Effect of topographic curvature on near-surface stresses and application to sheeting joints

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[1] Sheeting joints have attracted attention for more than two centuries, yet their cause has remained unclear. Sheeting joints are opening mode rock fractures that form subparallel to the topographic surface and develop to depths of at least 100 m. They are best developed beneath convex surfaces in massive rocks where the compressive stress parallel to the surface (P) is high. A tensile stress normal to a convex traction-free surface will arise if the product of P and the surface curvature (k) exceeds the product of the unit weight of rock (ρg) and the cosine of the slope ($\cos\beta$). The tensile stress contributes fundamentally to sheeting joints; erosion of overburden, by itself, does not. Rocks with high uniaxial compressive strengths host sheeting joints because they can sustain the high compressive stresses necessary to generate surface-normal tensile stresses given typical curvatures in landscapes. **Citation:** Martel, S. J. (2006), Effect of topographic curvature on near-surface stresses and application to sheeting joints, *Geophys. Res. Lett.*, *33*, L01308, doi:10.1029/2005GL024710.

1. Introduction

[2] Sheeting joints (i.e., "exfoliation" joints) have fascinated and puzzled geologists for more than two centuries [e.g., *Dale*, 1923]. They form subparallel to the topographic surface, are gently curved, and develop well in strong rocks such as granite, gneiss, massive sandstone, mafic intrusive rocks, and marble [*Matthes*, 1930; *Jahns*, 1943; *Holzhausen*, 1977, and references therein]. Sheeting joints occur in many parts of the world [*Cadman*, 1969, and references therein; *Twidale*, 1982; *Holzhausen*, 1989] and are renowned for their role in landscape development [*Gilbert*, 1904; *Matthes*, 1930; *Bradley*, 1963]. They strongly influence shallow groundwater systems [e.g., *LeGrand*, 1949; *Trainer*, 1988], either on their own or by intersecting other fractures to form a conductive hydrologic network [*Borchers*, 1996]. They contribute significantly to mass wasting hazards [e.g., *Terzaghi*, 1962; *Hermanns and Strecker*, 1999; *Wieczorek and Snyder*, 1999; *Chigara*, 2000]. They are associated with rock bursts in mines and quarries [e.g., *Niles*, 1871; *Dale*, 1923; *Twidale and Bourne*, 2000]. In spite of numerous studies, however, their cause has remained elusive. Commonly cited explanations either fail to account for key characteristics of sheeting joints or are untenable (or suspect) on physical grounds.

[3] This manuscript shows directly how compressive stresses parallel to convex portions of the Earth's topographic surface can generate tensile stresses normal to the surface, a mechanism considered previously by *Holzhausen* [1977] and *Miller and Dunne* [1996] for particular idealized geometries and boundary conditions. This manuscript describes key characteristics of sheeting joints, reviews traditional explanations for the joints, presents the fundamental theoretical requirements for the proposed mechanism, and tests it against existing data.

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2. Sheeting Joint Characteristics

[4] Sheeting joints form subparallel to the topographic surface (Figure 1). They occur most commonly where the topography is convex and can extend in plane at least 200 m [*Gilbert*, 1904; *Matthes*, 1930; *Jahns*, 1943]. They occur from within a meter of the present surface to depths in excess of 100 m, and with depth their spacing usually increases while their curvature decreases [*Jahns*, 1943]. They typically develop independent of grain scale structure of the rock (e.g., foliation) and cross cut contacts between different rock units [*Jahns*, 1943].

[5] Field observations of sheeting joints overwhelmingly favor an opening mode origin. The joints locally have apertures as great as several centimeters, although their walls typically are in contact, and show little or no evidence of slip [e.g., *Dale*, 1923; *Matthes*, 1930; *Holzhausen*, 1989]. The joints also end in a pattern of echelon fractures and display surface textures characteristic of opening mode fractures in rock [*Segall and Pollard*, 1983; *Pollard and Aydin*, 1988; *Holzhausen*, 1989].

[6] Sheeting joints are among the youngest structures in rock outcrops. They mimic the topography, which in many areas has developed in the last few million years. Some have formed since late Pleistocene glacial episodes, even during quarry operations [*Holzhausen*, 1989]. These findings indicate that analysis of current conditions might illuminate how sheeting joints form.

[7] High compressive stresses parallel to the surface consistently have been either inferred from field observations [e.g., *Niles*, 1871; *Dale*, 1923; *Matthes*, 1930; *Jahns*, 1943] or measured (Table 1) where sheeting joints exist. Surface-parallel compressive stresses of 10–30 MPa at depths of several meters or less are common. These levels exceed the uniaxial compressive strength of weak rocks but not massive rocks such as granites [*Bell*, 1993]. This manuscript focuses on an effect of surface-parallel compression along a curved surface instead of its causes [e.g., *Turcotte and Schubert*, 2002].

3. Traditional Explanations

[8] Erosion of overburden, although widely cited as the cause for sheeting joints, cannot be an essential cause by itself. First, eroded terrains generally lack sheeting joints.

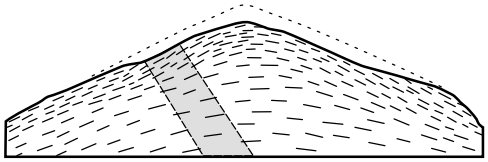

Figure 1. Cartoon of sheeting joints in cross section, modified from *Jahns* [1943] © 1943 by the University of Chicago. All rights reserved. The rock types in the white and gray regions differ. The dotted line represents the inferred topography when the joints formed.

Second, although a reduction of vertical compressive stress during erosion will lead to a vertical rebound (strain) of the rock, erosion by itself does not produce a tensile stress normal to the surface, something needed to open sheeting joints.

[9] Residual grain-scale stresses also are rejected as a cause. First, microscopic examinations show grains along joints typically are fresh and lack signs of grain-scale hydration and chemical alteration [*Jahns*, 1943]; this rules out chemical effects as a source for grain-scale residual stresses. Second, if residual tensile stresses from any source caused the joints to form in-place at a quarry site, then the fractures should continue form in the extracted rock after it is quarried, yet this does not occur [*Holzhausen*, 1989]. In situ factors must therefore control the formation of sheeting joints, not the intrinsic condition of the rock.

[10] Opening mode fractures can develop in solids with cavities or flaws in response to uniaxial compression, but experiments show that they tend to stabilize after attaining a length of a few cavity or flaw dimensions [*Brace and Bombolakis*, 1963; *Nemat-Nasser and Horii*, 1982]. Theoretical analyses indicate that a nearby free surface assists fracture growth [*Germanovich and Dyskin*, 2000], but whether fracture-parallel compression alone can account for sheeting joints hundreds of meters long is unclear, especially when the weight of overburden is accounted for.

4. Compression Along a Curved Surface

[11] Figure 2 shows how a compressive stress (P) parallel to the convex surface of an infinitesimal weightless element can induce a tension normal to the surface (N). The convex upper surface is traction-free, and no shear tractions (τ) exist on the boundary of the element. The compressive (negative) tractions on the canted element ends yield a net force directed away from the center of curvature. That force must be opposed by tensile tractions at the base of the element to maintain equilibrium.

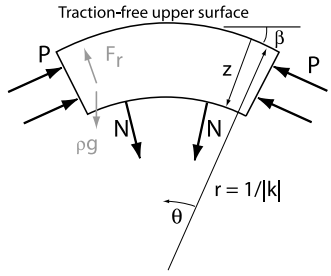

Figure 2. Infinitesimal convex element subject to compressive (negative) end loads (P). The upper surface of radius r is traction-free. The lower surface is subject to basal normal tractions (N). No shear tractions (τ) act on the boundary of this element. If body forces (e.g., gravity) are absent and if P is compressive, then N must be tensile (positive) for equilibrium. Fractures would form parallel to the surface where N becomes sufficiently large.

[12] Instead of solving a particular boundary value problem, I use the equations of equilibrium to provide general quantitative insight into near-surface stress conditions when a body force due to gravity is present. These equations are entirely independent of rheology. For a body in plane strain, the equations for a cylindrical reference frame are [*Chou and Pagano*, 1967]:

$$\frac{\partial N}{\partial r} + \frac{1}{r}\frac{\partial \tau}{\partial \theta} + \frac{N-P}{r} + F_r = 0, \tag{1a}$$

$$\frac{1}{r}\frac{\partial P}{\partial \theta} + \frac{\partial \tau}{\partial r} + \frac{2\tau}{r} + F_\theta = 0, \tag{1b}$$

where F_r and F_θ are the radial and tangential components of the gravitational body force, respectively, r is the radial distance, and θ provides the angular position (Figure 2). Only equation (1a) contains the critical term N, and it can be simplified to describe conditions at a traction-free surface by some substitutions. First, $N = \tau = 0$. Next, note that (a) the magnitude of curvature of the surface $|k| = 1/r$, and (b) $F_r = -\rho g \cos\beta$, where ρ is the rock density, g is gravitational acceleration, and β is the slope at the surface.

Table 1. Stresses, Curvatures, and Map-View Aspect Ratios Of Topography at Sites With Sheeting Joints[a]

Location	P_1, MPa	P_2, MPa	P_1, Trend	$\lvert k_{max}\rvert$, m^{-1}	$\lvert k_{min}\rvert$, m^{-1}	Length:Width Ratio
Cannital Quarry, Manitoba (a)	−19.5	−9.0	~N45E	0.0068	0.0067	0.8
Granite Mountain, Texas (b)	−15.3	−10.3	N33W	0.0019	0.0017	1.5
Kittredge Hill, New Hampshire (c)	−26.7	−4.5	N67W	0.0004	0.0004	1.1
Oak Hill Quarry, Massachusetts (b)	−22.6	−12.1	N60E	0.0003	0.0002	1.0
Pine Mtn., Georgia (b)	−7.2	−5.9	N54E	0.0009	0.0008	1.3
Stone Mountain, Georgia (b)	−10.3	−6.9	N08E	0.0023	0.0008	2.4
Tuolumne Meadows, California (d)	−13.9	−6.5	N30E	—	—	—
Olmstead Point				0.0021	0.0008	2.7
Cathedral Peak				0.0035	0.0008	1.4

[a]The maximum and minimum convex curvatures are from the curves of Figure 3. A circular dome has a length:width ratio of 1. References: (a) *Everitt et al.* [1994] and *Martin et al.* [2003], (b) *Hooker and Johnson* [1969], (c) *Plumb et al.* [1984] and *Jahns* [1943], and (d) *Cadman* [1969].

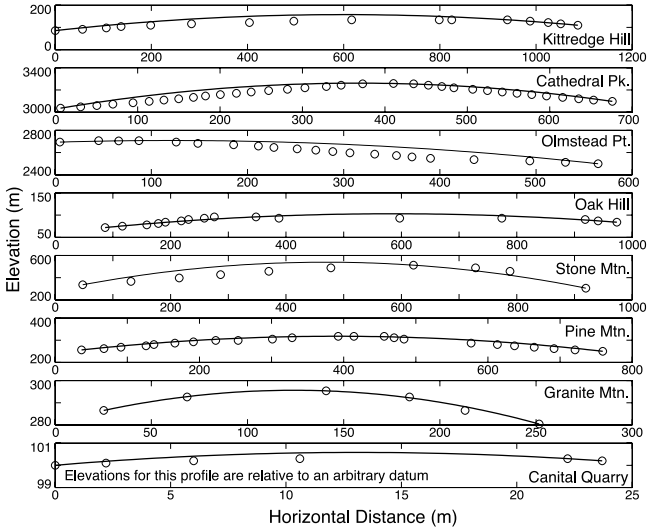

Figure 3. Topography in the direction of P_1 across sites of Table 1, with parabolic curves fit to the endpoints of the profiles and the profile highpoint. The horizontal and vertical scales are not equal.

Finally, let z be the depth normal to the surface, so $dz = -dr$. With the above substitutions, equation (1a) yields:

$$\left.\frac{\partial N}{\partial z}\right|_{z=0} = kP - \rho g \cos\beta. \tag{2}$$

This equation applies to all cylindrical surface geometries, and when body forces are absent it matches the corresponding Lamé-Maxwell equation of equilibrium [*Ramsay and Lisle*, 2000]. For a spherical surface [*Chou and Pagano*, 1967] the same equation results, except that P must be interpreted as sum of the two principal stresses parallel to the spherically curved surface (i.e., $P = P_1 + P_2$). For a flat horizontal surface, $k = \beta = 0$, and the lithostatic vertical stress inhibits sheeting fracture growth. A tensile stress normal to the surface *must* develop in the shallow subsurface, however, where $\partial N/\partial z$ is positive (e.g., where $kP > \rho g$), since $N = 0$ at the surface. For a convex surface, k is negative, and if P is also negative (compressive), then kP is positive. For the Earth as a whole, $k = -1.6 \times 10^{-7}$ m^{-1} and kP is negligible compared to the lithostatic gradient ($-\rho g$). In many places, however, $|k|$ locally exceeds 10^{-3} m^{-1}, kP can exceed ρg, and fractures would open subparallel to the surface. Where a convex slope has even a small angular bend, $|k|$ is very large and fractures could open even for tiny values of P.

5. Analysis of Current Conditions

[13] Table 1 shows principal stress magnitudes from near-surface measurements where sheeting joints exist, together with curvature values in the direction of P_1. The stress values for Tuolumne Meadows are applied to nearby Cathedral Peak and to Olmstead Pt., which is 13.5 km southwest. All other values are from sites on or adjacent to the respective slopes where the curvatures are measured. The most compressive stress (P_1) generally is between -10 and -30 MPa in Table 1. The peak in situ compressive stresses could exceed the tabulated values by as much as 7 MPa depending on thermal stresses [*Holzhausen*, 1989]. The curvature range is determined from a parabolic arc fit to the endpoints and the high point along a profile parallel to the trend of the most compressive stress (Figure 3).

[14] Equation (2) can be used to test whether current stresses and curvatures at or near the surface meet the minimum threshold for sheeting joints to open. Figure 4 shows P plotted against the curvature values for the sites of Table 1. The values of P range from P_1 to $P_1 + P_2$ to cover the topographic spectrum from ridges to domes, respectively. Conditions at five of the sites either meet or straddle the fracture threshold curve; sheeting joints might be forming there now. For the three sites farthest from the threshold (Kittredge Hill, Oak Hill, and Pine Mtn), kP is ~30%–50% of the threshold level. Two main possibilities emerge for why current conditions fall below threshold values, assuming that the stress measurements do not have systematic errors. First, the regional compressive stresses might have diminished since the time of jointing. Second, the topographic curvature might be less now than when the joints formed because of erosion. Independent evidence favors this option. *Jahns* [1943] found evidence of at least 8–13 meters of local glacial erosion after the joints formed at Kittredge Hill, and at least 33 meters at Oak Hill; these amounts are 30%–50% of the current relief of the respective hills (Figure 3). Although erosion can locally increase curvature, it generally flattens slopes and reduces the overall curvature, especially if the erosion is focused at a summit, where the curvature commonly is largest. The current curvatures at Kittredge Hill and Oak Hill almost surely are lower now then when the joints formed and do not preclude the proposed mechanism.

6. Discussion

[15] The proposed mechanism accounts for more than the presence and orientation of sheeting joints. It accounts for their subparallel nature and substantial overlap, a pattern

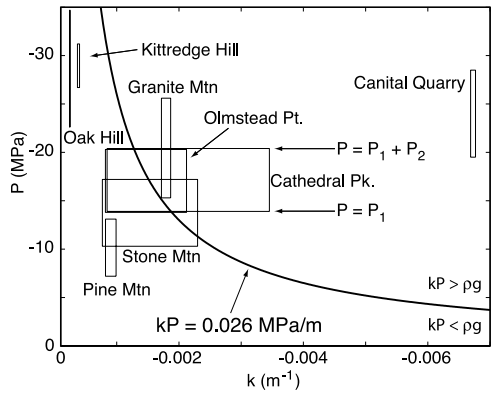

Figure 4. Plots of current surface parallel stresses for assumed cylindrical geometries (P_1 for lower bases of rectangles) and assumed spherical geometries ($P_1 + P_2$) for upper bases of rectangles) vs. current surface curvature along the trend of the most compressive stress for sites in Table 1. Fracturing is predicted for conditions above the threshold curve $kP = \rho g$.

predicted only for opening mode fractures that grow where the difference between the ambient principal stresses is large relative to the tensile stress fracturing the rock [*Olson and Pollard*, 1989]. Here, P is usually more negative than -10 MPa (Table 1 and Figure 4), and N is controlled by the tensile strength of the rock, which is no more than several MPa and positive [*Holzhausen*, 1989; *Bell*, 1993]. The difference between P and N thus is large relative to N, meeting the condition for subparallel, highly overlapped joints. The proposed mechanism can also account for the tendency of the joint spacing to increase with depth and for the joints to be closed. When each joint opens, N is reset to zero at the joint. The observed decrease in joint curvature with depth (Figure 1) will decrease the right side of equation (2), so the distance z required for the surface-normal tensile stress N to build to the level required for fracture will increase with depth. Erosion after the time of jointing will tend to lessen the surface curvature, and hence N, even if the surface-parallel compressive stresses are maintained. When N drops below zero the joint walls would close. The mechanism accounts for a broad variety of the essential characteristics of sheeting joints.

[16] Sheeting joints are well developed in massive rocks and are absent where the rock is heavily fractured or weathered. Reports of sheeting joints in weak rocks such as shales are conspicuous in their absence. These findings support the hypothesis that rock masses must be able to withstand high compressive stresses in order to develop sufficient tensile stresses normal to the rock mass surface to open sheeting joints. The proposed mechanism not only accounts for the presence of sheeting joints in strong rocks such as granites, gneisses, and massive sandstones, but also their scarcity or absence in weak or weathered rock.

[17] The vertical normal stress typically is assumed to be just due to the weight of the overburden, in spite of near-surface measurements to the contrary [e.g., *Martin et al.*, 2003]. Where the local relief is large and the horizontal compressive stresses are too, however, the vertical normal stress can depart substantially from lithostatic pressure [see also *Savage and Swolfs*, 1986; *Miller and Dunne*, 1996]. Predicting and controlling hydraulic fractures and the stability of wells will be difficult in certain cases if the interaction of the Earth's stresses with the topographic surface is not accounted for.

[18] Many issues bearing on the process of sheeting joint formation remain to be addressed. These include how sheeting joints interact, the shapes they assume as they propagate, the distances they extend beyond their region of nucleation, and why they exist in slopes that are now concave in cross section [*Gilbert*, 1904; *Matthes*, 1930; *Jahns*, 1943]. Detailed mapping of sheeting joints and the associated topography, together with three-dimensional mechanical analyses, would shed light on all these matters.

7. Conclusions

[19] The effect of topography on the stresses near the surface of the Earth can be profound [e.g., *Molnar*, 2004]. The equations of equilibrium show that a compressive (negative) stress parallel to a convex traction-free surface will cause near-surface tensile stresses to arise normal to the surface if the product of the curvature and the surface-parallel stress exceeds the product of the unit weight of the rock and the cosine of the slope. These stresses contribute fundamentally to sheeting joint formation.

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