which it is dropped, map out differences in the size of the gravitational acceleration experienced by the ball caused by variations in the underlying geology. A plot of the gravitational acceleration versus location is commonly referred to as a gravity profile.
The Relevant Geologic Parameter is Not Density, But Density Contrast

Contrary to what you might first think, the shape of the curve describing the variation in gravitational acceleration is not dependent on the absolute densities of the rocks. It is only dependent on the density difference (usually referred to as density contrast) between the ore body and the surrounding soil. That is, the spatial variation in the gravitational acceleration generated from our previous example would be exactly the same if we were to assume different densities for the ore body and the surrounding soil, as long as the density contrast, \(d2 - d1\), between the ore body and the surrounding soil were constant. One example of a model that satisfies this condition is to let the density of the soil be zero and the density of the ore body be \(d2 - d1\).
The only difference in the gravitational accelerations produced by the two structures shown above (one given by the original model and one given by setting the density of the soil to zero and the ore body to $d_2 - d_1$) is an offset in the curve derived from the two models. The offset is such that at great distances from the ore body, the gravitational acceleration approaches zero in the model which uses a soil density of zero rather than the non-zero constant value the acceleration approaches in the original model. For identifying the location of the ore body, the fact that the gravitational accelerations approach zero away from the ore body instead of some non-zero number is unimportant. What is important is the size of the difference in the gravitational acceleration near the ore body and away from the ore body and the shape of the spatial variation in the gravitational acceleration. Thus, the latter model that employs only the density contrast of the ore body to the surrounding soil contains all of the relevant information needed to identify the location and shape of the ore body.
Gravity Anomaly

I.e., the null hypothesis is it’s all granite!

Figure 19-26
Schematic illustration of a gravity anomaly. The value of gravity changes across the structure shown because the less dense sediments contain less mass than an equal volume of granite. The thicker the sedimentary deposit, the greater the decrease in gravity, as the curves show.
Plumb Bob Deflection

Initially discovered while surveying at base of Himalayas!

Figure 19-22
A plumb line ordinarily hangs in a vertical position. Near a mountain system we would expect the plumb bob to be deflected toward the mountains because of the gravitational attraction of their mass. The observed deflection is typically less than expected, a discrepancy whose explanation led to an important discovery. The diagram exaggerates the amount of deflection, which is small but readily measurable.
Plumb Bob Deflection

Figure 19-24
The discrepancy between the observed and expected deflection of the plumb bob in Figure 19-22 can be reconciled if the excess mass of the mountain is compensated by a deficiency of mass in a "light" crustal root below. The root provides buoyant support for the mountain, which otherwise would sink into the mantle.
Latitude Dependent Changes in Gravitational Acceleration

Two features of the earth's large-scale structure and dynamics affect our gravity observations: its shape and its rotation. To examine these effects, let's consider slicing the earth from the north to the south pole. Our slice will be perpendicular to the equator and will follow a line of constant longitude between the poles.

- **Shape**: To a first-order approximation, the shape of the earth through this slice is elliptical, with the widest portion of the ellipse aligning with the equator. This model for the earth's shape was first proposed by Isaac Newton in 1687. Newton based his assessment of the earth's shape on a set of observations provided to him by a friend, named Richer, who happened to be a navigator on a ship. Richer observed that a pendulum clock that ran accurately in London consistently lost 2 minutes a day near the equator. Newton used this observation to estimate the difference in the radius of the earth measured at the equator from that measured at one of the poles and came remarkably close to the currently accepted values.

(Pendulum clock as a precise gravimeter!)
Although the difference in earth radii measured at the poles and at the equator is only 22 km (this value represents a change in earth radius of only 0.3%), this, in conjunction with the earth's rotation, can produce a measurable change in the gravitational acceleration with latitude. Because this produces a spatially varying change in the gravitational acceleration, it is possible to confuse this change with a change produced by local geologic structure. Fortunately, it is a relatively simple matter to correct our gravitational observations for the change in acceleration produced by the earth's elliptical shape and rotation.

To first order*, the elliptical shape of the earth causes the gravitational acceleration to vary with latitude because the distance between the gravimeter and the earth's center varies with latitude. As discussed previously, the magnitude of the gravitational acceleration changes as one over the distance from the center of mass of the earth to the gravimeter squared. Thus, qualitatively, we would expect the gravitational acceleration to be smaller at the equator than at the poles, because the surface of the earth is farther from the earth's center at the equator than it is at the poles.
Rotation Correction

- **Rotation** - In addition to shape, the fact that the earth is rotating also causes a change in the gravitational acceleration with latitude. This effect is related to the fact that our gravimeter is rotating with the earth as we make our gravity reading. Because the earth rotates on an axis passing through the poles at a rate of once a day and our gravimeter is resting on the earth as the reading is made, the gravity reading contains information related to the earth's rotation.

We know that if a body rotates, it experiences an outward directed force known as a **centrifugal force**. The size of this force is proportional to the distance from the axis of rotation and the rate at which the rotation is occurring. For our gravimeter located on the surface of the earth, the rate of rotation does not vary with position, but the distance between the rotational axis and the gravity meter does vary. The size of the centrifugal force is relatively large at the equator and goes to zero at the poles. The direction this force acts is always away from the axis of rotation. Therefore, this force acts to reduce the gravitational acceleration we would observe at any point on the earth, from that which would be observed if the earth were not rotating.
Correcting for Latitude Dependent Changes

Correcting observations of the gravitational acceleration for latitude dependent variations arising from the earth's elliptical shape and rotation is relatively straightforward. By assuming the earth is elliptical with the appropriate dimensions, is rotating at the appropriate rate, and contains no lateral variations in geologic structure (that is, contains no interesting geologic structure), we can derive a mathematical formulation for the earth's gravitational acceleration that depends only on the latitude of the observation. By subtracting the gravitational acceleration predicted by this mathematical formulation from the observed gravitational acceleration, we can effectively remove from the observed acceleration those portions related to the earth's shape and rotation.

\[
g_n = 978.03185 (1.0 + 0.005278895 \sin^2 \phi - 0.000023462 \sin^4 \phi) \quad (\text{cm/s}^2)
\]

\[g_n = \text{Normal Gravity: Gravitational acceleration expected for a rotating ellipsoidal earth without any geologic complications and no surface features}
\]

\[\phi = \text{Latitude}
\]

The mathematical formula used to predict the components of the gravitational acceleration produced by the earth's shape and rotation is called the Geodetic Reference Formula of 1967. The predicted gravity is called the normal gravity.

How large is this correction to our observed gravitational acceleration? And, because we need to know the latitudes of our observation points to make this correction, how accurately do we need to know locations? At a latitude of 45 degrees, the gravitational acceleration varies approximately 0.81 mgals per kilometer. Thus, to achieve an accuracy of 0.01 mgals, we need to know the north-south location of our gravity stations to about 12 meters.

(Within the precision of modern GPS units)
Elevation or Free-Air Correction

Figure 19-27
Gravity is measured at A and B to see if there is a difference in subsurface mass. To emphasize subsurface effects, corrections are made to the value of gravity at B, as if to bring B to the same elevation as A and also to remove the obvious gravitational attraction of the mountain. Any remaining gravity difference between A and B is ascribed to a change in subsurface geology.
Variation in Gravitational Acceleration Due to Changes in Elevation

Imagine two gravity readings taken at the same location and at the same time with two perfect (no instrument drift and the readings contain no errors) gravimeters: one placed on the ground, the other place on top of a step ladder. Would the two instruments record the same gravitational acceleration?

No, the instrument placed on top of the step ladder would record a smaller gravitational acceleration than the one placed on the ground. Why? Remember that the size of the gravitational acceleration changes as the gravimeter changes distance from the center of the earth. In particular, the size of the gravitational acceleration varies as one over the distance squared between the gravimeter and the center of the earth. Therefore, the gravimeter located on top of the step ladder will record a smaller gravitational acceleration, because it is positioned farther from the earth's center than the gravimeter resting on the ground.

Therefore, when interpreting data from our gravity survey, we need to make sure that we don't interpret spatial variations in gravitational acceleration that are related to elevation differences in our observation points as being due to subsurface geology. Clearly, to be able to separate these two effects, we are going to need to know the elevations at which our gravity observations are taken.
Approximate the gravity anomaly observed at B due to the difference in topography between A and B, $h$, and the excess mass under B by assuming the excess mass can be approximated as a slab of material with thickness $h$ and density $\rho_b$. 
Variations in Gravity Due to Nearby Topography

Although the slab correction described previously adequately describes the gravitational variations caused by gentle topographic variations (those that can be approximated by a slab), it does not adequately address the gravitational variations associated with extremes in topography near an observation point. Consider the gravitational acceleration observed at point $B$ shown in the figure below.

![Diagram of gravity variations due to topography](image)

In applying the slab correction to observation point $B$, we remove the effect of the mass surrounded by the blue rectangle. Note, however, that in applying this correction in the presence of a valley to the left of point $B$, we have accounted for too much mass because the valley actually contains no material. Thus, a small adjustment must be added back into our Bouguer corrected gravity to account for the mass that was removed as part of the valley and, therefore, actually didn't exist.

The mass associated with the nearby mountain is not included in our Bouguer correction. The presence of the mountain acts as an upward directed gravitational acceleration. Therefore, because the mountain is near our observation point, we observe a smaller gravitational acceleration directed downward than we would if the mountain were not there. Like the valley, we must add a small adjustment to our Bouguer corrected gravity to account for the mass of the mountain.
Summary of Gravity Types

We have now described the host of corrections that must be applied to our observations of gravitational acceleration to isolate the effects caused by geologic structure. The wide variety of corrections applied can be a bit intimidating at first and has led to a wide variety of names used in conjunction with gravity observations corrected to various degrees. Let's recap all of the corrections commonly applied to gravity observations collected for exploration geophysical surveys, specify the order in which they are applied, and list the names by which the resulting gravity values go.

- **Observed Gravity (gobs)** - Gravity readings observed at each gravity station after corrections have been applied for instrument drift and tides.
- **Latitude Correction (gn)** - Correction subtracted from gobs that accounts for the earth’s elliptical shape and rotation. The gravity value that would be observed if the earth were a perfect (no geologic or topographic complexities), rotating ellipsoid is referred to as the *normal gravity*.
- **Free Air Corrected Gravity (gfa)** - The Free-Air correction accounts for gravity variations caused by elevation differences in the observation locations. The form of the Free-Air gravity anomaly, gfa, is given by;

\[
gfa = gobs - gn + 0.3086h \text{ (mgal)}
\]

where \( h \) is the elevation at which the gravity station is above the *elevation datum* chosen for the survey (this is usually sea level).
Summary of Gravity Types (cont.)

- **Bouguer Slab Corrected Gravity (gb)** - The Bouguer correction is a first-order correction to account for the excess mass underlying observation points located at elevations higher than the elevation datum. Conversely, it accounts for a mass deficiency at observation points located below the elevation datum. The form of the Bouguer gravity anomaly, $gb$, is given by:

$$gb = gobs - gn + 0.3086h - 0.04193\rho (mgal)$$

where $\rho$ is the average density of the rocks underlying the survey area.

- **Terrain Corrected Bouguer Gravity (gt)** - The Terrain correction accounts for variations in the observed gravitational acceleration caused by variations in topography near each observation point. The terrain correction is positive regardless of whether the local topography consists of a mountain or a valley. The form of the Terrain corrected, Bouguer gravity anomaly, $gt$, is given by:

$$gt = gobs - gn + 0.3086h - 0.04193\rho + TC (mgal)$$

where $TC$ is the value of the computed Terrain correction.

Assuming these corrections have accurately accounted for the variations in gravitational acceleration they were intended to account for, any remaining variations in the gravitational acceleration associated with the Terrain Corrected Bouguer Gravity, $gt$, can now be assumed to be caused by geologic structure.
Buoyancy and Isostasy

Examples of buoyancy
- Iceberg
- Boat
- Mountain

Crust
Mantle

(a)
(b)
(c)
(d)

Ongoing
Isostatic Uplift
(mm/year)
From Flint (1971)
Transcontinental Gravity Survey

Figure 19-28
Transcontinental gravity survey from the Pacific Ocean to the Atlantic Ocean. The negative gravity anomalies over the mountainous regions, the near-zero values at low elevations, and the positive values over deep oceans, mirroring the topography, the role of isostatic compensation in the surface features.
Figure 5-13  Model of crustal structure based on gravity survey across the central Aleutian Arc, North Pacific. Free-air gravity anomalies shown at top. Calculated gravity values are plotted as circles. Density of various layers shown in g cm$^{-3}$. (From J. A. Grow, Geol. Soc. Amer. Bull., Vol. 84, p. 2181, 1973. courtesy The Geological Society of America)
Simplified Model of Subduction

Fig. 3.21 The convergence of a continental plate and an oceanic plate.
Fig. 16. Gravity anomalies and seismically determined structure across the North Mid-Atlantic Ridge. The continuous gravity data were obtained on Vema cruise 17. Bouguer anomalies were obtained assuming two-dimensionality and a density of 2.60 gm/cm³ for the basement layer. A correction was also made for the sediment layer. The seismic section is obtained by projecting the structure at seismic stations along the gravity profile. Seismic horizons are represented by dots. Values of compressional wave velocities in km/sec are indicated. Numbers within parentheses denote assumed seismic velocities. (After Talwani et al., 1965.)
Gravity Profiles have Contributed to our Models of the Earth’s Interior
Earth-Core Formation Models

FORMATION OF THE EARTH is visualized as having been by one or the other of the processes depicted here. In the homogeneous-accretion model silicate (black) and iron (color) accumulate to form the complete planet (top left). Subsequently the core forms by the separation of the metal from the silicate (top right). During the formation of the core the iron sinks to the center of the planet and heat is generated by the release of gravitational energy. In the heterogeneous-accretion model the metallic core is accumulated first and the silicate mantle accretes around it. The sequence could occur during or after the condensation of solids out of the solar nebula, depending on whether chemical or physical processes are involved. In each model the accretion of the planet is viewed as resulting from the infall of meteoritic bodies.
Space Probe Gravity Models of Planets => Estimates of Europa’s deep structure

From: NASA Galileo Jupiter Mission Images
Micro-gravity surveys at Poas Volcano, Costa Rica

Rymer et al. (2000)
High-Res. Regional Gravity Maps by Tandem Satellite Surveys

Global Gravity Anomalies Reveal Deep Earth Structure & History

Gravity Summary

Gravity Anomalies

1) Earth not a sphere

2) Shape approximates that of fluid balance (at the equatorial radius):
   1. gravitational forces tend to make earth spherical
   2. centrifugal forces of rotation tend to flatten it

3) As a result, the equatorial radius is about 21 km greater than the polar radius

4) At the equator, the center of mass is further from the surface, making the gravity smaller, but also at the equator, the centrifugal force, which is opposite to gravitational acceleration, is a maximum. Difference is 5 gals

   \[ 1 \text{ gal} = 1 \text{ cm/sec}^2 \]

Gravitational Acceleration

Newton's formula

\[ F = G \frac{m_1 m_2}{r^2} \]

- \( m_1 \) - body on earth
- \( m_2 \) - earth-gravity correlated at center of earth
Also, a force acting on a body of mass \( m \) may also be defined by Newton's second Law of Motion.

\[ F = m a \]

\( a \) is the acceleration that would be caused by the gravitational attraction of the earth if a body were allowed to fall.

Therefore,

\[ a = \frac{F}{m} = \gamma \frac{m_2}{r^2} \]

**Gravitational Constant**

Determined first by Bouguer, then by Mitchell in "Cavendish Experiment", 1799.

\[ \gamma = \frac{200}{3} \cdot 10^{-9} \text{ cgs units} \]

\[ = 66.67 \cdot 10^{-9} \]

**Units**

1 gal = 1 cm/sec\(^2\)

1 mgal = 1 milligal = 0.001 gal = 0.001 cm/sec\(^2\)
Since normal gravitational acceleration at the surface of the earth is 980 gals, 1 mgal is approximately 1 ppm of the earth's gravitational field.

Shape of the Earth

Expressed in terms of ideal spheroid of reference with a degree of flattening \( f \),

\[
f = \frac{a-b}{a}
\]

where \( a \) = equatorial radius, \( b \) = polar radius

Gravity Formula

Gives normal or theoretical gravity field over earth as a function of latitude. The 1930 U.S. C&GS formula - the "international gravity formula"

\[
g_o = 978.049 \left( 1 + 0.0052884 \sin^2 \varphi - 0.0000059 \sin^2 2 \varphi \right)
\]

\[
a = 6,378,388 \text{ meters}
\]
\[
b = 6,356,909 \text{ meters}
\]
\[
f^{-1} = 297.0
\]

If the earth were a perfect fluid, with no lateral variations in density, its surface would correspond to an ideal spheroid represented by the gravity formula.