SCATTERING OF INTERNAL WAVES AT FINITE TOPOGRAPHY

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Abstract

The scattering of internal waves at finite bottom topography is studied theoretically and numerically. For the two-dimensional problem, a numerical scheme has been developed to calculate the distribution of scattered wave energy flux in wavenumber space for a given incident wave. Results are used to assess the partition of energy flux between reflected and transmitted waves and the effectiveness of various types of bottom topography in transferring internal wave energy to higher modenumbers through the interaction.

We consider two types of topography: a slope-shelf and a ridge. In the slope-shelf configuration, a shallow flat bottom is connected to a deep one by a slope, while in the ridge configuration, a finite bump sits on an otherwise flat bottom. Calculations are carried out for various depth ratios and bottom slopes to explore the effect of the topographic parameters. For a monochromatic incident wave, results show that the energy partition between reflected and transmitted waves strongly depends on the height and the type of topography. For supercritical topography, a significant part of the incident wave energy flux is reflected, especially for low modenumber incident waves which satisfy $n^i\delta < 1$, where δ is the depth ratio and n^i is the modenumber. The distribution of scattered wave energy flux in modenumber space shows peaks at modenumbers which can be roughly estimated from a reflection law using the maximum slope of the bottom. It also shows that more energy is transferred to higher modenumber than to lower modenumber, especially at near critical slope.

Experiments with different topographic shapes show that a convex slope is much more effective in transferring energy to higher modenumbers than a concave slope,

which is consistent with the result of previous studies. Near the critical slope, a curved bottom is far less effective in transferring energy to higher modenumbers than a straight slope. Thus, results obtained with a straight infinite slope may well overestimate the energy transfer rate.

Both the energy and the energy flux spectrum of the scattered wave field are calculated from the incident wave spectrum, which is specified by projecting Garrett-Munk spectrum onto a two-dimensional spectrum. The reflected energy spectrum shows that internal wave energy is transferred from low wavenumber to high wavenumber with most of the energy redistribution at subcritical frequencies. Assuming a symmetric ridge, the scattered wave field is the sum of the reflected and transmitted wave field. The redistributed energy flux is less than 10% of the incident energy flux from our calculation. For the slope configuration, about 90% of the incoming energy flux is transmitted to the shelf. The energy transfer from low to high wavenumbers results in great amplification of the inverse Richardson number for scattered wave fields. Based on a calculation with 512 resolved modes, for a linear and convex bottom the R_i^{-1} ratio between reflected wave field and the incident wave field can be as large as several hundred while that for a concave bottom is less than 100.

For three-dimensional internal wave scattering, we derive a general formula for the scattered wave field using a Green's function method. The scattered wave is expressed as an integral of internal waves generated by a boundary source. The boundary source itself satisfies an integral equation. The source term for the integral equation is the incident wave field, and its kernel is related to the Green's function for internal waves in an infinite domain. The system is successfully applied to both the internal wave reflection from a straight slope and the scattering by an infinitesimal rough bottom.