EFFECTIVE STRESS AND MOHR-COULOMB FAILURE (26)

I Main Topics

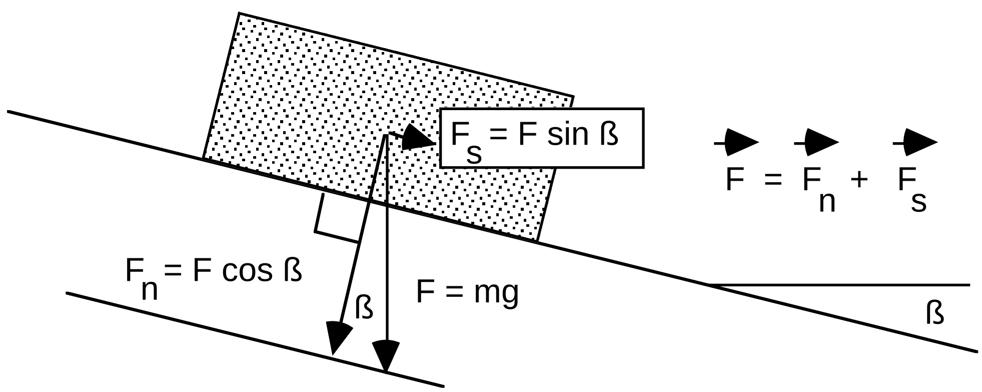
A Driving and resisting stresses at the base of an inclined block

B Factor of safety

C Effective stress

D Mohr-Coulomb failure

II Driving and resisting stresses at the based of a dry inclined block



A Driving stress: $\sigma_s = F_s/A$

Slope-parallel component of block weight / block area

B Resisting stress

1 Shear strength = Shear strength of slip surface under no normal stress + frictional resistance (which depends on F_n!)

2 $\tau = c + (F_n \tan \phi)/A$

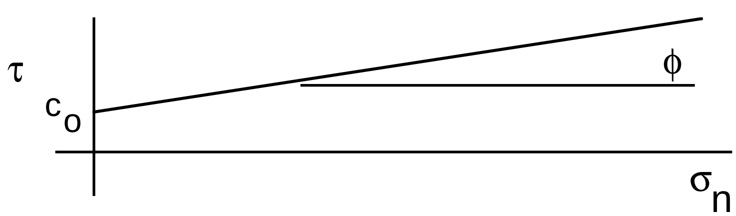
a c = cohesion

i Tape sticks to a vertical wall because of cohesion

ii Dry sand has virtually no cohesion

b $\tan \phi$ = coefficient of friction = $\Delta\tau / \Delta\sigma_n$

c ϕ = angle of internal friction



Shear strength at the base of sliding block as a function of normal stress **(Compression taken as positive here!)**

III Factor of safety (F.S.)

1 $F.S. = \frac{F_{resisting}}{F_{driving}} = \frac{F_{resisting}/A}{F_{driving}/A} = \frac{\sigma_{resisting}}{\sigma_{driving}}$

2 If resisting forces (stress) < driving forces (stresses) $\Rightarrow$ failure

3 If factor of safety < 1 $\Rightarrow$ failure

4 For dry cohesionless (sandy) soils

Factor of safety = $\{(F \cos ß)(\tan \phi)\}/(F \sin ß)$

= $\tan \phi / \tan ß$

Factor of safety < 1 if ϕ < ß (i.e. if angle of internal friction < slope)

IV Effective stress

A For soils: $\sigma' = \sigma_{total} - P$

1 σ' = Effective stress = load supported by solid framework

2 σ_{total} = total normal stress porous material is subjected to

3 P = pore pressure of pore fluid = load supported by fluid

B General case (For soil and rock): $\sigma' = \sigma_{total} - \alpha P$

(Nur and Byerlee, 1971, JGR, v. 76, p. 6414-6419)

1 $\alpha = 1 - (K_{agg}/K_s)$

a K = Bulk modulus = 1/B = 1/compressibility = Δpressure/(ΔV/V)

b Low bulk modulus = high compressibility; low K = high B

c K_{agg} = bulk modulus of dry aggregate;

d K_s = bulk modulus of the solid component of the aggregate

2 $\alpha = 1 - (B_s/B_{agg})$

3 For soils, $K_{agg} << K_s$ (or $B_{agg} >> B_s$) so $\alpha = 1$

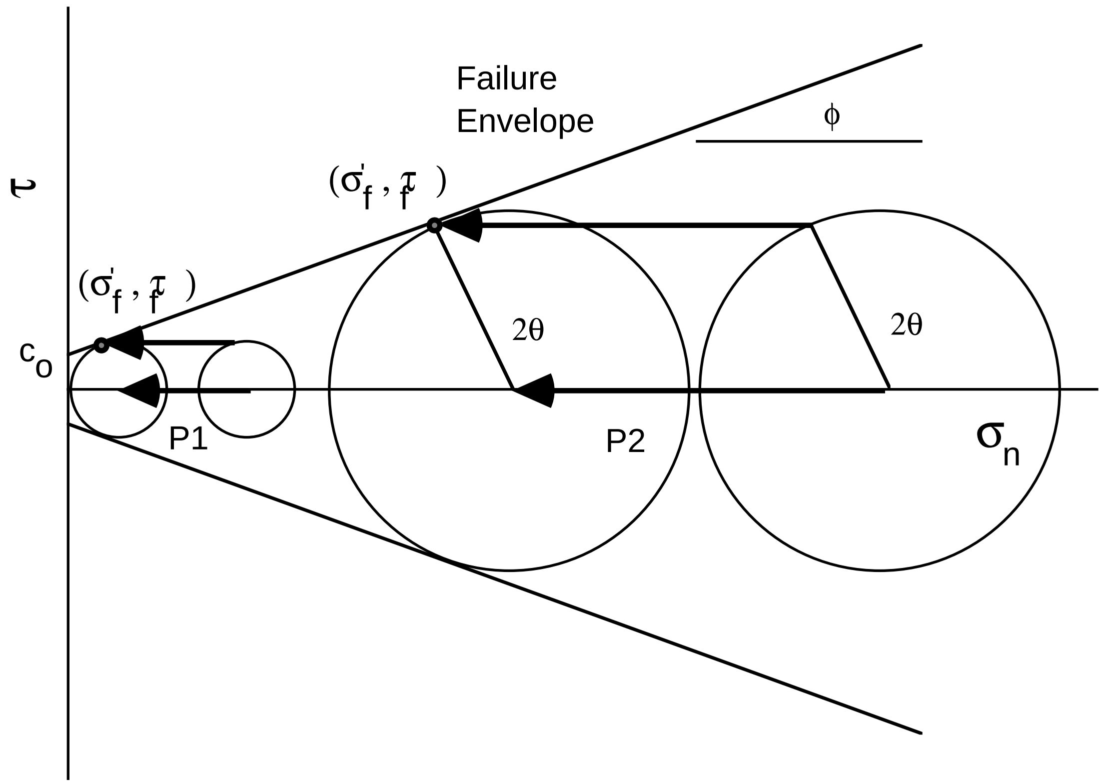
4 For some rocks $\alpha \neq 1$ $(0 < \alpha < 1)$

C Pore pressure does not "lubricate" the failure surface; the pore pressures acts to float the material overlying the failure surface

D Until the pore fluid has time to flow, increased loads on a saturated "soil" are supported by an increase in the pore pressure.

V Mohr-Coulomb failure

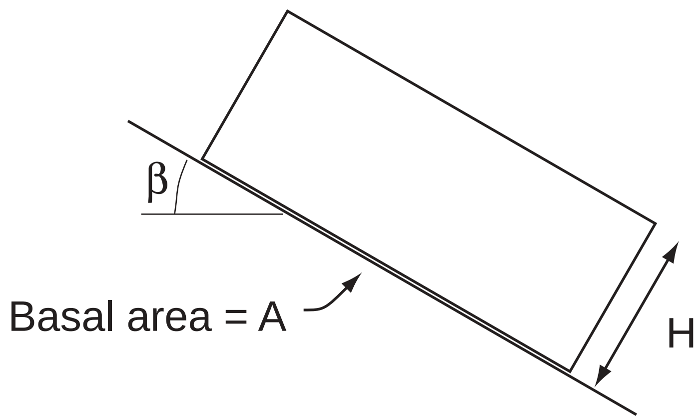
A $\tau_{failure} = c + \sigma'_{failure} \tan\phi$, where σ' = effective normal stress



$(\phi + 90°)/2 = \theta$ If $\phi = 30°$, then $\theta = 60°$

B For soils: $\tau_{failure} = c + (\sigma_n - P) \tan\phi$

C Factor of safety for a tilted block accounting for pore pressure



$$F.S. = \frac{F_{resisting}}{F_{driving}} = \frac{F_{resisting}/A}{F_{driving}/A} = \frac{\sigma_{resisting}}{\sigma_{driving}}$$

$$F.S. = \frac{c + (\sigma_n - P)\tan\phi}{\tau}$$

$$\sigma_n = \left((m)g/A\right)\cos\beta = \left((\rho HA)g/A\right)\cos\beta = \rho g H \cos\beta$$ (Here, compression is positive)

$$\tau = \left((m)g/A\right)\sin\beta = \left((\rho HA)g/A\right)\sin\beta = \rho g H \sin\beta$$

$$F.S. = \frac{c + (\rho g H \cos\beta - P)\tan\phi}{\rho g H \sin\beta}$$

Factors That Increase Shear Driving Stress

Removal of Support

- Erosion of toe areas by streams, surf; lowering of reservoirs
- Holocene oversteepening of slopes by glacial erosion
- Weathering promotes erosion
- Mass wasting of supporting lower slope
- Quarries, canals, road cuts, tunnels

Surcharge

- Weight of precipitation
- Accumulation of talus
- Construction of fill
- Stockpiles of ore, waste
- Weight of buildings
- Weight of water from leaking pipes, lawn watering, pools, etc.

Transitory Earth Stresses

- Earthquakes
- Vibrations from machinery, traffic, blasting, etc.

Volcanic Processes

- Inflation of magma chambers (Mt. St. Helens)
- Harmonic tremors, earthquakes

Increase in Lateral Pressure

- Water in cracks
- Swelling (ice, clays)

Factors That Contribute To Low or Reduced Shear Strength

- Inherently weak geologic materials (clay, shale, organic material, schists, soft tuffs, talc, serpentine
- Discontinuities (Bedding planes, faults, joints)
- Strata inclined towards free face
- Dip slopes
- Weathering of rocks
- Chemical exchange or leaching (e.g. quick clays)
- Hydration of clay minerals
- Dissolution of cement
- Shrink/swell effects

INCREASED PORE PRESSURE

- Rain, reservoir level rise, alteration of drainage

Effective Stress Example

Suppose a block of impermeable, zero porosity quartz of density 2.67 g/cm^3 rests on a horizontal surface. What is the total normal stress and effective stress at the base of a 10m tall block of pure quartz?

$\sigma_n = (\rho_{quartz})(g)(h)$

$= (2.67 \times 10^3 \ kg/m^3)(9.81 \ m/sec^2)(10m)$

$= 2.62 \times 10^5 \ kgm/sec^2/m^2 = 2.62 \times 10^5$ Pa

$\sigma_n' = \sigma_n$ (no pore pressure at base)

Now suppose a dry block of quartz sandstone rests on a horizontal surface. Further assume that the porosity of the sandstone is 20%. What is the total normal stress at the base of a 10m tall block of this sandstone ?

$\sigma_n' = \{\rho_{bulk}\}(g)(h)$

$= \{(2.67 \times 10^3 \ kg/m^3)(0.80) + (0 \ kg/m^3)(9.81 \ m/sec^2)(10m)(0.20)$

$= (2.14 \times 10^3 \ kg/m^3)(9.81 \ m/sec^2)(10m)$

$= 2.10 \times 10^5 \ kgm/sec^2/m^2 = 2.10 \times 10^5$ Pa (i.e., 80% of first case)

Now suppose a saturated block of quartz sandstone rests on a horizontal surface. Further assume that the porosity of the sandstone is 20%. What is the total normal stress and total effective stress at the base of a 10m tall block of this sandstone?

$\sigma_n = (\rho_{sat})(g)(h)$

$= \{(2.67 \times 10^3 \ kg/m^3)(0.80) + (10^3 \ kg/m^3)(9.81 \ m/sec^2)(10m)(0.20)$

$= (2.34 \times 10^3 \ kg/m^3)(9.81 \ m/sec^2)(10m)$

$= 2.30 \times 10^5 \ kgm/sec^2/m^2 = 2.30 \times 10^5$ Pa

NOTE: $\rho_{sat} \neq \rho_{sat} + \rho_{water}$!! $(2.30 \neq 2.13 + 1.00)$

So the total stress is slightly greater for the saturated block because the saturated block is denser than the dry block. But the pore pressure increase at the base of the block more than compensates for the total stress increase resulting from the higher bulk density:

$\sigma_n' = \sigma_n - P = (\rho_{bulk})(g)(h) - (\rho_{water})(g)(h)$

$= 2.30 \times 10^5 \text{ Pa} - (10^3 \ kg/m^3)(9.81 \ m/sec^2)(10m)$

$= 2.30 \times 10^5 \text{ Pa} - 0.98 \times 10^5 \text{ Pa} = 1.32 \times 10^5$ Pa

So saturating the block increases the total stress but decreases the effective stress.