RECURRENCE INTERVALS AND PROBABILITY (18)

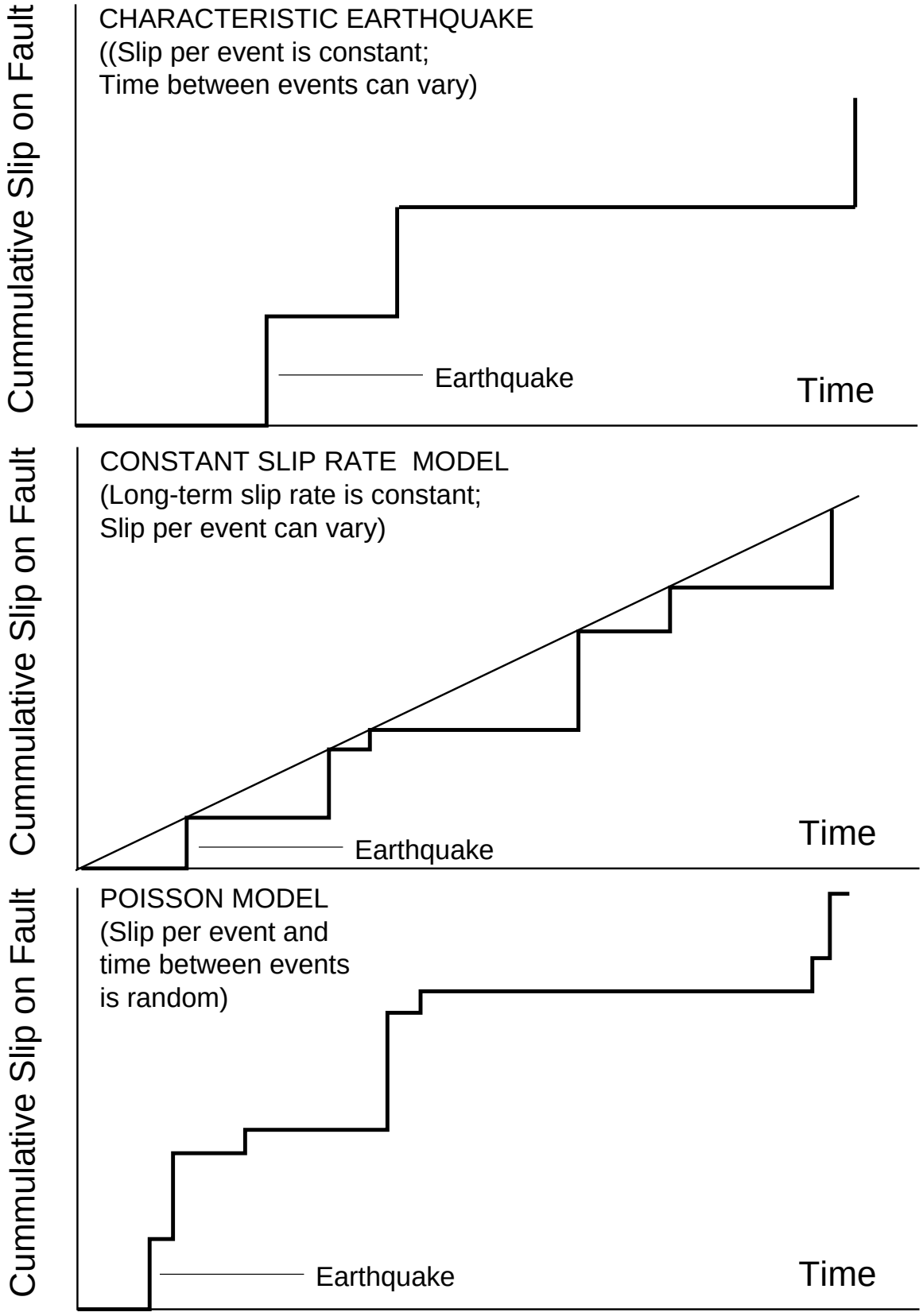
I Main Topics
- A Recurrence intervals
- B Simple empirical earthquake recurrence models
- C Seismic gaps
- D Probability distributions
  - http://www.seismo.berkeley.edu/seismo/hayward/probabilities_new.html
  - http://quake.usgs.gov/prepare/ncep/
- E Exercise on probability of "The Big One" in So. Cal. in next 30 years
- F Recognition, Characterization, Risk Evaluation, Risk Assessment

II Recurrence interval:
- A Used to evaluate <u>when</u> an earthquake is likely to occur
- B Recurrence interval = time between consecutive earthquakes (usually with reference to earthquakes of a given magnitude)
- C Can be determined by geologic means
  - 1 Dating of individual events (e.g. data from trench study)
  - 2 Average recurrence int. = Average slip per event/average slip rate

III Simple empirical earthquake recurrence models
- A Characteristic Earthquake Model
  - 1 Same rupture length and slip distribution (and seismic moment)
  - 2 Recurrence interval can vary through time
- B Constant slip rate (time-predictable) Earthquake Model
  - 1 Slip rate across fault is constant
  - 2 Recurrence interval depends on slip during earthquake
- C Random (Poisson) Model
  - 1 Historical record too short to separate any patterns from "noise"
  - 2 Earthquakes might best be considered as random events in time
- D Problems with resolving dates of events

IV Seismic gaps
- A Used to evaluate <u>where</u> an earthquake is likely to occur
- B Along an active fault, the probability of an earthquake occurring is ~highest where the most time has elapsed since the last rupture

COMPARISON OF THREE EARTHQUAKE MODELS

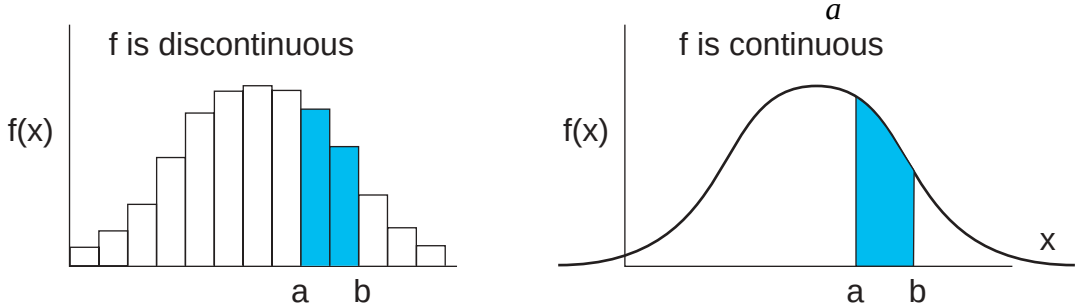


V Probability distributions

A Probability density functions [PDF = f(x)]: general comments

1 Probability (a<X<b) = probability of an outcome between a and b

= P(a<X<b) = area under f(x) from a to b = $\int_a^b f(x)\ dx$



a Example 1: Probability of Micheal Jordan scoring 25-35 points

b Example 2: Probability of quake (M_W = 7.5) in next 25-35 years

2 (P(-∞<X<∞) = area under f(x) from -∞ to ∞ = $\int_{-\infty}^{\infty} f(x)\ dx$ = 1 =100%

3 For <u>continuous</u> distributions, P(x=a) = area under f(x) from a to a

$= \int_a^a f(x)\ dx = \underline{0}$

B The normal distribution ("The bell-shaped curve): one kind of PDF

1 Described by mean μ and standard deviation σ

$$\mu = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i} \qquad \sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n-1}}$$

2 P(μ–σ<x<μ+σ) ≈ 2/3; P(μ–2σ<x<μ+2σ) ≈95% P(μ–3σ<x<μ+3σ) ≈99%

VI Exercise on probability of "The Big One" in So. Cal. in next 30 years

VII Recognition, Characterization, Risk Evaluation, Risk Assessment

A Probabilistic assessment allows the likelihood of given effects (e.g. intensities), and hence potential damages, to be estimated for a given area for a given time frame. **This is what is meant by evaluating the level of risk.**

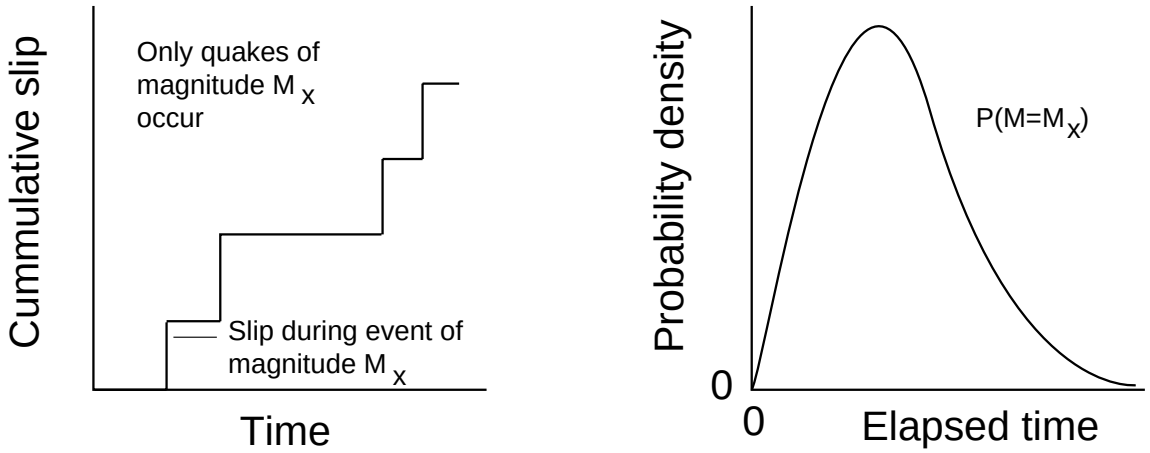
B Steps 1 and 2 must be done in order to get to step 3 (and then 4)

C Outcome probabilities are sensitive to the model one chooses

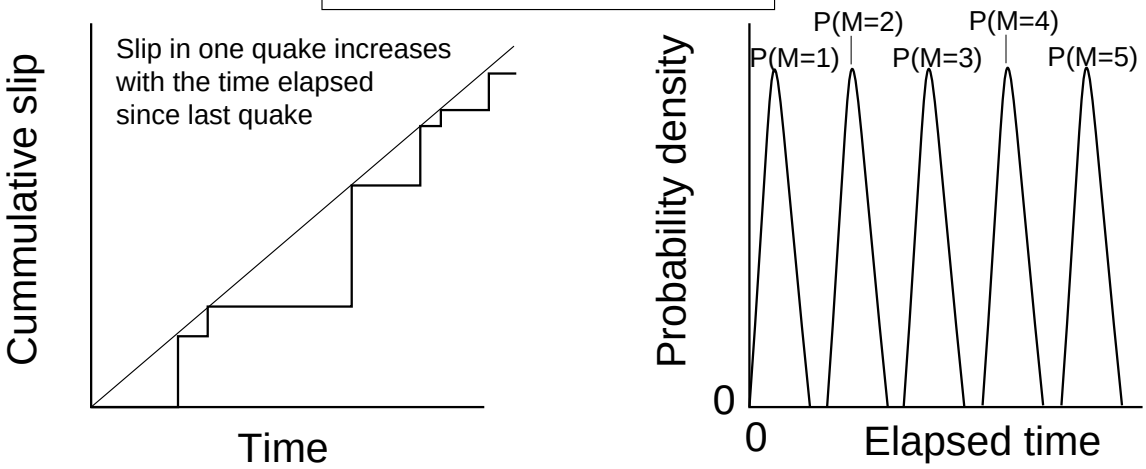
D This approach can be (and has been) applied to many phenomena

Probability distribution curves for three earthquake models

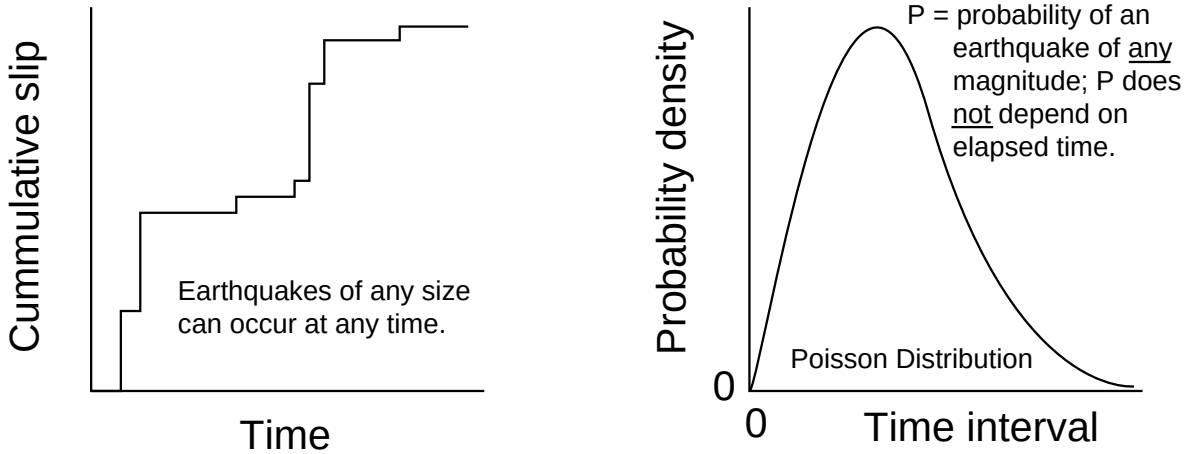
Characteristic Earthquake model



Time-predictable model



Random model



Method for Predicting the Annual Likelihood of "The Big One" at Pallet Creek

Kerry Sieh, a professor at Cal Tech, has done more than any other single person to document the hazard presented by recurring large earthquakes on the San Andreas fault in Southern California. We will use some of Kerry's results to estimate the probability of a large earthquake on the San Andreas fault in Southern California.

Here are Kerry's estimates (from his 1984 JGR paper) on the time of the last 12 large earthquakes at Pallet Creek (the uncertainties associated with these events are dropped):

1857, 1720, 1550, 1350, 1080, 1015, 935, 845, 735, 590, 350, 260

1 Based on the time between the oldest event listed above and the 1857 quake, calculate the average (mean) recurrence interval for large earthquakes at Pallet Creek.

Mean Recurrence Interval = (1857-260)years/11 intervals = 1597 yr/11 = 145 years

2 The earthquakes are not occurring at a perfectly regular pace. Calculate the recurrence times between each successive pair of earthquakes.

137, 170, 200, 270, 65, 80, 90, 110, 145, 240, 90

3 Calculate the standard deviation of the 11 recurrence intervals associated with the 12 quakes. The equation to use is:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (R_i - R^*)^2}{n-1}}$$

where σ is the standard deviation, R_i is the recurrence time between a given pair of events, R* is the mean recurrence interval, and n is the number of recurrence intervals (<u>not</u> the # of quakes!).

68 years

4a Assuming the year is 1993, how many years have elapsed since the last large San Andreas earthquake in southern California?

1993-1857 = 136 years

4b How many years shy of the mean recurrence interval would we be?

145-136 = 9 years

4c How many standard deviations shy of the mean recurrence interval would we be?

9 years/68 years = 0.13 standard deviations

5 We will now suppose the distribution of recurrence intervals is normally distributed about the mean recurrence interval. On the supplied paper, plot the equation

$$f(t)=\frac{1}{\sigma\sqrt{2\pi}}\exp\frac{-(t-t^*)^2}{2\sigma^2}$$

where f(t) is the normal distribution, t is time, t* is the mean, and σ is the standard deviation.

Plot this for $0 \le t \le 250$ years.

6 What is the probability of an earthquake on the San Andreas fault at Pallet Creek in the next 30 years from 1993 given our model? This probability is the area under the curve from 1993 to 30 years hence divided by the area from 1993 to infinity.

Suppose the year is 1993 - 9 years (0.13 standard deviations) <u>shy</u> of the mean recurrence interval. In 30 years we would be 21 years (or 21/68 = 0.31 standard deviations) <u>past</u> the mean recurrence interval. The area under the probability density curve from the mean to 0.13 standard deviations shy of the mean is 0.0517. The area under the probability density curve from the mean to 0.31 standard deviations past the mean is 0.1217. The area under the probability density curve from 0.13 standard deviations shy of the mean to ∞ is 0.5 + 0.0517. So:

P = (0.0517 + 0.1217) / (0.5 + 0.0517) = 0.1734/0.5517 = 31%

Even though Kerry doesn't think he missed a quake, <u>suppose</u> there were circumstantial evidence (e.g. Indian legends) for one large quake in the year 490 and another in 1215.

7 What would the new mean recurrence interval and standard deviation be?

New mean recurrence interval = 1597 years/13 intervals = 122.8 years =123 years
New standard deviation = 38 years (larger % change in standard deviation than in mean!)

8 Assuming the year is 1993, what would the recalculated probability be for a large quake at Pallet Creek in the next 30 years?

The year 1993 would be 13 years (or 13/38 = 0.34 standard deviations) past the mean recurrence interval. In 30 years we would be 43 years (or 43/38 = 1.13 standard deviations) past the mean recurrence interval. The area under the probability density curve from the mean to 0.34 standard deviations from the mean is 0.1331. The area under the probability density curve from the mean to 1.13 standard deviations past the mean is 0.3708. The area under the probability density curve from 0.34 standard deviations past the mean to ∞ is (0.5 - 0.1331). So:

P = (0.3708 - 0.1331) / (0.5 - 0.1331) = 0.2377/0.3669 = 65%

Now suppose we consider the earthquakes to be distributed randomly (i.e. they are characterized by a Poisson distribution). Then the probability of an earthquake occurring does not depend on how much time has elapsed since the last earthquake. The probability of "x" number of earthquakes occurring in a given interval of time t is given by:

$$P(x) = \frac{(vt)^x \; e^{-vt}}{x!}$$

where "v" is the average rate of occurrence. So if the average recurrence interval is 145 years, the probability of getting 1 event in 145 years is:

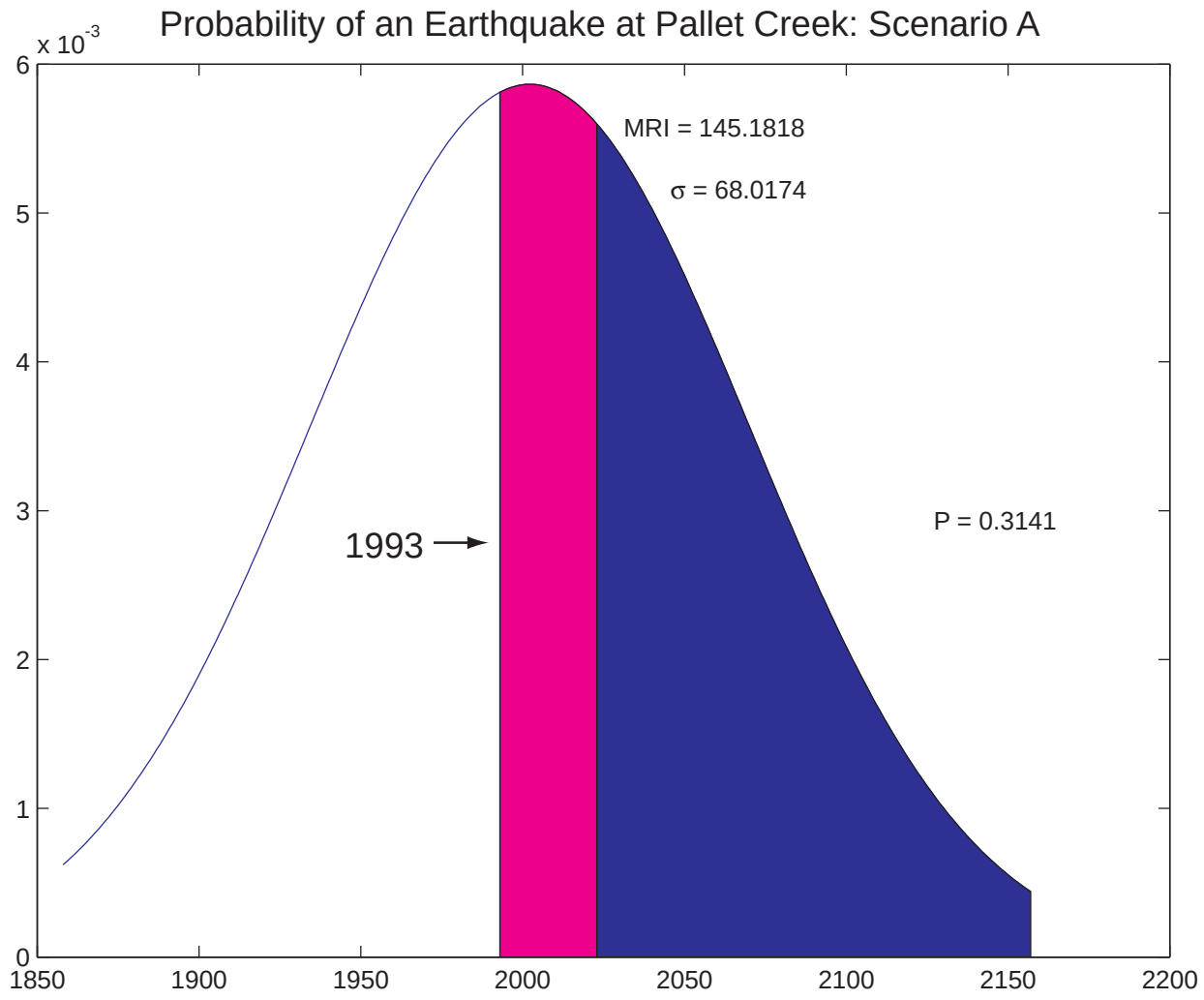
$$P(1) = \frac{(\frac{1\ event}{145\ yrs}145\ yrs)^1 \; e^{-(\frac{1\ event}{145\ yrs}145\ yrs)}}{1!} = e^{-1} = 37\%$$

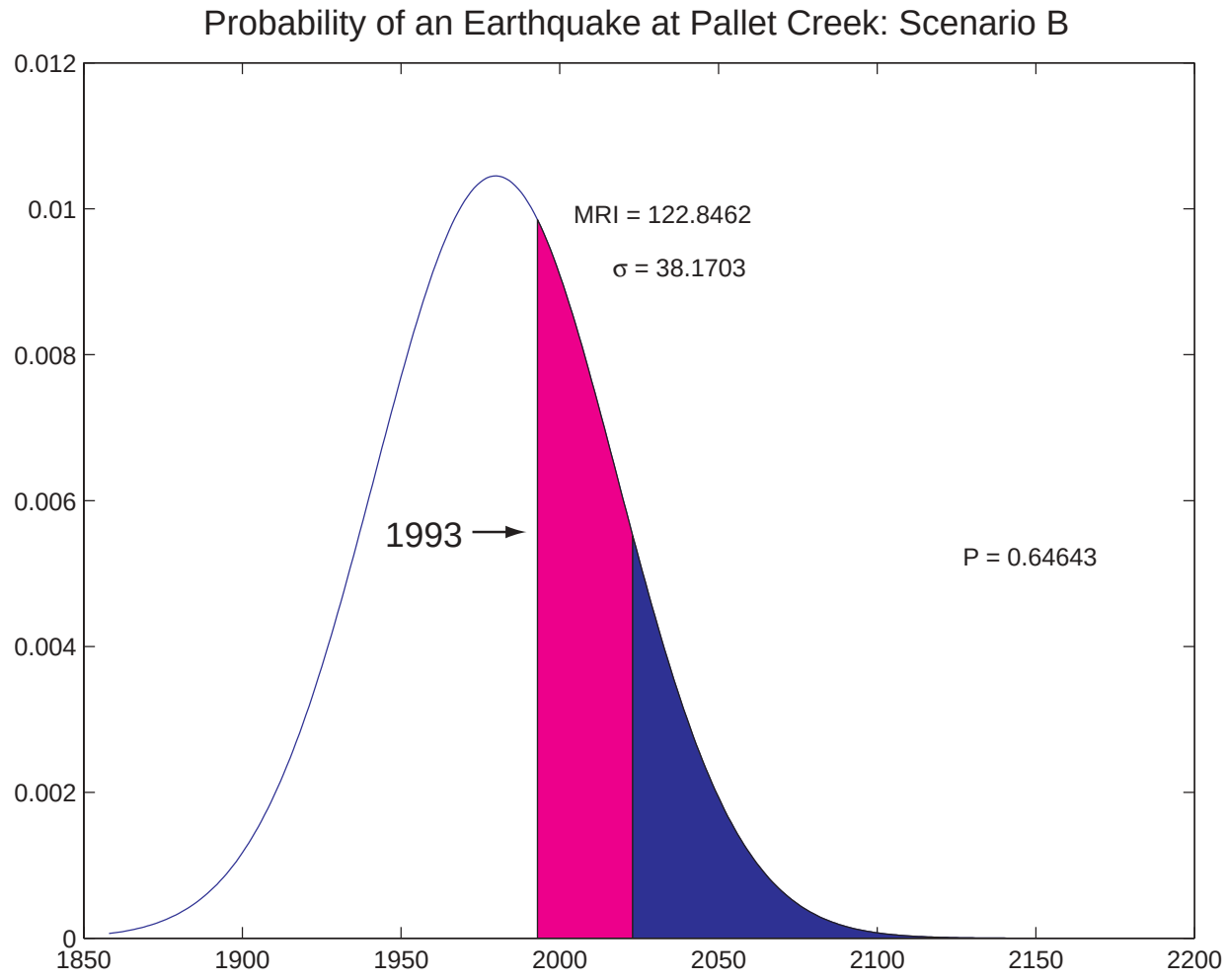
The probability of getting one event in 30 years is:

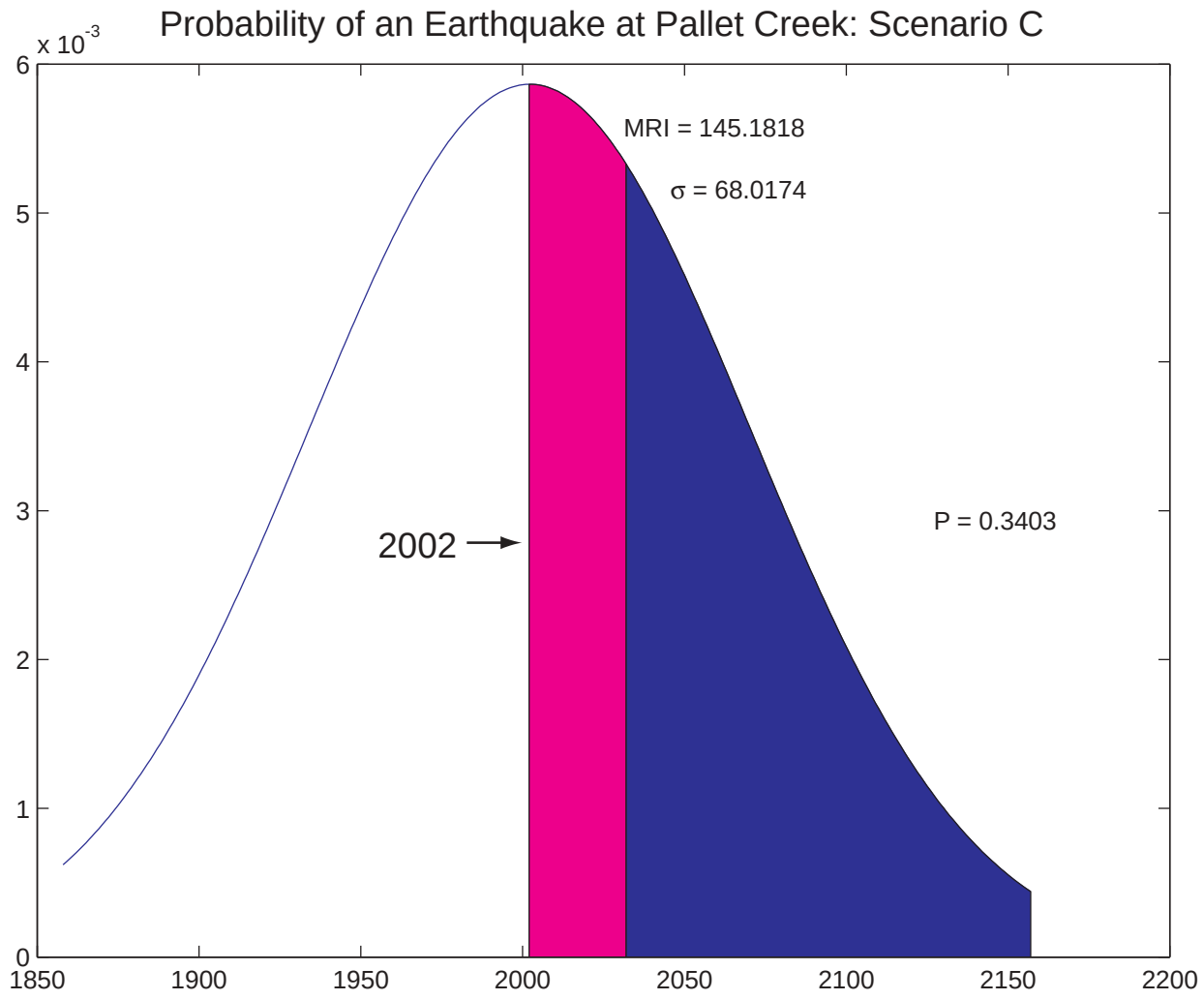
$$P(1) = \frac{(\frac{1\ event}{145\ yrs}30\ yrs)^1 \; e^{-(\frac{1\ event}{145\ yrs}30\ yrs)}}{1!} = (30/145)(e^{-30/145}) = 17\%$$

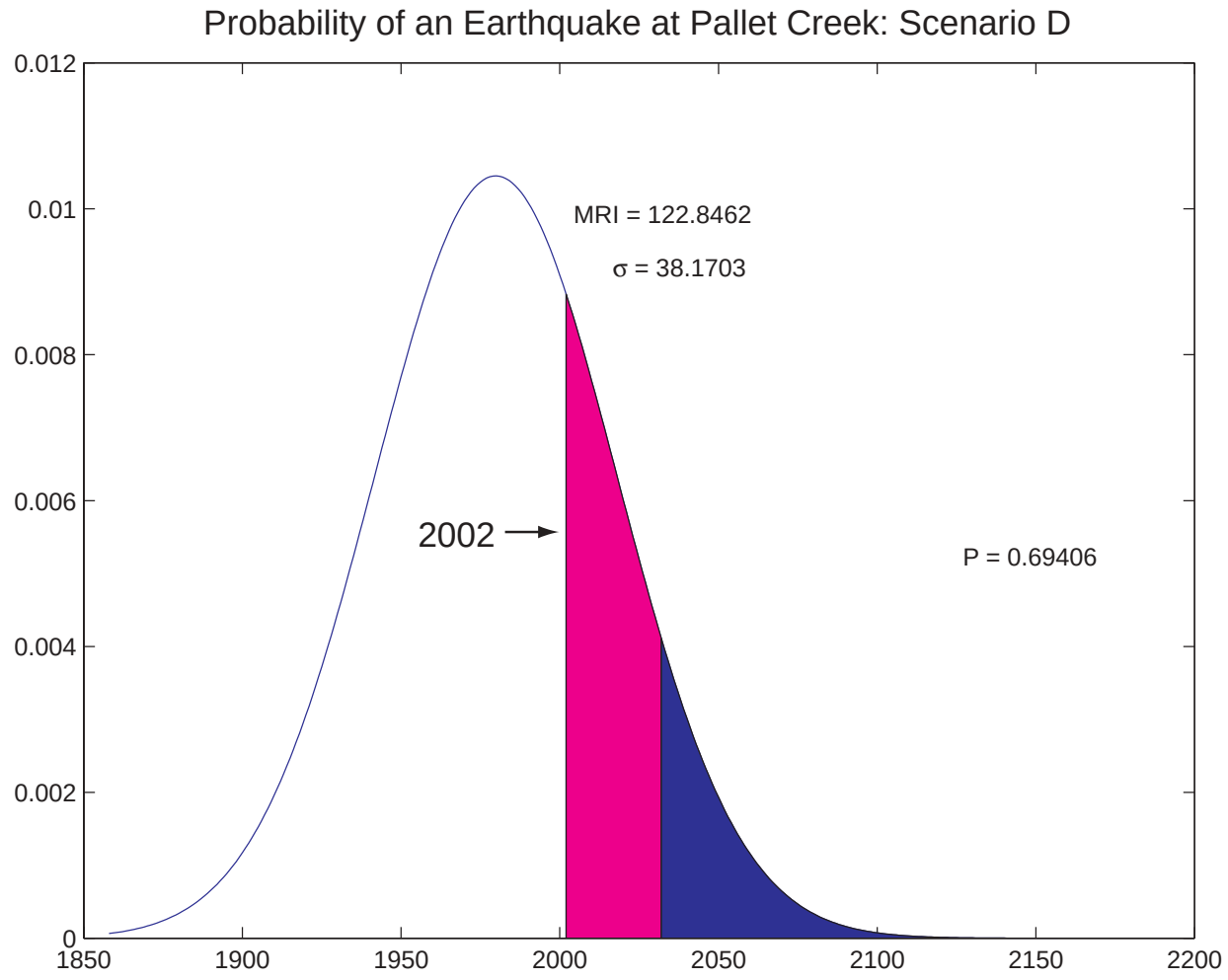
The probability of getting no event in 30 years is:

$$P(1) = \frac{(\frac{1\ event}{145\ yrs}30\ yrs)^0 \; e^{-(\frac{1\ event}{145\ yrs}30\ yrs)}}{0!} = (e^{-30/145}) = 81\%$$


Probability of an Earthquake at Pallet Creek: Scenario A
x 10^-3
6
5
4
3
2
1
0
1850
1900
1950
2000
2050
2100
2150
2200
MRI = 145.1818
σ = 68.0174
1993
P = 0.3141


Probability of an Earthquake at Pallet Creek: Scenario B
0.012
0.01
0.008
0.006
0.004
0.002
0
1850
1900
1950
2000
2050
2100
2150
2200
MRI = 122.8462
σ = 38.1703
1993
P = 0.64643


Probability of an Earthquake at Pallet Creek: Scenario C
x 10^-3
6
5
4
3
2
1
0
1850
1900
1950
2000
2050
2100
2150
2200
MRI = 145.1818
σ = 68.0174
P = 0.3403
2002


Probability of an Earthquake at Pallet Creek: Scenario D
0.012
0.01
0.008
0.006
0.004
0.002
0
1850
1900
1950
2000
2050
2100
2150
2200
MRI = 122.8462
σ = 38.1703
2002
P = 0.69406