CHARACTERIZING EARTHQUAKE SOURCES (13)

I Main Topics
   A Elastic rebound theory
   B Slip on a fault with a uniform stress drop
   C Seismic moment
   D Energy budget during an earthquake
   E Seismic energy equivalents

II Elastic rebound theory
(H.F. Reid, 1908, v. 2 of 1906 Earthquake report)
   • A Founded by comparing pre- and post-quake survey lines across SAF
   • B Seismic energy source: elastic potential energy of rock around fault
II Elastic rebound theory
(H.F. Reid, 1906 earthquake report)

Sequence of survey lines:
actual (solid) and hypothetical (dashed)

1 Hypothetical survey line (1806)
   • ~3m of far-field relative displacement likely between “1806” and 1851-1865
2 Real survey (1851-1865)
   • ~1-2m far-field relative displacement from 1851-1865 to 1874-1892
3 Real survey (1874-1892)
   • ~1-2m far-field relative displacement from 1874/1892 - 1906
   • ~3m far-field relative displacement in ~50 years from 1851-1865 to 1906.
4 Hypothetical survey in 1906 before earthquake
   • 6m slip in 1906 (2x the far-field relative displacement in previous ~50 years*).
5 Real survey after 1906 quake (note apparent “overshoot” along fault relative to 1851-1865)

Diagrammatic survey lines across San Andreas fault (SAF)

III Slip (Δu) on a 2D fault with a uniform shear stress drop (Δτ = τ_1 - τ_2)

A Rock is elastic, homogenous, isotropic, isothermal material
B Shear stress on fault in direction of slip prior to slip = τ_1; post-slip shear stress = τ_2
C Slip profile is related to the shape and size of the rupture
D Slip distribution is particularly sensitive to the short dimension
E For a "2-D" rupture (one dimension >> other dimension) suppose the short dimension = 2a
III Slip ($\Delta u$) on a 2D fault with a uniform shear stress drop ($\Delta \tau = \tau_1 - \tau_2$)

1. $\Delta u = 2(1-\nu) (\Delta \tau / \mu) (a^2 - x^2)^{1/2}$
2. $\Delta u_{\text{max}} = \Delta u(x = 0) = 2(1-\nu) (\Delta \tau / \mu) a \approx 3 \times 10^{-4} a$
3. $\Delta u_{\text{min}} = \Delta u(x = \pm a) = 0$
4. $\Delta u_{\text{ave}} = \int_{-a}^{a} \Delta u \, dx = \frac{\pi}{4} \Delta u_{\text{max}}$

IV Seismic moment $M_o$

A. $M_o = \mu \Delta u_{\text{ave}} A$
   $\mu$ = shear modulus
   $\Delta u_{\text{ave}}$ = rupture area

B. $M_o$ has dimensions of energy; used to measure earthquake size

C. $\mu$ has been measured; geologists can estimate $\Delta u_{\text{ave}}$ and $A$

D. Seismic moments can be predicted

E. Moment can characterize a faulting earthquake of any size
IV Seismic moment $M_o$

1. Elasticity theory predicts seismic wave amplitudes as a function of the wave period for fault ruptures of different size and slip.
2. These amplitude curves can be shifted and calibrated against real seismic records of previous earthquakes of assigned magnitudes.
3. Waves with very long periods yield the seismic moment.
   
   $$M_w = \frac{2}{3} \log M_o - 10.73$$
   where $M_o$ is in dyne-cm.

4. Waves with $M_w = \frac{2}{3} \log M_o - 6.07$
   where $M_o$ is in Nm.

5. Amplitude hits a ceiling (saturates) for periods greater than a certain value for a given seismic moment.

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V Energy budget during an earthquake
(From Scholz, 1990)

\[ E_{\text{seismic}} + \Delta E_{\text{strain}} + \Delta E_{\text{friction}} + \Delta E_{\text{fracture}} + \Delta E_{\text{chemical}} + ? = 0 \]

A. Kinetic energy ($E_{\text{seismic}}$) in seismic waves

1. $E_{\text{seismic}}$ varies with amplitude and wavelength.
2. Waves of a frequency of zero ("ultra-long period and wavelength") correspond to a static situation ("permanent" deformation).
3. Seismic moment, which describes the "permanent" deformation after an earthquake, should be related to seismic energy release.
4. Empirical relationship of seismic energy ($E_{\text{seismic}}$) to magnitude ($M_w$): $E_{\text{seismic}}$ (joules)$= 10(4.8 + 1.5 M_w)$. 

Before

\[ \tau_1 \]

\[ E_{\text{strain}} \text{(initial)} \]

After

\[ \tau_2 \]

\[ E_{\text{friction}} \]

\[ E_{\text{seismic}} \]

\[ E_{\text{fracture}} \]

\[ E_{\text{strain}} \text{(final)} \]
V Energy budget during an earthquake  
(From Scholz, 1990)

B  Strain energy ($\Delta E_{\text{strain}}$)
1  Energy in a linear spring
   a  Force in a spring
       $F = kx$
       $k = \text{spring constant}$
       $x = \text{displacement}$
   b  Strain energy ($\Delta E_{\text{strain}}$) equals area under a force-disp.
       curve
       $\Delta E_{\text{strain}} > 0$ if spring stretches
       $\Delta E_{\text{strain}} < 0$ if spring contracts

\[
\Delta E_{\text{strain}} = \int_0^x F \, dx = \int_0^x kx \, dx = \frac{1}{2} kx^2 \\
= \left( \frac{kx}{2} \right) (x)
\]

Average force on spring during displacement
Displacement of spring end

V Energy budget during an earthquake  
(From Scholz, 2002)

1  $E_{\text{seismic}} + \Delta E_{\text{strain}} + \Delta E_{\text{friction}} + \Delta E_{\text{fracture}} = 0$
   - The strain energy in the earth decreases after a quake, so $\Delta E_{\text{strain}} < 0$.
   - Energy appears in the form of heat, so $\Delta E_{\text{friction}} > 0$.
   - If the energy to create fractures is assumed to be negligible*, then

2  \[ E_{\text{seismic}} \approx -\Delta E_{\text{strain}} - \Delta E_{\text{friction}} \]

3  \[ E_{\text{seismic}} \approx \frac{1}{2} [\tau_1 + \tau_2] [\Delta u_{\text{ave}}] [A] - (1/2)(2)[(\tau_2) [\Delta u_{\text{ave}}] [A] \]
   - Average shear stress during slip
   - Assumed shear stress during slip

4  \[ E_{\text{seismic}} \approx (1/2)[\tau_1 - \tau_2] [\Delta u_{\text{ave}}] [A] = (1/2)[\Delta \tau] [\Delta u_{\text{ave}}] [A] \]
   - Shear stress drop during slip

5  So the seismic kinetic energy depends on the strength change on
   the fault $\Delta \tau$
   * Substantial uncertainty

\[ 2/6/17 \quad GG303 \]
V Energy budget during an earthquake

C Formulas relating seismic energy release ($E_{\text{seismic}}$), moment ($M_o$), and moment magnitude ($M_w$)

1. $M_w \approx \frac{2}{3} \log M_o - 6.067$ \hspace{1cm} $M_o$ in Nm
2. $E_{\text{seismic}} \approx 10^{(4.8 + 1.5 M_s)}$ \hspace{1cm} $E_s$ in joules
3. $E_{\text{seismic}} \approx \frac{M_o}{20,000}$
4. $\log E_{\text{seismic}} \approx \log M_o - 4.3$
5. $\log M_o \approx 1.5 M_w + 9.1$

1. To dovetail with magnitude from surface waves
2. Calibration between magnitude and energy release done using nuclear tests

D Energy Release vs. Magnitude

The empirical relationship between energy content in radiated seismic waves and magnitude is (Richter, 1958; Bolt, 1989):

$E_s$ (joules) $= 10^{(4.8 + 1.5 M_s)}$

$M_s$ is the surface wave magnitude.

Consider two earthquakes that differ in magnitude by 1, where $M_{s1} = 1 + M_{s2}$. Then

$E_{s1}/E_{s2} = \frac{10^{(4.8 + 1.5 M_{s2})}}{10^{(4.8 + 1.5 M_{s2})}}$

$= (10^{4.8})(10^{1.5 M_{s2}})/(10^{4.8})(10^{1.5 M_{s2}})$

$= 10^{1.5} \approx 31.6$

A unit increase in magnitude corresponds to (a) a factor of 10 increase in amplitude of shaking, and (b) a factor of 31.6 increase in energy release. One magnitude 8 quake releases the energy of 1000 magnitude 6 quakes.
V Energy budget during an earthquake

Relationship between seismic energy ($M_w$) and seismic moment ($M_o$)

$$M_w = 2/3 \log M_o - 6.067, \text{ where } M_o \text{ is measured in Nm}$$

This empirical relation dovetails magnitudes from surface waves and seismic moment (i.e. $M_s = M_w$). What is the relationship between $M_o$ and $E_{\text{seismic}}$?

$$E_{\text{seismic}} = 10^{(4.8 + 1.5 \log M_o - 6.067)} \text{ where } E_s \text{ is in joules}$$

$$E_{\text{seismic}} = 10^{(4.8 + 9.1)} = 10^{9.9}$$

Alternatively, $E_{\text{seismic}} = [\Delta \tau / 2] [\Delta u_{\text{ave}}] [A]$ p. 165 of Scholz

$$M_o = [\mu] [\Delta u_{\text{ave}}] [A]$$

$E_s / M_o = [\Delta \tau / 2 \mu] p. 179 \text{ of Scholz}$

Typically $\Delta \tau = 3 \text{ MPa}$, and $2 \mu = (2)(3 \times 10^4 \text{ MPa})$, so $E_s / M_o = 1/20,000$

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VI Seismic energy equivalents

<table>
<thead>
<tr>
<th>Approximate Magnitude</th>
<th>kg TNT for Seismic Energy Yield</th>
<th>Joule equivalent</th>
<th>Example</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>0.480</td>
<td>$2.0 \times 10^5$</td>
<td>Stick of dynamite</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>480</td>
<td>$2.0 \times 10^9$</td>
<td>Oklahoma City bomb</td>
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<tr>
<td>3.87</td>
<td>$9.5 \times 10^3$</td>
<td>$40 \times 10^9$</td>
<td>Chernobyl explosion</td>
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<tr>
<td>6</td>
<td>$15 \times 10^3$</td>
<td>$63 \times 10^{12}$</td>
<td>Hiroshima bomb</td>
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<tr>
<td>6.3</td>
<td>$43 \times 10^3$</td>
<td>$180 \times 10^{12}$</td>
<td>Christchurch, 2011</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>$480 \times 10^3$</td>
<td>$2.8 \times 10^{15}$</td>
<td>Haiti, 2010</td>
<td>Java, 2009</td>
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<tr>
<td>8.35</td>
<td>$50 \times 10^6$</td>
<td>$210 \times 10^{15}$</td>
<td>Tsar Bomba</td>
<td>(H-bomb)</td>
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<tr>
<td>9</td>
<td>$480 \times 10^6$</td>
<td>$2.0 \times 10^{18}$</td>
<td>Japan, 2011</td>
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<tr>
<td>9.2</td>
<td>$950 \times 10^6$</td>
<td>$4.0 \times 10^{18}$</td>
<td>Alaska, 1964</td>
<td>Sumatra, 2004</td>
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<td>9.5</td>
<td>$2.7 \times 10^9$</td>
<td>$11 \times 10^{21}$</td>
<td>Chile, 1960</td>
<td></td>
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<tr>
<td>13</td>
<td>$100 \times 10^{12}$</td>
<td>$420 \times 10^{24}$</td>
<td>Chicxulub impact</td>
<td></td>
</tr>
</tbody>
</table>

http://en.wikipedia.org/wiki/Richter_magnitude_scale