NUMERICAL SIMULATION OF FREE OSCILLATIONS OF ENCLOSED BASINS ON A ROTATING EARTH

A DISSERTATION SUBMITTED TO THE GRADUATE DIVISION OF THE UNIVERSITY OF HAWAII IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

IN OCEANOGRAPHY

AUGUST 1972

By

Niels Christensen, Jr.

Dissertation Committee:

Gordon W. Groves, Chairman
Brent S. Gallagher
M. S. Longuet-Higgins
Harold G. Loomis
Edward D. Stroup
Klaus Wyrtki

CHAPTER 1. INTRODUCTION

The purpose of this dissertation is to study the normal modes of water oscillations in enclosed basins on a rotating earth. Barotropic modes are treated for basins similar to the earth's oceans. Analytic solutions are computed for a hemispherical basin centered on the equator using the method of Longuet-Higgins and Pond (1970). The main effort, however, is in computing solutions by strictly numerical techniques. That is, after solving Laplace's equations for a single dependent variable, derivatives are replaced by finite differences. Boundary conditions are also expressed in finite differences, and the result is a set of linear homogeneous algebraic equations. Solutions of this set of equations are approximate normal modes of the basin being modeled. The formulation of the problem and the geometry of the basin determine the solutions. Analytic solutions for various basins provide important 'bench marks' for comparison with these numerical solutions. The flexibility of the numerical techniques is exploited in computing modes of a Pacific-sized basin on a rotating earth and of a basin with a coastal shelf.

The simplest solutions, the gravity modes with lowest frequencies and the planetary modes with highest frequencies, can be described more accurately with coarse grids. They are therefore most amenable to treatment by numerical techniques. Also, they are more likely to be activated in the oceans since their frequencies are nearest those of the semidiurnal and diurnal tides. And, since they are the simplest modes, there is more likely to be an ocean-wide disturbance coherent with their wave form. (Consider the relative difficulty in plucking a guitar string so that it oscillates nearest one of its higher modes. Also, high modes are more quickly damped, similar to the oceans.)

A brief note on the history of this research, with an account of some of the successes and failures, will serve as a general conclusion. The initial concept was to first test numerical techniques by generating solutions for cases where analytic solutions are available, starting with simple geometries and formulations and progressing to more complicated ones. For simple geometry, a small rectangular basin was chosen and, for a simple formulation, the case where horizontal divergence is neglected was taken. A divergent formulation and a hemispherical basin provided the most complex test case by way of comparison.

The problem was kept in a general form to facilitate deformations and grid refinements in relating known solutions to solutions of other basin geometries. The effects of (1) a coastal shelf, (2) altering the size and symmetry to that of the Pacific basin, (3) steady circulation, and (4) a mid-ocean ridge were to be studied by deforming from a hemispherical basin while following a solution, and then refining the grid.

Comparisons for simple cases were adequately successful. However, a major part of the effort was devoted to an unsuccessful attempt to compare numerical and analytical divergent solutions of a hemispherical basin. Had this difficulty been anticipated, a less general numerical formulation could have been used. By taking advantage of symmetries of the hemispherical basin, the number of equations for a given grid could have been reduced and the comparison carried to finer grids. For the hemisphere it was rarely possible to relate solutions from grid to grid.

Four solutions were successfully followed from a hemispherical basin with constant depth to a hemispherical basin with a coastal shelf. But all attempts to follow a solution while deforming the basin from a hemisphere to a Pacific-sized basin were unsuccessful. Plans to investigate other effects were abandoned because of lack of confidence in the divergent numerical calculations.