The Direct Response to Tropical Heating in a Baroclinic Atmosphere

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(Manuscript received 23 August 1993, in final form 15 June 1994)

ABSTRACT

The global response to tropical heating is studied by performing a time integration of a 15-level primitive equation model, starting with a basic flow maintained by a constant forcing. The direct, quasi-steady response to the tropical heating is seen during the first 20 days before baroclinic instability dominates. This technique enables the investigation of a variety of basic flows, from a resting state to a December–February 3D time-mean flow; of the timescales for establishing remote responses; and of nonlinear effects. It also allows the determination of timescales for the establishment of the response. The Gill-type response is seen in the lower troposphere in all cases. In the upper troposphere, depending on the basic conditions, the simple tropical quadrupole response of the Gill model shows considerable modification. The anticyclonic pair can be centered over the heating and can vary substantially in magnitude and vertical extent. The Rossby wave source and the upper-tropospheric divergence above the heating region is always found, but the existence and relative magnitudes of local Hadley and Walker cells as measured by upper-tropospheric convergence are strong functions of the flow. Both the Rossby wave source and the Rossby wave propagation are also strongly influenced by the ambient flow. Wave patterns extend to the equator in the regions in which the basic westerlies extend to the equator. Significant tropical zonal flow variations, which are also very dependent on the basic flow and the position of the heating, are also produced. The tropical and midlatitude response is generally established within a week. In an additional week the high-latitude pattern is determined and the subtropical wave pattern propagates back into the Tropics in the westerly wind regions. Nonlinear effects are found to be minor in all cases before the middle-latitude transients develop. On the two-week timescale of interest here, the sensitivity of steady-state models to the dissipations employed and to the existence of low-frequency modes is not found.

1. Introduction

Numerous observational, theoretical, and modeling studies have shown that tropical convective heating has important influences on both tropical and extratropical circulations. The variation of the local Hadley and Walker circulations associated with the Southern Oscillation was related by Bjerknes (1966, 1969) to the convective heating associated with sea surface temperature anomalies. The Pacific/North American (PNA) teleconnection pattern discovered by Horel and Wallace (1981) was viewed by them as a remote, extratropical response to the perturbed equatorial heating. Much progress has been made in our understanding of these features in the past decade. With very simple linear models in the style of Matsuno (1966), Gill (1980) and Lau and Lim (1982) showed that, for isolated equatorial heating in a resting atmosphere, Walker and Hadley type circulations could be described. The middle-latitude response has been widely studied by a number of investigators (Egger 1977; Opsteegh and Van den Dool 1980; Hoskins and Karoly 1981; Simmons 1982; Webster 1981, 1982) using mostly linearized steady-state baroclinic models. All these studies found that, in the presence of a basic westerly zonal flow, an equivalent barotropic Rossby wave train can be generated by tropical heating. The role of zonal asymmetries in the climatological flow in configuring the remote barotropic response was further discussed by Simmons et al. (1983). In particular, they considered the barotropic instability of the nonzonal flow when it is maintained by a fixed forcing. Webster and Holton (1982) and Webster and Chang (1988) studied the effect of local westerlies over the equatorial region. The actual forcing of Rossby wave motion by specified equatorial divergence was investigated by Sardeshmukh and Hoskins (1988). Although these studies have covered many aspects of the problem, the picture is still not complete. Hoskins and Jin (1991, hereafter HJ) recently used time integration of a numerical model to study initial value problems in a baroclinic

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atmosphere for equatorial perturbations to a variety of basic flows. The temporal and spatial evolution of both equatorial wave patterns and extratropical Rossby wave trains were then discussed.

In this paper the same time integration technique is applied to a study of the global response to steady tropical heating, which is turned on at the initial instant. As in HJ the evolution of the direct response to the heating is seen before baroclinic instability eventually dominates the solution. The numerical model and the parameters used are described in some detail in HJ. The model is T31 spectral with 15 σ-coordinate levels, which are indicated in Fig. 1. There is Newtonian cooling, which for historical reasons has timescales (12 × 10, 9, 8, 7) days and (4, 6, 10, 18, 8 × 25, 18, 12, 8) days, respectively, on the zonal and wave components at the 15 model levels. There was also a drag at the lowest two levels with 4 and 1.5 day timescales, respectively. In this study a \( \nabla^6 \) hyperdiffusion is included as well as a biharmonic internal diffusion with coefficient \( 2.338 \times 10^{-16} \) m\(^4\) s\(^{-1}\). In each case, if required, a constant forcing is applied that would maintain the basic flow in the absence of the additional heating.

The time integration approach used in this paper has a number of advantages over standard stationary wave calculations:

1) it permits calculations for basic states ranging in complexity from the Gill–Matsuno resting atmosphere to 3D observed flows,
2) it allows calculation for finite-amplitude heating,
3) there is relative insensitivity to damping representations and to the existence of low-frequency modes (cf. Branstator 1992), and
4) it gives timescales for setting up patterns.

Without a substantial increase in complexity, however, the model does not provide any insight into the important interaction of midlatitude transient with the tropically forced wave: only the direct response to the heating is determined. Thus, the nonlinear effects discussed in this paper are only associated with interaction of the tropically forced wave with itself and with the basic state.

In section 2 the heating is applied to a resting, stratified atmosphere to give a new look at the Gill–Matsuno, resting basic-state problem. A climatological zonal flow is used in section 3. In section 4 a climatological flow with longitudinal asymmetries is posed and the response to heatings at various longitudes determined. In section 5 a detailed discussion of the upper-tropospheric divergence and the Rossby wave source in each integration is given.

2. Gill–Matsuno problem

We first consider solutions for a resting stratified atmosphere forced by tropical heating switched on at \( t = 0 \). The heating has a vertical profile that is shown in Fig. 1. It peaks at \( \sigma \approx 0.4 \) with a magnitude of 5 K day\(^{-1}\), and the column average is 2.5 K day\(^{-1}\), equivalent to the latent heating associated with 1 cm of precipitation per day. In the horizontal the heating has a cosine squared profile in an elliptical region, which is indicated in Fig. 2b.

The streamfunctions at an upper \( (\sigma = 0.24) \) and a lower \( (\sigma = 0.78) \) tropospheric level at day 15 of the integration are shown in Figs. 2a and 2b. To the west of the heating there is the pair of anticyclones above the pair of cyclones, associated with the equatorially trapped Rossby wave response. To the east is the Kelvin wave response with equatorial westerlies and a pair of cyclones above equatorial easterlies and a pair of anticyclones. The vertical structure of the zonal wind field on the equator is shown more clearly in Fig. 3. Above the heating the easterly and westerly maxima are about 6 and 5 m s\(^{-1}\), respectively. The lower-tropospheric westerlies peak at more than 3 m s\(^{-1}\), but the easterlies are only half this strength. It is clear that the inclusion of a zonally averaged component to the heating has led to a Hadley cell response and zonally averaged westerlies in the equatorial upper troposphere. If this zonal response is removed from the upper-tropospheric streamfunction, as in Fig. 2c, the asymmetric response is clearly seen to have a quadrupole structure.

The solution shown in Figs. 2 and 3 appears to be qualitatively consistent with the simple model solution exhibited by Gill (1980). The opposite senses of the circulation in the upper and lower troposphere indicate the dominance of the first internal mode in the troposphere. The interpretation in terms of simple equatorial waves is supported by Fig. 4, which shows the temporal development of the upper-tropospheric equatorial wind field. Associated with the Rossby wave, the easterlies spread westward at about 10° day\(^{-1}\). The Kelvin wave
westerly wind front moves eastward at about $34^\circ$ day$^{-1}$. The cyclic nature of the domain becomes important at about day 7, with interference between the two waves. The changes after day 8 are small.

Two further experiments have been performed with a resting basic state. When the imposed heating peaks lower in the troposphere, the low-level response becomes significantly stronger. To study the role of nonlinearity, a linear run was performed by reducing the heating amplitude by 1000. The scaled solution (not shown) is very similar to that shown in Figs. 2-4. As might be anticipated, the cyclones are slightly intensified and the anticyclones slightly weakened in the nonlinear run. The upper-tropospheric easterly maxima are 6.6 and 5.7 m s$^{-1}$ in the linear and nonlinear runs, respectively, and the westerly maxima are 5.1 and 5.3 m s$^{-1}$.

3. A December–February zonal flow

We now consider the same problem as in section 2 except that the basic flow is a December–February climatological, zonally averaged zonal flow taken from six years of European Centre for Medium-Range Weather Forecasts (ECMWF) data. The day 15 solution, corresponding to Figs. 2 and 3, is shown in Figs. 5 and 6. It is clear that the tropical portion of the solution shows considerable similarity with the resting atmosphere solution. In the upper troposphere, the Northern Hemisphere (winter) anticyclone strengthens considerably, while its Southern Hemisphere (summer) counterpart weakens slightly. The easterlies between them increase from 5.7 to 7.4 m s$^{-1}$. The westerlies also increase from 5.3 to 6.1 m s$^{-1}$. The Northern Hemisphere anticyclone, in particular, is displaced toward and almost over the heating region. The time-longitude plot of equatorial upper-tropospheric $u$ (not shown) is very similar to that in Fig. 4.
The most striking difference between the two solutions is the equivalent barotropic Rossby wave trains that are able to propagate on the subtropical and middle-latitude westerlies. The equivalent barotropic nature of these is indicated in Figs. 5a,b by the same sign that is implied for the asymmetric streamfunction perturbation at the two levels. To demonstrate the vertical structure more clearly, a vertical section of $\nu$ at 32°N on day 15 is given in Fig. 7. There is a slight westward tilt in the lowest levels associated with the drag at those levels. Elsewhere there is a simple vertical structure with a single maximum on the tropopause. To highlight the Rossby wave propagation the $\nu$ field at $\sigma \approx 0.24$ is given in Fig. 8. The results are consistent with the simple wave theory and ray tracing given in Hoskins and Karoly (1981). Wavenumbers greater than 4 are refracted toward low latitudes by the poleward flank of the subtropical jet. Low wavenumbers (less than 4) can propagate to high latitudes and move equatorward about 180° downstream. In Fig. 8 the subtropical wave train is seen to have a scale of about wavenumber 6, whereas that in the high-latitude train is about 3. The propagation into the summer hemisphere shows a single weaker wave train, consistent with the broader band of easterly winds to the south of the equator and a weaker subtropical jet.

The nature of the developing Northern Hemisphere wave activity is illustrated by the longitude–time plots of $\sigma \approx 0.24 \nu$ shown in Fig. 9. For the first 20 days, the dominant wave pattern along 32°N (Fig. 9a) has stationary phase and eastward energy propagation with an almost constant group velocity, as shown by the suc-
cessive downstream development. There is little evidence of transient waves generated by the forcing interfering with the stationary phase pattern, which suggests that the stationary solutions obtained with simple models such as the barotropic vorticity equation and the steady baroclinic models forced by tropical heating in realistic zonal basic flows are indeed relevant. The very different signature of baroclinically unstable waves with eastward phase speed and group velocity is seen after about 2 weeks and dominates in 3–4 weeks. At 69°N the development of the lower-wave-number pattern is seen. As zonal wavenumbers 2 and 1 reach this latitude, the negative and positive regions expand in longitude. With the implied eastward phase speed on the eastern flank it appears that the pattern merges into baroclinically unstable waves.

The implication of this analysis is that in a timescale of two weeks, equivalent barotropic Rossby wave trains lead to a response at middle and high latitudes to anomalous tropical heating. Of course this result relies on the absence of preexisting baroclinic waves whose behavior will be influenced by the developing wave trains and will, in turn, feed back onto them. For middle latitudes, the height field perturbation is similar to that of streamfunction, with the contour interval in Fig. 5a being equivalent to about 1 dam. The perturbations are, therefore, small but significant. A linear integration yielded very similar results, with the differences reflecting those found in the resting atmosphere case.

Two plots for another integration with an anomaly heating at (15°S, 180°) are given in Figs. 10 and 11. The upper-tropospheric streamfunction plot in Fig. 10 shows some similarity with that for equatorial heating (Fig. 5). The Northern Hemisphere wave patterns are seen with a weaker amplitude, but the Southern Hemisphere pattern is slightly stronger. The longitude–time

Fig. 9. Longitude–time pictures of the $\sigma \approx 0.24$, meridional wind perturbation at (a) 32° and (b) 69°N. The ordinate is the time in days. The contour interval is 1 m s$^{-1}$, starting at ±0.5 m s$^{-1}$. Negative contours are dashed.

Fig. 10. Day 15 perturbation streamfunction field at $\sigma \approx 0.24$ for heating centered at 15°S, 180° in a Dec–Feb zonal flow. The conventions are as in Fig. 2.

Fig. 11. Longitude–time plot of the $\sigma \approx 0.24$, meridional wind perturbation at 32°N for heating centered at 15°S, 180° in a Dec–Feb zonal flow. The conventions are as in Fig. 9 except that the contours are at ±0.25 m s$^{-1}$ and every 0.5 m s$^{-1}$. 
plot of $v$ at 32°N and $\sigma = 0.24$ in Fig. 11 indicates that the NH higher wavenumber, subtropical wave train is initiated after about a week but only persists for a similar period, much as in the initial value solution in Fig. 6a of HJ. A notable difference between Figs. 9 and 11 is the westward phase speed, particularly in the first week. Simple theory predicts that only just such a wave can propagate through the equatorial easterly belt into the Northern Hemisphere and that a steady, stationary wave train could not persist in an atmosphere with damping.

4. A December–February 3D flow

As in HJ, the December–February time-mean 3D vorticity and divergence are specified in the basic state, and the temperature and surface pressure are obtained by a balancing procedure. A constant forcing is added to the model, which would maintain the basic state in the absence of the diabatic heating perturbation. The solution now depends on the longitude of this perturbation. For all of the cases looked at, the first 15–20 days appear to be dominated by a developing quasi-stationary response, with baroclinic instability dominating after this period.

The streamfunction fields at $\sigma = 0.24$ and $\sigma = 0.78$ for day 15 of an integration with the heating centered on 0°N, 180°W are illustrated in Fig. 12. Comparing with Fig. 5, it is seen that the overall structure of the response is similar. The Northern Hemisphere upper-tropospheric anticyclone is now centered over the heating region. The Rossby wave train is distorted so that there is a cyclonic center almost due poleward of the anticyclone. This dipole, together with the downstream wave pattern, is reminiscent of the PNA pattern of Wallace and Gutzler (1981) and was prominent in the earlier study of HJ. The zonally averaged upper-tropospheric westerly winds are stronger in the present run than for the zonal basic state.

The vertical structure of the $u$ field on the equator at day 15 is shown in Fig. 13. Comparing with Fig. 6, the upper-tropospheric structure reflects the generally stronger westerlies that have developed. Discrete maxima occur in the westerly wind regions of the east Pacific and Atlantic and in the Indian Ocean. The lower-tropospheric structure is, however, still very similar to that of the zonal flow and, indeed, the resting atmosphere case.

The longitude–time plot of equatorial $u$ at $\sigma = 0.24$ (not shown) exhibits the Kelvin wave propagation to the east in a manner very similar to that seen in Fig. 4. However, consistent with the anticyclones remaining over the heating, there is no sign of the westward propagating Rossby wave. This propagation is probably inhibited by the near-zero poleward gradient of absolute vorticity in the Indonesian region on the western flank of the heating.

For heating centered on the equator at any longitude, the upper-tropospheric development in the tropical region during the first few days is always a simple quadrupole similar to that in Fig. 2c. However, the asymmetric basic flow then starts to modify the perturbation. Figure 14 shows the zonally asymmetric $\sigma = 0.24$ streamfunction at day 15 for heatings on the equator at
tropospheric pattern keeps its Gill-type response in all cases. The vertical extent, relative longitude, and magnitude of the upper-tropospheric easterlies associated with the anticyclone pair varies with the longitude of the heating. It is clear also that the magnitude of the zonally averaged, upper-tropospheric equatorial westerlies is also dependent on this longitude, being much reduced for the 90°W and 60°E heatings.

To provide a clearer picture of wave propagation, Fig. 16 shows the $\sigma \approx 0.24$ $v$ field at day 15 for the four heating longitudes discussed here. The 180° heating solution picture (Fig. 16a) gives evidence of the high-latitude wave train across North America and subtropical wave activity in both hemispheres. The extent of the equatorial easterly winds is a minimum near the tropopause level, and the region of such easterly winds at 150 mb is indicated in the panels of Fig. 16. It is evident in all cases that the wave activity extends to the equator in the westerly wind regions. In the equatorial region the wave patterns tend to have geographically fixed nodes. Thus, the equatorial east Pacific pattern in Fig. 16a is also present in Figs. 16c,d. The

![Diagram](image)

**Fig. 14.** Day 15 asymmetric, $\sigma \approx 0.24$ perturbation streamfunction fields for heating on the equator at (a) 90°W, (b) 60°E, and (c) 120°E. As in Fig. 2c the zonal average has been removed and the conventions are as in Fig. 2.

(a) 90°W, (b) 60°E, and (c) 120°E. The 90°W day 15 heating response still shows the simple tropical quadrupole with the anticyclones centered some 10° west of the heating maximum. Consistent with the ambient westerlies near 90°W, there is clearly significant wave propagation into both hemispheres. For the 60°E heating the Northern Hemisphere anticyclone is enhanced and almost over the heating maximum. The propagation is mainly into the Northern Hemisphere, and there is evidence of the north-south dipole in the North Pacific with the opposite sign from that produced by the heating on the date line. The reversal in this response has been commented on previously in Hsu et al. (1990). The 120°E heating initially develops a simple tropical response, but by day 15 the $\sigma_{0.24}$ tropical anticyclones have decreased in extent and, near the equator, are centered to the east of the heating. There is propagation into both hemispheres, and the North Pacific response is similar to that for the date line heating.

The vertical structure of the equatorial $u$ response at day 15 for these cases is shown in Fig. 15. The lower-

![Diagram](image)

**Fig. 15.** Day 15 perturbation zonal wind on the equator for heating on the equator at (a) 90°W, (b) 60°E, and (c) 120°E. The conventions are as in Fig. 3.
Western Hemisphere westerlies encroach into the equatorial region in the Indian Ocean, and the same small-scale pattern is found there in Figs. 16a,b,d.

The 90°W heating (Fig. 16b), in a region of equatorial westerlies, gives arching wave patterns into both hemispheres. The Northern Hemisphere wave train moves down through the eastern Mediterranean, triggering the equatorial Indian Ocean response, and then along the southern Asian region and into the Pacific. This behavior is reminiscent of the waveguide seen in the analysis of Hsu and Lin (1992) and the barotropic modeling of Hoskins and Ambrizzi (1993). The 60°E heating (Fig. 16c) leads only to a Northern Hemisphere wave train. At 120°E, there are easterlies in the equatorial upper troposphere. However, there are strong subtropical westerly jets in each hemisphere, which allow a remote Rossby wave source as described in Sardeshmukh and Hoskins (1988). Figure 16d does indeed show wave trains in both hemispheres whose characteristic Rossby wave eastward arching is only apparent from the subtropics.

The timescales for developing the patterns in these cases with longitudinally varying flows are similar to those for zonal flows. The wave train propagates into and through middle latitudes in the first week, and both into higher latitudes and back into the westerly regions in the Tropics in the second week. The barotropic modal description of Simmons et al. (1983) appears to be relevant to the pattern that is triggered in the North Pacific. However, on the timescales discussed here, there is little evidence of the growth of such modes.

The integrations described here have been repeated for an infinitesimal heating. It is found that none of the features discussed here are crucially dependent on nonlinearity.

5. Rossby wave sources

As in the above discussion, much of the impact of the basic flow can usefully be separated into its effect on the forcing of Rossby motion by the equatorial heating and on the propagation of these Rossby waves. The latter has been investigated in the barotropic context in Hoskins and Ambrizzi (1993), and subsequent work will discuss the baroclinic case in detail. Here we will investigate the variations in the upper-tropospheric divergence response and the consequent generation of Rossby wave motion.

Sardeshmukh and Hoskins (1988) discussed the generation of Rossby waves at an upper-tropospheric level in the case of a specified tropical divergence. They showed that the vorticity equation may be approximated and written in the form of the forced barotropic vorticity equation:

$$\left( \frac{\partial}{\partial t} + \mathbf{v}_* \cdot \nabla \right) \zeta = S.$$

Here the so-called Rossby wave source (RWS)

$$S = -\zeta D - \mathbf{v}_* \cdot \nabla \zeta = -\nabla \cdot (\mathbf{v}_* \zeta).$$
where $\psi$ is the rotational flow, $\zeta$ is the absolute vorticity, and $v_\phi$ and $D$ are the specified divergent velocity and divergence, respectively. Sardeshmukh and Hoskins (1988) emphasized the importance of the $v_\phi$ term for specified divergence in regions of equatorial easterlies. The advection by the divergent wind blowing out from such a region can lead to a significant RWS in westerly wind regions in which Rossby wave motion is possible.

In the present context it is equatorial diabatic heating that is specified. This heating rapidly leads to local equatorial ascent and upper-tropospheric divergence. Fast gravity and slower Rossby waves emanate from the switch on, and, unlike the barotropic model, divergence patterns can develop elsewhere.

The day 5, $\sigma \approx 0.20$ divergence fields for the resting atmosphere, zonal flow, and zonally asymmetric flow cases are given in Fig. 17. This level, which is the one above those shown elsewhere in this paper, has been chosen because it generally exhibits the maxima in tropical divergence. However, for the resting atmosphere the tropopause is at a lower global level, and the divergence peaks below this $\sigma \approx 0.20$ level. Consequently the divergence shown in Fig. 17a is relatively weak compared with those in the other panels. Thus, the contour interval has been halved for this case. The field is dominated by the divergence in the heating region. The equatorial convergence near the equator and 0°E is moving steadily eastward at a speed consistent with a 10-day circumpolar time and is clearly associated with the transient equatorial Kelvin wave. The subtropical convergences west of the heating may be considered as part of the transient equatorial Rossby wave response: they are not present 5 days later.

For the zonally symmetric basic state the equatorial divergence is now closely bounded by convergence, implying local Hadley-type cells. The Northern Hemisphere cell is some three times stronger than its Southern Hemisphere counterpart and extends both westward and southeastward. These features exist still at day 15. The transient Kelvin wave signature is again seen at $\sim 180^\circ$ from the heating. The Northern Hemisphere subtropical divergence to the northeast of the heating is probably better viewed as a sign of the Rossby wave propagation rather than as part of the forcing. Equivalent barotropic Rossby waves have ascent in the poleward moving flow. In Fig. 5 near 20°N and 30°E of the heating positive $v$ at lower levels going to about zero at upper levels can be seen. Ascent and upper divergence is consistent with this. Consistently, a finer-contour interval shows convergence in the Baja California region where the reversed sign in $v$ is found.

The figures for the zonally asymmetric basic flow (Figs. 17c–f) all show the divergence in the heating region. Each has a Kelvin wave signature at about 180° from the heating, though its magnitude is slightly below the contour interval for the 60°E case (Fig. 17e). The local Hadley cell intensities, as measured by the convergence to north and south, vary markedly. They are both strong for the 90°W case (Fig. 17d) and both weak for 120°E (Fig. 17f). The northern cell is much enhanced for 60°E (Fig. 17c), and the latitudinal asymmetry is reversed for 180°E (Fig. 17c). The 60° and 120°E cases (Figs. 17e,f) have interesting equatorial convergence maxima some 30°–50° east of the heating, indicative of a local Walker-type circulation.

The Rossby wave sources (RWS) at $\sigma \approx 0.20$ on day 5 for the various integrations are given in Fig. 18. In cases where the basic flow has a source $S_0$ associated with it, then the RWS associated with the additional diabatic heating is $S - S_0$, and it is this quantity that is shown in the figure. In each case the equatorial divergence in the heating region leads to a dipole anticyclonic forcing. The resting atmosphere case (Fig. 18a) shows the broadening of this dipole by the $v_\phi$ term, which in this case is approximately $- \beta v_x$. In the longitude of the heating $v_\phi$ is strongly poleward. In each case the transient Kelvin wave signature of a cycloonic dipole associated with the equatorial convergence is seen about 180° from the heating.

The resting atmosphere case (Fig. 18a) also shows cyclonic forcing in the subtropical convergence regions to the west of the heating. However, these are transient features, as are the anticyclonic forcings near 40° and the cyclonic forcings to the east.

For the zonally symmetric basic state the broadening of the Southern Hemisphere anticyclonic RWS is clear, though its long zonal wavelength at higher latitudes implies very little generation of zonally propagating Rossby waves from there, consistent with Fig. 5a. The strong Hadley cell convergence in the Northern Hemisphere only acts to reduce the meridional broadening. However, the equatorial pattern extends strongly north-eastward to become part of the signature of the extratropical Rossby wave response. The westward extension of the Northern Hemisphere subtropical convergence seen in Fig. 17b leads to the cyclonic RWS to the west of the heating near 30°N. Unlike the resting atmosphere case, this is not a transient feature. Both it and the much weaker Southern Hemisphere feature appear to be associated with the poleward/western flanks of the equatorial anticyclones.

The relative intensities of the equatorial anticyclonic forcings in the asymmetric cases depend both on the divergence and on the local ambient vorticity. The basic state has very small and even reversed poleward vorticity gradients equatorward of the Asian–North Pacific jet from 90° to 180°E and strong gradients in the jet (Hoskins and Ambrizzi 1993, Fig. 3b). Consistent with this and in agreement with Sardeshmukh and Hoskins (1988) the 180° and 120°E cases (Figs. 18c and 18f) both have almost separated anticyclonic forcings centered near 10° and 30°N. The complexity of the Rossby wave patterns in these cases (Figs. 16a,d) is possibly partly associated with the importance of these subtropical forcings. Because the ambient vorticity is
Fig. 17. The day 5, $\sigma \approx 0.20$ divergences for heating at $180^\circ$ in (a) resting atmosphere, and (b) Dec-Feb zonal flow, and for heating in a 3D Dec-Feb flow at (c) $180^\circ$, (d) $90^\circ$W, (e) $60^\circ$E, and (f) $120^\circ$E. The divergences have been smoothed using a spectral filter of the form $\exp(-K(n+1)^2)$ with $K$ chosen so that the highest wavenumber spectral coefficients are multiplied by 0.1 (for details see Sardeshmukh and Hoskins 1984). The contour interval is $2 \times 10^{-7}$ s$^{-1}$ in (a) and $4 \times 10^{-7}$ s$^{-1}$ otherwise. The zero contours are suppressed, and negative contours are dashed.
Fig. 18. The day $5, \sigma \approx 0.20$ Rossby wave sources for the same cases as in Fig. 17: heating at $180^\circ$ in (a) resting atmosphere, and (b) Dec–Feb zonal flow, and a 3D Dec–Feb flow with heating at (c) $180^\circ$, (d) $90^\circ$W, (e) $60^\circ$E, and (f) $120^\circ$E. The same exponential spectral filter has been applied. The contour interval is $5 \times 10^{-12}$ s$^{-2}$. Zero contours are suppressed, and negative contours are dashed.
very small north of the equator near 120°E, in this case the equatorial anticyclonic forcing is small. It was this case and the 60°E case that gave Walker cell signatures in the divergence. Consequently, the near-equatorial RWS for both cases (Figs. 18c, f) exhibit a sign of a quadrupole pattern, the anticyclonic forcings being accompanied by cyclonic forcings to the east. The weak equatorial quadrupole seen in the streamfunction for the 120°E case (Fig. 14c) agrees well with these comments. However, in the 60°E case the strong anticyclonic forcing appears to overwhelm the weak cyclonic part of the quadrupole in producing the strong Northern Hemisphere anticyclone seen in Fig. 14b.

The hypothesis that the subtropical convergence to the northwest of the heating is associated with the westerly extension of the anticyclone is supported by its weakness in the 120°E case in which this anticyclone is also weak.

It is of interest to compare the evidence for a Southern Hemisphere Rossby wave source given by the \( v \) field in each case (Figs. 16a–d) with the source actually determined (Figs. 18c–f). For 180°E heating, the positive \( v \) stretching northeast from north of New Zealand in Fig. 16a is just outside the easterly wind region and is consistent with Fig. 18c and a simple Sverdrup balance:

\[
\beta v' \approx S - S_0.
\]

This is true in magnitude as well as pattern with 1.5 m s\(^{-1}\) in \( v' \) corresponding to \( 3.5 \times 10^{-11} \) s\(^{-2}\) in RWS. For the 90°W case the heating is in an upper-tropospheric westerly regime so that the Rossby wave propagation seen in Figs. 14a and 16b can be directly forced. The positive \( v \) maximum near 15°S, 80°W marks the eastern flank of the anticyclone and is consistent with the positive sign in RWS (Fig. 18d). The 60°E case (Fig. 16c) shows no significant zonally propagating Rossby wave train. The RWS (Fig. 18e) outside the easterlies appears to have such a long zonal wavelength that only meridional propagation is possible. Finally, the shorter zonal wavelength of the RWS for the 120°E case (Fig. 18f) is generally consistent with the generation of the propagating pattern seen in Fig. 16d.

6. Conclusions

The initial value technique has enabled us to illustrate the direct response to tropical heating in a baroclinic atmosphere for a hierarchy of basic flows up to a 3D seasonal-mean flow. The timescales for establishing portions of the response are immediately apparent, and the effects of nonlinearity can be determined. As is usual in such studies, where a basic flow is used that does not satisfy the equations, a constant forcing has been added to these equations to make it a stationary solution. It is not believed that, in the relatively short timescale of interest here, the solutions would be very different if flow-dependent forcings were used.

Branstator (1992) used a steady-state model linearized about a zonally asymmetric flow to diagnose the maintenance of low-frequency anomalies in a GCM. As discussed by him, such models exhibit sensitivity to the dissipations employed and to the existence of low-frequency modes. From a wider range of calculations than those exhibited here, it appears that such sensitivity is not relevant to the quasi-stationary response to tropical heating on the timescale of up to two weeks.

The Gill-type solution was evident in the tropical lower troposphere in all of the cases studied, in agreement with the argument of Neelin (1988). The nonlinear solution for a resting atmosphere was similar to Gill’s picture. The speeds of the outgoing Rossby and Kelvin wave fronts mean that the cyclic nature of the earth becomes important within 8 days, and upper-tropospheric zonally averaged westerlies are established.

With a Northern Hemisphere winter zonal flow, Rossby wave propagation occurs into the winter hemisphere, in particular. In middle and high latitudes, the signature is equivalent barotropic, having the same sign throughout the troposphere and lower stratosphere and maximum amplitude on the tropopause. The results, including the low wavenumber train propagation to high latitudes and the higher wavenumber train being reflected by the northern flank of the winter subtropical jet, are consistent with the study of Hoskins and Karoly (1981).

The response to tropical heating in a 3D December–February basic state depends on the longitude of the heating. The general nature of the tropical upper-tropospheric quadrupole remains, but the relative magnitudes of the different centers and their positioning vary with this longitude. For example, for heating in regions such as the west Pacific, the upper-tropospheric anticyclones are centered over the heating. There is no westward propagation because the basic state has a region of almost zero absolute vorticity (and thus poleward absolute vorticity gradient) there associated with the seasonal mean heating.

A detailed study of the upper-tropospheric divergence and Rossby wave source (RWS) has shown many interesting but quite complicated variations from case to case. Although the basic balance associated ascent and upper-tropospheric divergence with equatorial heating remains valid, the details are strongly dependent on the ambient flow. As measured by upper-tropospheric convergence, local Hadley and Walker cells of differing relative magnitudes can be generated depending on the position in realistic basic states. Extratropical equivalent barotropic Rossby waves also have a signature in the divergence field. Thus, the interpretation of the RWS is much more complex than in the barotropic case with specified divergence as discussed by Sardeshmukh and Hoskins (1988). How-
ever, many of the results are in agreement with that study. Due to the term describing the advection of vorticity by the divergent wind, the anticyclonic RWS is generally much broader than a simple $f\Delta$ argument would suggest. Indeed, there is a separate anticyclonic maximum in RWS in the Asian–North Pacific jet for heating to the south or southeast of it. The magnitude of this term also means that the Hadley cell convergences are mostly not accompanied by cyclonic forcing. Cyclonic forcing is found in the subtropics to the west of the heating and appears to be linked to the poleward/westward flank of the equatorial anticyclones.

The wave propagation from the source regions is influenced by the longitudinally varying basic state, showing preferred paths and propagation into the equatorial westerly regions, as well as preferred structures such as the dipole in the North Pacific. Such features have been illustrated by the observational study of Hsu and Lin (1992) and examined in the context of a barotropic model by Hoskins and Ambrizio (1993). It is clear that many of the features shown here are in agreement with those studies, but a detailed investigation of such propagation in a baroclinic model will be described elsewhere.

In all cases, for the realistic amplitudes used here, the nonlinear interactions involving the tropically forced wave and the basic state are found to be of minor importance. Clearly nonlinearity would be important in subsequent interaction with middle-latitude transients.

The timescale for establishing the tropical and middle-latitude patterns in all cases is about a week. Within a further week the higher-latitude pattern is established, and propagation back into the westerly tropical regions occurs.

Another feature of interest in the solutions exhibited here is the zonal flow generation, which is also apparent in linear integrations. For the resting and zonal basic states the perturbation zonal flow is part of the Hadley cell response. For the asymmetric basic flow there are also momentum fluxes ($\bar{u}'\bar{v}' + \bar{u}\bar{v}'$), which depend on the longitude of the heating. The upper-tropospheric westerly flow generated in the region of the equator is strong in some cases and weak in others. Again, a detailed investigation of such zonal flow generation will be given elsewhere.

Acknowledgments. This study was performed in 1987/88 while one of us, FFJ, was visiting the Department of Meteorology with a Royal Society Fellowship for China. We are very grateful to Tercio Ambrizio for recreating the figures and for the calculations performed by Yang Guiying and him for section 5.

REFERENCES


