

**REFLECTION OF NONLINEAR  
BAROCLINIC ROSSBY WAVES  
AT A NON-ZONAL BOUNDARY  
AND THE DRIVING OF  
SECONDARY MEAN FLOWS**

**A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF  
THE UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT  
OF THE REQUIREMENTS FOR THE DEGREE OF**

**DOCTOR OF PHILOSOPHY**

**IN**

**OCEANOGRAPHY**

**AUGUST 1990**

**By**

**Federico Graef Ziehl**

**Dissertation Committee:**

**Lorenz Maggaard, Chairman**

**Eric Firing**

**Dennis Moore**

**Peter Muller**

**Klaus Wyrтки**

**Chester Vause**

# Abstract

The reflection of weakly nonlinear Rossby waves (RWs) from a vertical wall has been studied analytically using perturbation methods with the  $\beta$ -Rossby number ( $\varepsilon$ ) being the small parameter. A uniformly valid solution up to  $O(\varepsilon^3)$  was constructed using multiple scales.

At  $O(\varepsilon)$ , the nonlinear interaction between an incident and a reflected RW leads to: 1) an Eulerian steady flow,  $u_s^{(1)}$ , parallel to the (non-zonal) wall; 2) a transient flow oscillating with a frequency of twice ( $2\omega$ ) that of the RW pair. The steady forcing, whose response is  $u_s^{(1)}$ , can never be resonant, implying that  $u_s^{(1)}$  is stable for  $\varepsilon < 1$  since instability proceeds only via resonant interactions. The transient forcing can be resonant only if  $0 < |\sin \alpha| \leq 1/3$ , where  $\alpha$  is the angle between the wall and the circles of latitude; the wave amplitudes are then slowly varying periodic functions of  $y$ , the offshore coordinate.

At the next order, the nonlinear interaction between the RW pair and  $u_s^{(1)}$  plus the transient flow produces, in general, resonant forcing, leading (using multiple scales) to a modification of the RWs' phases: a shift in their wavenumbers. Whenever the leading order solution  $\psi^{(0)}$  is *any* superposition of RWs, the second order perturbation equation for  $\psi^{(2)}$  will almost always have resonant forcing.

The steady flow occurring at  $O(\varepsilon^3)$  is driven by the modified RWs as well as through interactions of several components of the solution up to second order. The correction to  $u_s^{(1)}$  can be negligible, of the order of  $u_s^{(1)}$ , or unrealistically large.

To make a fair comparison between the theoretical steady current and observations, it is necessary to have data that would allow elimination of all transients. The transients that appear at orders one to three have been disregarded in previous studies. The transient flow at  $O(\varepsilon)$  is of the same order of magnitude as  $u_s^{(1)}$  and could very well be the reason why the agreement between the predicted mean current and occasional observations is not equally good in all cases (for the Hawaiian Ridge). The comparisons made so far are inconclusive.