STRAIN (06)

I Main topics

A General deformation

B Tensor notations for strain

C Relationship between stress and strain

II General deformation (changes of position of points in a body)

A Rigid body translation

1 All points displaced by an equal vector (equal magnitude and direction); no displacement of points relative to one another

2 [X'] = [u] +[X] matrix addition

B Rigid body rotation

1 All points rotated by an equal amount about a common axis; no angular displacement of points relative to one another

2 [X'] = [a][X] matrix multiplication; rows in [a] are dir. cosines!

C Change in shape (distortional strain)

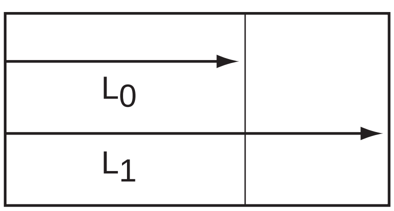
1 Change in linear dimension (normal strain or elongation)

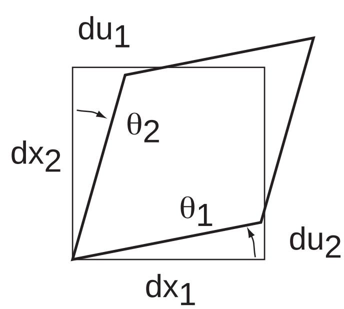
$$\varepsilon = \frac{\Delta L}{L_o} = \frac{L_1 - L_o}{L_o}$$ dimensionless! (6.1)

2 Change in angles (shear strain)

$$\gamma = \theta_1 + \theta_2 = 2\varepsilon_{xy} = 2\varepsilon_{yx}$$ dimensionless! (6.2)

For small angle changes, $\theta_1 = \tan^{-1}(du_2/dx_1) \approx du_2/dx_1$





A positive shear strain corresponds to a **decrease** in the right angle

D Change in volume (dilation)

$$\Delta = \frac{\Delta V}{V_o} = \frac{V_1 - V_o}{V_o}$$ dimensionless! (6.3)

III Tensor notations for infinitesimal strain

Displacement gradient matrix = Strain matrix + rotation matrix

$J_u = E + \Omega$ (6.4)

$$J_{ij} = \left(\frac{\partial u_i}{\partial x_j}\right) \qquad \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

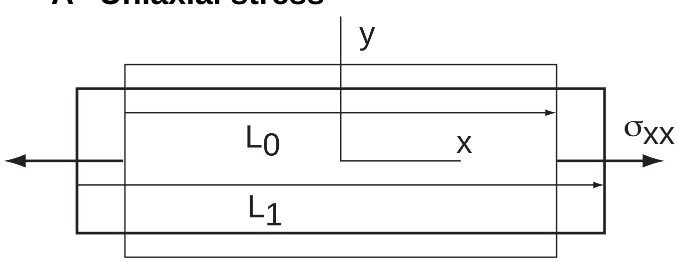
In 2-D

$$\begin{bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} \end{bmatrix} =$$

$$\begin{bmatrix} \frac{1}{2}\left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_1}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_2} + \frac{\partial u_2}{\partial x_2}\right) \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\left(\frac{\partial u_1}{\partial x_1} - \frac{\partial u_1}{\partial x_1}\right) & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right) & \frac{1}{2}\left(\frac{\partial u_2}{\partial x_2} - \frac{\partial u_2}{\partial x_2}\right) \end{bmatrix} \quad (6.5)$$

IV Relationship between stress and strain

A Uniaxial stress



$\sigma_{xx} = E\varepsilon_{xx}$ (6.6)

E = Young's modulus; dimensions of stress

Analog:

$F = kx$

k = spring constant; dimensions of stress*length

$\varepsilon_{yy} = \varepsilon_{zz} = -\nu\varepsilon_{xx}$ (6.7)

ν = Poisson's ratio (dimensionless)

B General conditions

$$\varepsilon_{xx} = \frac{1}{E}\left[\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right] \tag{6.8}$$

$$\varepsilon_{yy} = \frac{1}{E}\left[\sigma_{yy} - \nu\left(\sigma_{zz} + \sigma_{xx}\right)\right] \tag{6.9}$$

$$\varepsilon_{zz} = \frac{1}{E}\left[\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right] \tag{6.10}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = \frac{1}{2G}\sigma_{xy} \tag{6.11}$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \frac{1}{2G}\sigma_{yz} \tag{6.12}$$

$$\varepsilon_{zx} = \varepsilon_{xz} = \frac{1}{2G}\sigma_{zx} \tag{6.13}$$

For isotropic materials, the principal stress parallel principal strains!

Relationships Among Elastic Constants for an Isotropic Solid
(from Mal and Singh, 1991, p. 16)

Pair	λ,G (λ,μ)	λ,ν	G,k (μ,k)	μ,E	k,ν	E,ν
λ =			$k - \frac{2G}{3}$	$\frac{G(E-2G)}{3G-E}$	$\frac{3k\nu}{1+\nu}$	$\frac{E\nu}{(1+\nu)(1-2\nu)}$
G = (μ =)		$\frac{\lambda(1-2\nu)}{2\nu}$			$\frac{3k(1-2\nu)}{2(1+\nu)}$	$\frac{E}{2(1+\nu)}$
k =	$\lambda + \frac{2G}{3}$	$\frac{\lambda(1+\nu)}{3\nu}$		$\frac{GE}{3(3G-E)}$		$\frac{E}{3(1-2\nu)}$
E =	$\frac{G(3\lambda+2G)}{\lambda+G}$	$\frac{\lambda(1+\nu)(1-2\nu)}{\nu}$	$\frac{9kG}{3k+G}$		$3k(1-2\nu)$	
ν =	$\frac{\lambda}{2(\lambda+G)}$		$\frac{3k-2G}{2(3k+G)}$	$\frac{E}{2G} - 1$		

References

Barber, J.R., 1993, Elasticity: Kluwer Academic Publishers, Boston, p. 11-19.

Chou, P.C., and Pagano, N.J., 1967, Elasticity, Dover, New York, p. 170-179.

Malvern, L.E., 1969, Introduction to the mechanics of a continuous medium: Prentice-Hall, Englewood Cliffs, p. 120-127.

Timoshenko, S.P., and Goodier, J.N., 1971: Theory of elasticity, McGraw-Hill, New York