

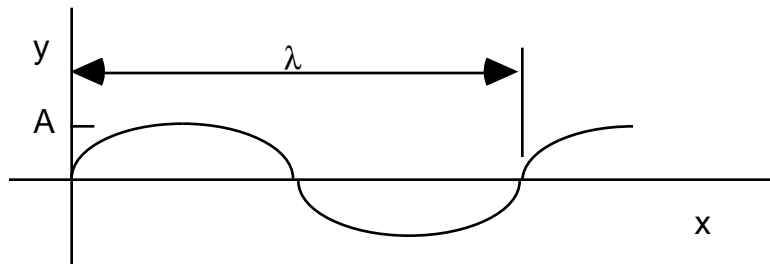
RESPONSE OF STRUCTURES (16)

I Main Topics

- A Acceleration, velocity and displacement spectra
- B Resonance and natural frequencies
- C Response of structures

II Displacement, velocity, and acceleration spectra

- A Spectra represent parameters as a function of wave frequency (or period), not time, and reveal the most energetic/forceful waves
- B Examples of shaking vs. time and shaking vs. frequency plots
- C Displacement, velocity, and acceleration spectra



A = amplitude

Displacement as a function of position along a wave

(1) $y(x,t) = A \sin \{(2\pi/\lambda)(x + vt)\}$ where $v = \lambda/T = f\lambda$, so $\lambda = v/f$

(2) $y(x,t) = A \sin \{(2\pi f/v)(x + vt)\}$

Consider the displacement at a fixed point on the ground (e.g. $x = 0$)

(3) $y(t) = A \sin \{(2\pi f/v)(vt)\}$ or $y = A \sin (2\pi f t)$

Displacement given as a function of the frequency and wave amplitude

Let $\omega = 2\pi f =$ angular frequency

(4) $y = [A] \sin (\omega t)$ The term in square brackets gives y_{max}

The shaking velocity of the ground (not the velocity of the wave) = dy/dt

(5) $y' = d(A \sin (\omega t))/dt = [\omega A] \cos (\omega t)$

The acceleration of the ground = dy'/dt

(6) $y'' = d(\omega A \cos (\omega t))/dt = -[\omega^2 A] \sin (\omega t) = -\omega^2 y$

Note that $(|y''/y|)^{1/2} = \omega$. For a given frequency $(|y''/y|)^{1/2} = \text{constant}$.

Velocity and Acceleration vs. Frequency (Displacement is constant)

Frequency (f) Hz	0.01*	0.1*	1 *	10*	100
(ω) Hz	0.06	0.6	6	63	628
Maximum Displacement (y) (m)	10^{-3}	10^{-3}	10^{-3}	10^{-3}	10^{-3}
Maximum Velocity y' (= ωy) (m/sec)	6×10^{-5}	6×10^{-4}	6×10^{-3}	6×10^{-2}	6×10^{-1}
Max. Acceleration y'' (= $ \omega^2 y $) (m/sec ²)	39×10^{-5}	39×10^{-4}	39×10^{-3}	39×10^{-2}	39×10^{-1}

*Significant frequency for design of large engineering structures

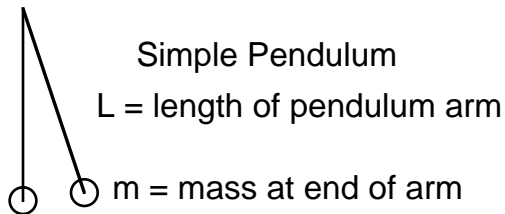
C Max. accelerations (forces) commonly at 2-10 Hz ($T=0.1-0.5$ sec), max. velocities (kinetic energy) at 0.5-2 Hz ($T=0.5-2$ sec), and max. displacements at 0.006-0.5 Hz ($T=2-160$ sec)

- 1 High frequency (small period) waves: high amplitudes of acceleration, small amplitudes of displacement
- 2 Low frequency (long period) waves: low amplitudes of acceleration, large amplitudes of displacement

D Avoid structure designs that are sensitive to wavelengths where seismic forces/energy is concentrated (i.e. avoid structure designs with natural periods that match the natural period of the underlying materials [or the source]).

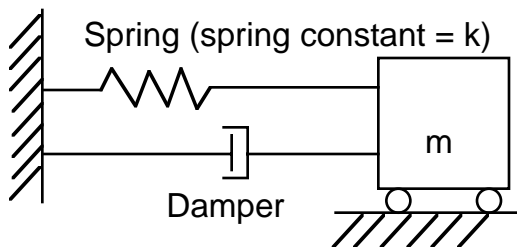
III Resonance and natural frequencies

- A Resonance: vibration of large amplitude due to arrival of energy at a particular frequency
- B Natural frequency: The frequency at which a structure will resonate
- C Natural frequency of a pendulum



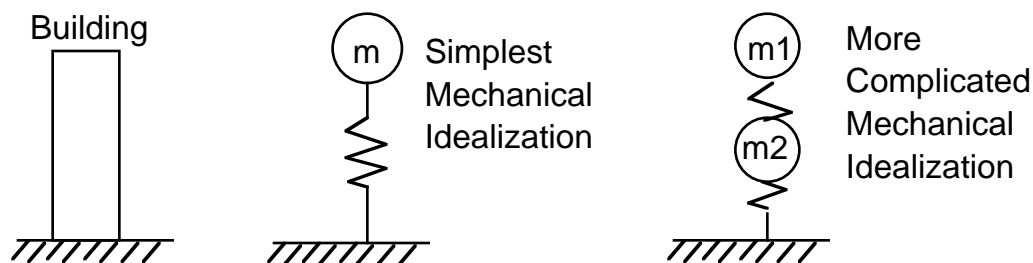
- 1 Natural period: $T = 2\pi(L/g)^{1/2}$ (Can use a pendulum to measure g!)
- 2 Natural frequency: $f = 1/T = (g/L)^{1/2}/(2\pi)$
- 3 Natural angular frequency: $\omega = 2\pi f = (L/g)^{1/2}$

- D Natural frequency of a mass on a spring (simple harmonic oscillator)



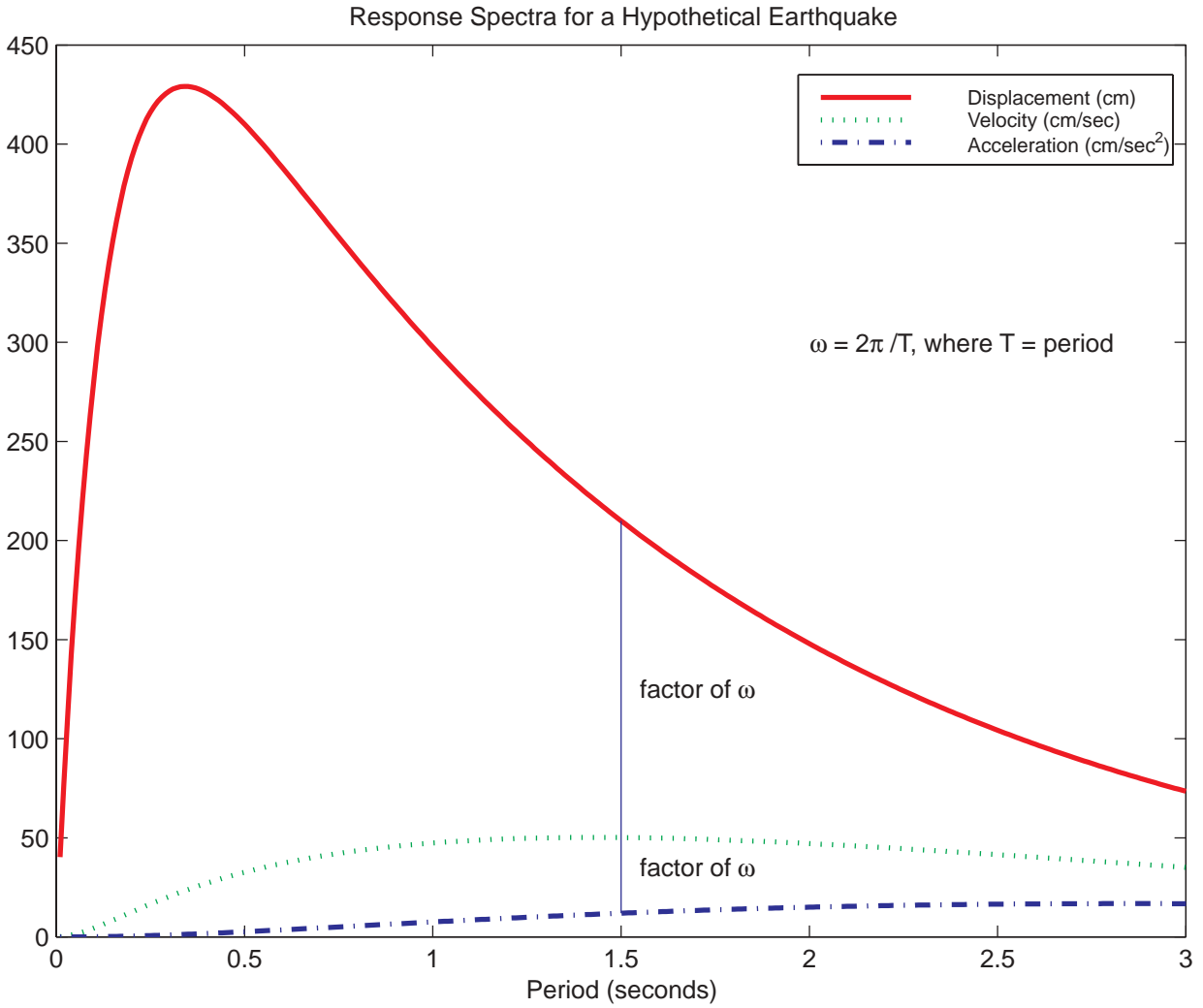
- 1 Natural period: $T = 2\pi(m/k)^{1/2}$
- 2 Natural frequency: $f = 1/T = (k/m)^{1/2}/(2\pi)$
- 3 Natural angular frequency: $\omega = 2\pi f = (k/m)^{1/2}$

IV Response of structures

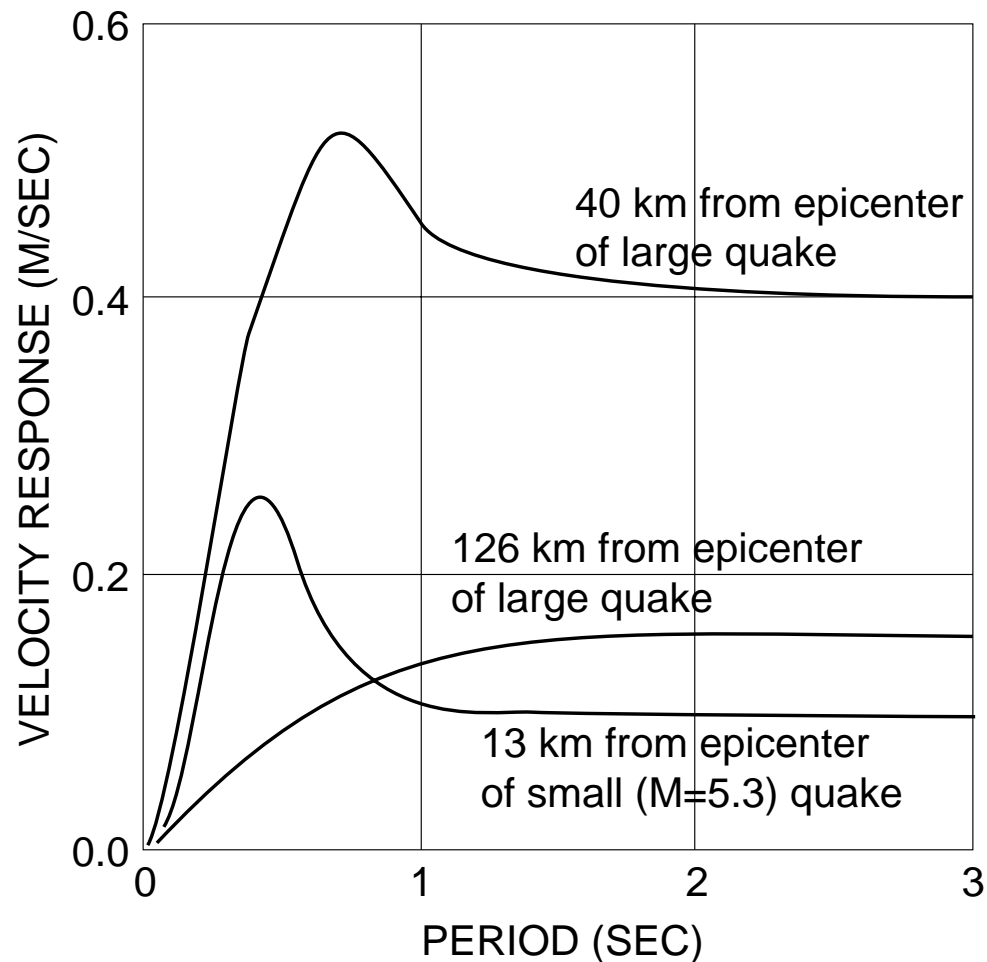


- A To first-order, a structure can be idealized as one mass on a spring
- B Natural period: $T = 2\pi(m/k)^{1/2}$, where $m = \text{mass}$ and $k = \text{stiffness}$
- C $k = f(\text{building dimension, shape, elasticity})$
- D Rule of thumb: $T (\text{seconds}) \approx \# \text{ of stories}/10$ _

- E Example: In Mexico City, the high rise buildings that were the most damaged were 10-30 story (100'-300' tall) high-rise buildings. These were calculated to have resonant period of 1-3 sec.
- F Real buildings are sensitive to several different wave frequencies, but will tend to be most sensitive to one frequency



IDEALIZED UNDAMPED VELOCITY SPECTRUM CURVES



GROUND RESPONSE = $f(\text{SOURCE, PATH})$

PATH = DISTANCE, GEOLOGY (ESPECIALLY NEAR-SURFACE GEOLOGY)

Source and distance effects highlighted here

A SMALL, NEARBY EARTHQUAKE MAY AFFECT SHORT-PERIOD STRUCTURES MORE THAN A LARGE, DISTANT QUAKE

A LARGE, DISTANT EARTHQUAKE MAY AFFECT LONG-PERIOD STRUCTURES MORE THAN A SMALL, NEARBY QUAKE

From Housner, 1970, in Wiegel, 1970)