RESPONSE OF STRUCTURES (16)

I Main Topics
A Seismic records
B Acceleration, velocity and displacement spectra
B Resonance and natural frequencies
C Response of structures

II Seismic time series records
A Accelerogram (plot of acceleration vs. time)

B Time series records of acceleration, velocity, and displacement

1. Acceleration (a)
   a. $a = \frac{dv}{dt}$
   b. Peak force at peak acceleration
   c. Peak acceleration at right: ~1100 cm/sec²
   d. Time of peak acceleration: ~7.5 sec

2. Velocity (speed) (v)
   a. $v = \frac{du}{dt}$
   b. Peak kinetic energy at peak velocity
   c. Peak velocity at right: ~115 cm/sec
   d. Time of peak velocity: ~3 sec

3. Displacement (u)
   a. Peak displacement at right: ~40 cm
   b. Time of peak displacement: ~5 sec


Fig. 31 from USGS Prof. Paper 1360

III Seismic spectra

A. Spectra represent parameters (e.g., displacement, velocity, and acceleration) as a function of wave frequency (or period), not time

B. Reveal the frequency (or period) and amplitude of the most energetic/forceful waves
III Seismic Spectra

C Example: Simav earthquake, Turkey, 2011, M = 5.7-5.9
Time series records

Seismic spectra for Simav event

N-S  E-W  U-D

Disp.  Vel.  Acc.

Time  Time  Time

Envelopes give maximum amplitudes

D Seismic spectra envelopes
Example: Simav earthquake, Turkey, 2011, M = 5.7-5.9

Spectra for Simav event with envelopes
N-S  E-W

Disp.  Vel.  Acc.

Period  Period

III Seismic spectra

E Relationships among spectral envelopes

1 \[ y = A \sin\left(\frac{2\pi}{\lambda}(x + vt)\right) \]
\[ v = \frac{\lambda}{T} = f \lambda; \quad \lambda = v/f \]
\[ y = A \sin\left(\frac{2\pi f}{v}(x + vt)\right) \]

2 Now consider a particular point
\[ y_0 = y(x = 0) \]
\[ = A \sin\left(\frac{2\pi f}{v}(vt)\right) \]
\[ = A \sin\left(2\pi ft\right) \]

3 Let \( \omega = 2\pi f = 2\pi/T = \) angular frequency
\[ y_0 = A \sin(\omega t) \]
\[ y'_0 = \frac{d\left[A \sin(\omega t)\right]}{dt} = A\omega \cos(\omega t) \]
\[ y''_0 = \frac{d\left[A\omega \cos(\omega t)\right]}{dt} = -A\omega^2 \sin(\omega t) = -\omega^2 y_0 \]
\[ \omega = \left(\frac{y''_0}{y_0}\right)^{1/2} \]

F Amplitude relationships among displacement, velocity, and accelerations for a single angular frequency

Let \( \omega = 2\pi f = 2\pi/T = \) angular frequency
\[ y_0 = A \sin(\omega t) \]
\[ y'_0 = A\omega \cos(\omega t) \]
\[ y''_0 = -A\omega^2 \sin(\omega t) = -\omega^2 y_0 \]
### III Seismic spectra

**F Relationships among spectral envelopes for a frequency range**

Let $\omega = 2\pi f = 2\pi/T = \text{angular frequency}$

$$
y_0 = A\sin(\omega t) \quad y_0' = A\omega \cos(\omega t) \quad y_0'' = -A\omega^2 \sin(\omega t) = -\omega^2 y_0
$$

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**G Max. acceleration (forces) commonly at 2-10 Hz (T=0.1-0.5 sec), max. velocities (kinetic energy) at 0.5-2 Hz (T=0.5-2 sec), and max. displacements at 0.006-0.5 Hz (T=2-160 sec)**

1. **High frequency (small period) waves**: high amplitudes of acceleration, small amplitudes of displacement
2. **Low frequency (long period) waves**: low amplitudes of acceleration, large amplitudes of displacement

*Important frequency for design of large engineering structures*
III Seismic spectra

H Effects of source and distance

1. A small, nearby earthquake can affect short-period structures more than a larger, distant quake
2. A large, distant earthquake can affect long-period structures more than a smaller, nearby quake
3. Short-period (high frequency) waves attenuate with distance more readily than long-period (low frequency) waves

IV Resonance and natural frequencies

A. Resonance: vibration of large amplitude due to arrival of energy at a particular frequency
B. Natural frequency: The frequency at which a structure will resonate
C. Natural frequency of a pendulum
   1. Natural period:
      \[ T = 2\pi \sqrt{\frac{L}{g}} \]
      *Natural period increases with length*
   2. Natural frequency:
      \[ f = \frac{1}{T} = \frac{(g/L)\sqrt{2}}{2\pi} \]
   3. Natural angular frequency:
      \[ \omega = 2\pi f = \frac{(g/L)\sqrt{2}}{2\pi} \]
IV Resonance and natural frequencies

D Natural frequency of a mass on a spring (simple harmonic oscillator)

1. Natural period:
   \[ T = 2\pi \left(\frac{m}{k}\right)^{1/2} \]

2. Natural frequency:
   \[ f = \frac{1}{T} = \left(\frac{k}{m}\right)^{1/2}/(2\pi) \]

3. Natural angular frequency:
   \[ \omega = 2\pi f = \left(\frac{k}{m}\right)^{1/2} \]

* No damping in these expressions

Diagram of a simple harmonic oscillator with damping

E Rule of thumb for buildings:
  natural period = # of stories/10

F Avoid structural designs with natural periods that match the natural period of the underlying materials (or the source)

G Previous experience helpful for step F
V Response of structures

A Earthquakes commonly impart large shear forces at the base of a building
B Bolt buildings to foundations and have sufficiently stiff ground floors
C Asymmetric designs susceptible to twisting
D Sophisticated models are used now to help design critical structures (beyond the scope of this course)