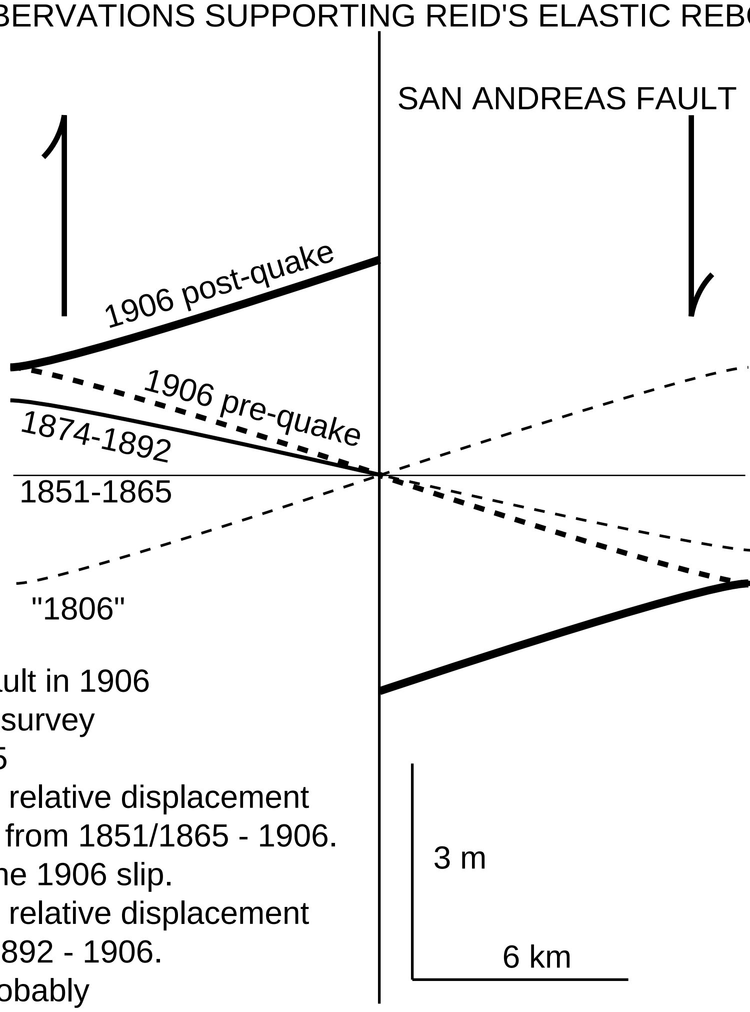
CHARACTERIZING EARTHQUAKE SOURCES (13)

I Main Topics
- A Elastic rebound theory
- B Slip on a fault with a uniform stress drop
- C Seismic moment
- D Energy release during an earthquake

II Elastic rebound theory (H.F. Reid, 1908, v. 2 of 1906 Earthquake report)
- A Founded by comparing pre- and post-quake survey lines across SAF
- B Seismic energy source: elastic potential energy of rock around fault

GEODETIC OBERVATIONS SUPPORTING REID'S ELASTIC REBOUND THEORY



1 6m slip on fault in 1906
2 Straight-line survey in 1851-1865
3 ~3m far-field relative displacement in ~50 years from 1851/1865 - 1906. This is half the 1906 slip.
4 ~2m far-field relative displacement from 1874/1892 - 1906.
5 3m of slip probably accumulated before 1851/1865.
6 Last quake 100 (= 50+50) years ago?

III Slip (Δu) on a fault with a uniform shear stress drop ($\Delta\tau = \tau_1 - \tau_2$)

A Rock is elastic, homogenous, isotropic, isothermal material

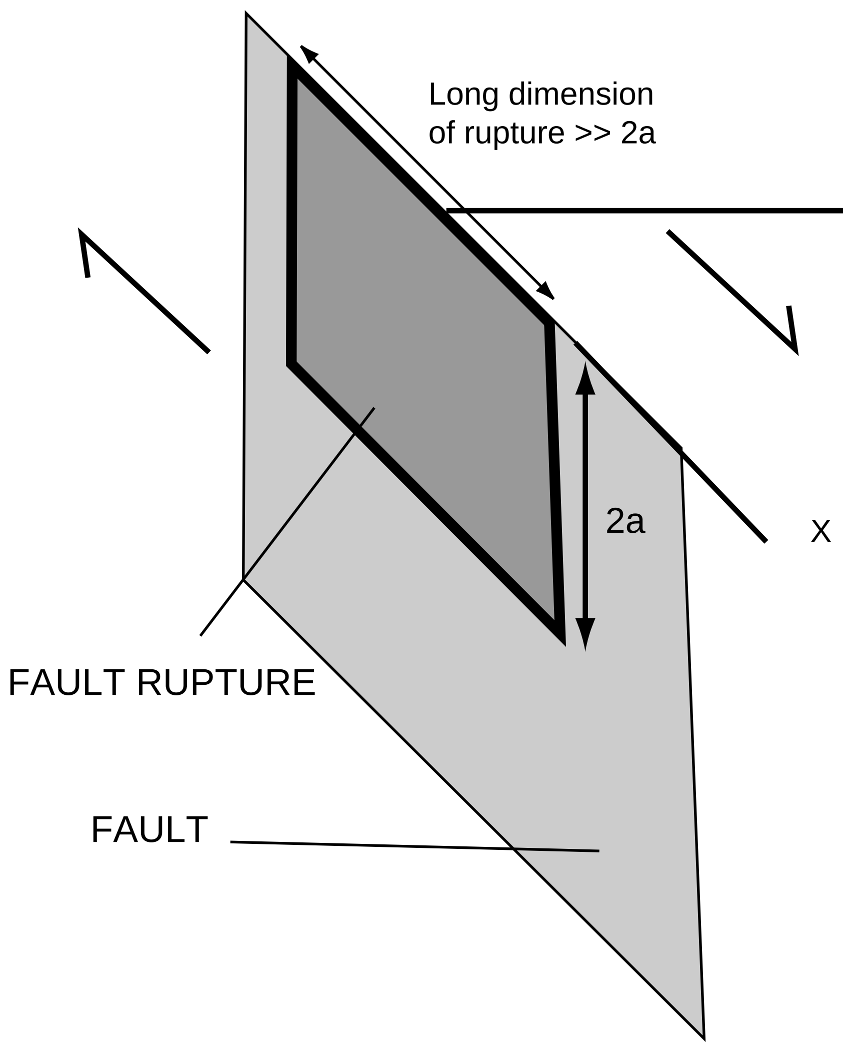
B Shear stress on fault prior to slip = τ_1; post-slip shear stress = τ_2

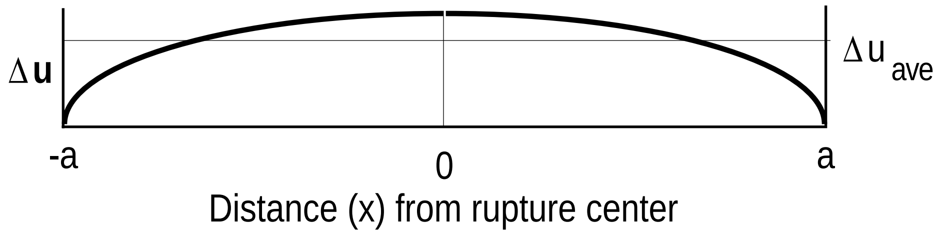
C Slip profile is related to the shape and size of the rupture

D Slip distribution is particularly sensitive to the short dimension

E For a "2-D" rupture (one dimension >> other dimension = 2a)

FAULT GEOMETRY AND FAULT RUPTURE GEOMETRY





1 $\Delta u = 2(1-\nu)\,(\Delta\tau/\mu)\,(a^2 - x^2)^{1/2}$ ν = Poisson's ratio

2 $\Delta u_{max} = \Delta u(x = 0) = 2(1-\nu)\,(\Delta\tau/\mu)\,a \approx 3 \times 10^{-4}\,a$

3 $\Delta u_{min} = \Delta u(x = \pm a) = 0$

4 $\Delta u_{ave} = (\int \Delta u\,dx)/2a = (\pi/4)(\Delta u_{max})$

IV Seismic moment $\mathbf{M_o}$

A $\mathbf{M_o} = \mu\,\Delta u_{ave}\,A$ μ = shear modulus; A = rupture area

B $\mathbf{M_o}$ has dimensions of energy; measures deformation

C A larger area of rupture or a larger slip $\Rightarrow$ larger earthquake

D μ has been measured; geologists can estimate Δu_{ave} and A

E Seismic moments ("earthquake size) $\mathbf{M_o}$ can be predicted

V Radiated seismic kinetic energy E_s (From Scholz, 1990)

$E_s + \Delta E_{strain} + \Delta E_{friction} + \Delta E_{fracture\ surface} + \Delta E_{chemical}? = 0$

A Kinetic energy (E_s) in seismic waves from a dynamics viewpoint

1 E_s varies with amplitude and L wavelength

2 Waves of a frequency of zero ("ultra-long wavelength") correspond to a static situation ("permanent" deformation).

3 Seismic moment, which describes the "permanent" deformation after an earthquake, should be related to seismic energy release.

4 Empirical relationship of seismic energy (E_s) to magnitude (M_w): E_s (joules)= $10^{(4.8 + 1.5 M_w)}$

B Strain energy (ΔE_{strain})

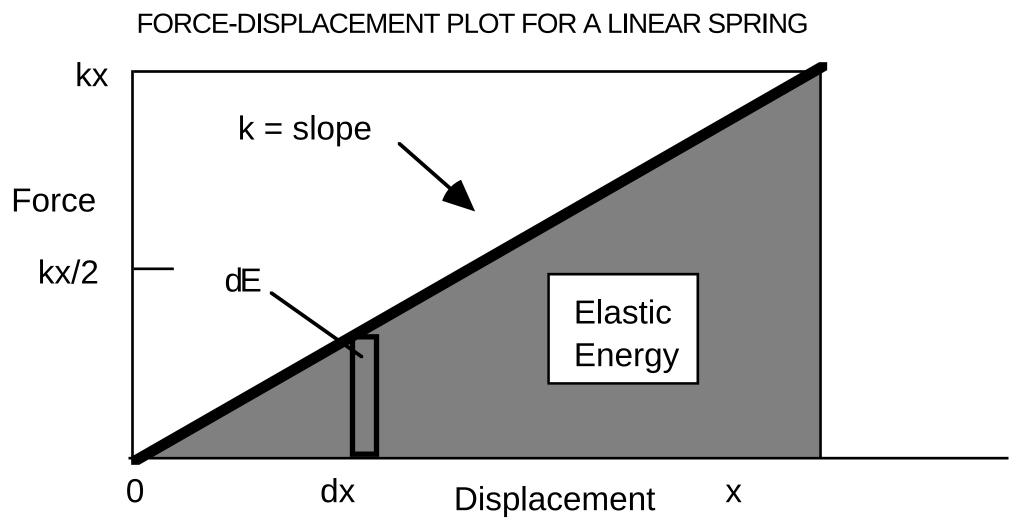
1 Energy in a linear spring

a Force in a spring (F):F=kx; k=spring constant, x=displacement

b Strain energy (ΔE_{strain}) equals area under a force-disp. curve:

$$\Delta E_{strain} = \int_0^x F\,dx = \int_0^x kx\,dx = \left[\frac{1}{2}kx^2\right]_0^x = \frac{1}{2}kx^2$$

ΔE_{strain} = ("Average force on spring")(total displacement of spring)



2 Energy of deformation (ΔE_{strain}) in an earthquake (Method of Reid)

a The change in strain energy = work of faulting

b Consider energy needed to restore rock to pre-quake conditions

c ΔE_{strain} = (1/2)(Peak shear stress)(Area of rupture)(slip)

(If the shear stress after slip occurred is zero, the relevant average stress $\bar{\tau}$ is half the maximum stress. This is where the factor of 1/2 comes from.)

d $\Delta E_{strain} = \bar{\tau}\ A\ \Delta u_{ave}$

Note the similarity between this and $\mathbf{M_o}$ (eq. IV.A)

e τ = (Shear strain)(Shear modulus of rock)

f Example: San Francisco, 1906

$\tau = (1/1500)(2 \times 10^{10}\ J/m^2) = 1.33 \times 10^7 J/m^2)$

$\Delta E_{strain} = (10^7\ N/m^2)(20 \times 10^3 m)(435 \times 10^3 m)(1/2 \times 4\ m)$

$\approx 10^{17}$ J

For comparison, $\Delta E_{Bikini,\ 1946} = 10^{12}$ J

C Heat due to friction ($\Delta E_{friction}$)

1 $\Delta E_{friction} = ([\tau_{friction}]\ [A]\ [\Delta u_{ave}])$

2 Assuming $\tau_{friction} = \tau_2$, $\Delta E_{friction} = [\tau_2]\ [\Delta u_{ave}]\ [A]$

D Energy in seismic waves (E_S) from a mechanics viewpoint

1 $E_S + \Delta E_{strain} + \Delta E_{friction} + \Delta E_{fracture\ surface} = 0$

The strain energy in the earth decreases after a quake, so $\Delta E_{strain}<0$. Energy appears in the form of heat, so $\Delta E_{friction}>0$.

The energy to create fracture surfaces is assumed to be negligible.

2 $E_S \approx -(\Delta E_{strain}) - (\Delta E_{friction})$

3 $E_S \approx (1/2[\tau_1 + \tau_2] [\Delta u_{ave}] [A]) - (1/2)(2)([\tau_2] [\Delta u_{ave}] [A])$??

4 $E_S \approx (1/2[\tau_1 - \tau_2] [\Delta u_{ave}] [A]) = (1/2[\Delta\tau] [\Delta u_{ave}] [A])$??

5 So the seismic kinetic energy depends on the strength change on the fault $\Delta\tau$.

VI Formulas relating seismic energy release (E_S), moment ($\mathbf{M_O}$), and moment magnitude (M_W)

A) $M_W \approx 2/3 \log \mathbf{M_O} - 6.067$ — $\mathbf{M_O}$ in Nm

B) $E_S \approx 10^{(4.8 + 1.5\, M_S)}$ — E_S in joules

C) $E_S \approx \mathbf{M_O}/20{,}000]$ — E_S in joules, $\mathbf{M_O}$ in Nm

D) $\log E_S \approx \log \mathbf{M_O} - 4.3$ — E_S in joules, $\mathbf{M_O}$ in Nm

E) $\log \mathbf{M_O} \approx 1.5\, M_W + 9.1$ — $\mathbf{M_O}$ in Nm

Energy Release vs. Magnitude

The empirical relationship between energy content in radiated seismic waves and magnitude is (Richter, 1958; Bolt, 1989):

$$E_S \text{ (joules)} = 10^{(4.8 + 1.5\, M_S)}.$$

M_S is the surface wave magnitude.

Consider two earthquakes, where $M_{S1} = 1 + M_{S2}$. Then

$$E_{S1}/E_{S2} = 10^{(4.8 + 1.5\, [M_{S2} + 1])}/10^{(4.8 + 1.5\, M_{S2})}$$

$$= \{(10^{4.8})(10^{1.5\,[M_{S2}]})(10^{1.5})\}/\{(10^{4.8})(10^{1.5\, M_{S2}})\}$$

$$= 10^{1.5} \approx 31.6$$

A unit increase in magnitude corresponds to (a) a factor of 10 increase in amplitude of shaking, and (b) a factor of 31.6 increase in energy release. One magnitude 8 quake releases the energy of 1000 magnitude 6 quakes.

Relationship between moment magnitude (M_W) and seismic moment ($\mathbf{M_O}$)

$M_W = 2/3 \log \mathbf{M_O} - 6.067$, where $\mathbf{M_O}$ is measured in Nm   p. 249 of Bolt

This empirical relation has been setup to dovetail with the surface wave magnitude (i.e. $M_S = M_W$). What is the relationship between $\mathbf{M_O}$ and E_S?

$E_S = 10^{(4.8 + 1.5\, M_S)}$ where E_S <u>is in joules</u>   p. 179 of Scholz

$$= 10^{(4.8 + 1.5\, [2/3 \log \mathbf{M_O} - 6.067])}$$

$$= 10^{(4.8 + \log \mathbf{M_O} - [1.5][6.067])}$$

$$= 10^{(4.8 + \log \mathbf{M_O} - 9.1)}$$

$$= 10^{(\log \mathbf{M_O} - 4.3)}$$

$$= [10^{\log \mathbf{M_O}}][10^{-4.3}]$$

$E_S = \mathbf{M_O}/20{,}000]$

Alternatively, $E_S = [\Delta\sigma/2][\Delta u_{ave}][A]$ p. 165 of Scholz

$\mathbf{M_O} = [\mu][\Delta u_{ave}][A]$

$E_S/\mathbf{M_O} = \{[\Delta\sigma/2\mu][\Delta u_{ave}][A]\} / \{[\mu][\Delta u_{ave}][A]\} = [\Delta\sigma/2\mu]$ p. 179 of Scholz

Typically $\Delta\sigma \approx 3$ MPa, and $2\mu = (2)(3 \times 10^4$ MPa), so $E_S/\mathbf{M_O} = 1/20{,}000$ √