IMPORTANT ENGINEERING & HYDROGEOLOGIC PROPERTIES OF ROCKS (02)

I Main Topic: Variety of qualities and quantities relevant to the mechanical and hydrologic behavior of rocks

II Density (ρ)

- A Density: mass/volume (e.g. kg/m^3). ρ_{water} = 10^3 kg/m^3
- B Specific gravity (ρ/ρ_{water}) and unit weight (γ= weight/volume)
- C Factors affecting density: mineralogy and porosity

III Strength and stress

- 1 Strength: maximum stress that a rock can withstand
- 2 Compressive strength, tensile strength, & shear strength
- 3 Dimensions of stress (s): Force/area (e.g. N/m^2)
- 4 Conversion of units: 1 MPa = 10^6 N/m^2 = 145 psi ≈ 10 atm
- 5 Pressure at bottom of a 10-m-deep pool ≈ 0.1 MPa ≈1 atmosphere
- 6 Stress as a tensor (quantities with 2 associated directions)
  - a σ_{ij} acts on a plane ⊥ to the i direction, and in the j direction.
  - b Normal stress acts ⊥ to surfaces; shear stress acts || to them
- 7 Factors affecting rock strength
  - a Rock type (crystalline vs. clastic) and mineralogy
  - b Weathering: generally decreases rock strength
  - c Discontinuities: these decrease rock strength
  - d Sample size: small samples stronger than large samples

IV Hardness

- A Mohs hardness scale (1= talc, 3 = calcite, 7 = quartz, 10 = diamond)
- B Factors affecting rock hardness
  - 1 Mineralogy
  - 2 Weathering: generally decreases rock hardness

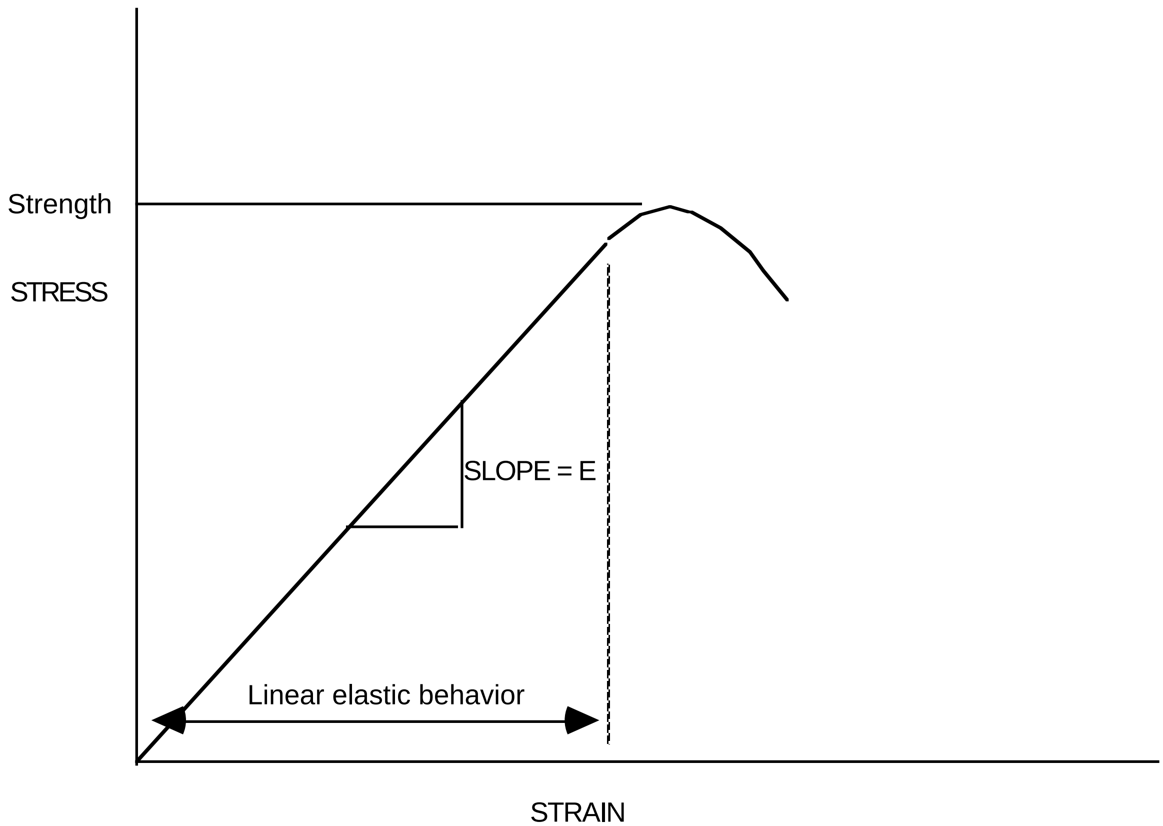
V Strain (ε) and Poisson's ratio (ν)

- A Strain (ε) = $(L_1-L_o)/L_o = \Delta L/L_o$. Strain measures deformation.
- B ε is dimensionless
- C Poisson's ratio (ν): $\varepsilon_1/\varepsilon_2$ (strain in one direction/strain in another)

VI Hook's law of linear elasticity and Young's modulus (E)

- A Hook's law of linear elasticity: $\sigma_x = E\,\varepsilon_x$ (or $\varepsilon_x = \sigma_x/E$)
- B E has dimensions of stress
- C Significance of Young's modulus
  - 1 For engineering projects: low E ⇒ large deformations from loads

STRESS-STRAIN CURVE:
RESULT FROM A ROCK STRENGTH TEST

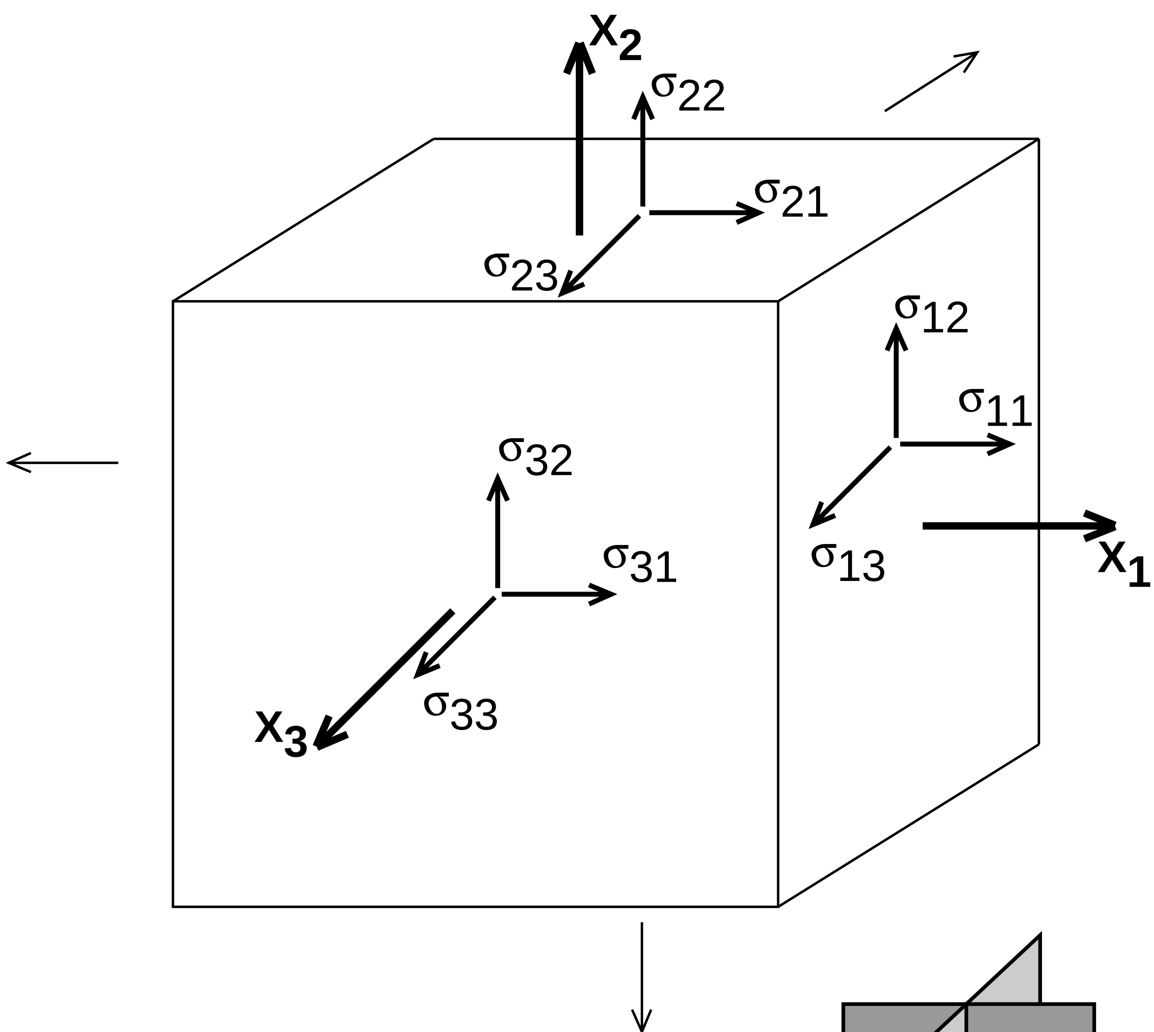


Components of stress at a point (the "on-in rule")

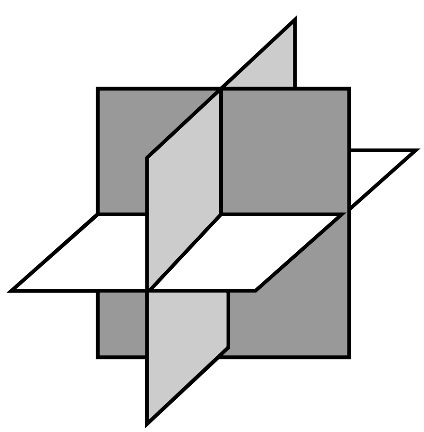
σ_{ij} acts on the plane perpendicular to the x_i axis and in the x_j direction

If i = j: normal stress: σ_{11}, σ_{22}, σ_{33}

If i ≠ j:shear stress: σ_{12}, σ_{13}, σ_{21}, σ_{23}, σ_{31}, σ_{32}



$$\sigma_{ij} = \begin{vmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{vmatrix}$$



2 For seismology: Young's modulus affects seismic velocity

D Factors affecting Young's modulus

1 Rock type

2 Mineralogy

3 Weathering

4 Fractures

VII Creep behavior

A Creep: "slow" inelastic deformation at low stress

B Rocks that creep

1 Ice

2 Salt

VIII Porosity (n)

A Definition: volume of voids/total volume of rock

B Void ratio (engineering term) = volume of voids/volume of solids

C Fluids (e.g. air, water, petroleum, natural gas) occupy voids

D Factors affecting porosity

1 Rock type

2 Weathering

3 Sorting: excellent sorting (or poor grading) ⇒ high porosity

IX Hydraulic conductivity (K) See handout

A Hydraulic conductivity = how readily rock conducts fluid ≠ porosity!

B

Q	=	-K	A	$(\Delta h/\Delta L)$
Discharge		Hyd. cond.	Area	head gradient
m^3/sec	=	(m/sec)	(m^2)	(m/m)

C Factors affecting hydraulic conductivity

1 Dynamic viscosity and density of fluid

2 Character of rock ("intrinsic permeability")

a Interconnection and apertures of pores

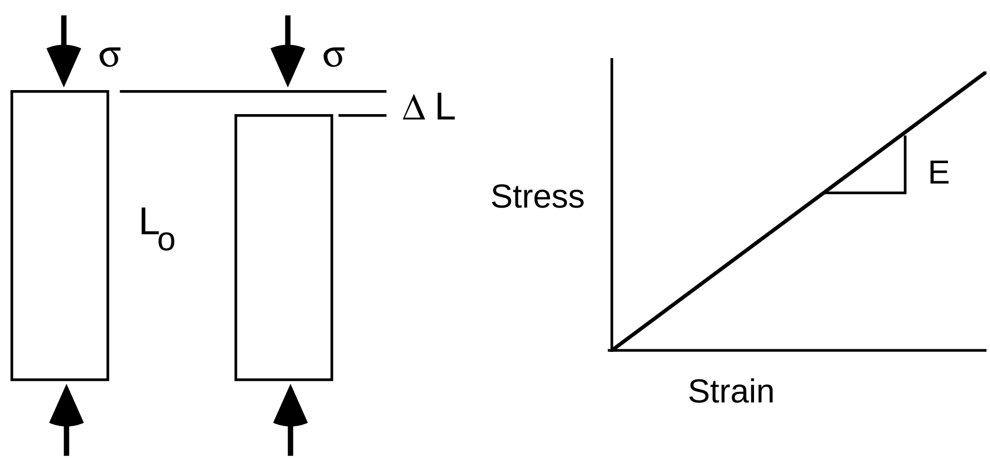
b Interconnection and apertures of fractures

X Chemical stability and reactivity

Conversions between the elastic moduli of rocks
(See Fung. p. 216-218; Turcotte and Schubert, p. 104-112)
Under relatively low loads of relatively short duration, nearly all solid materials, including rocks, deform in a recoverable manner. In other words, if the loads are relaxed, the material rebounds to its original shape. This is called elastic deformation. Under sufficiently large loads or over sufficiently large time frames, the materials will fail or flow, and they will not recover their original shapes when the loads are removed.

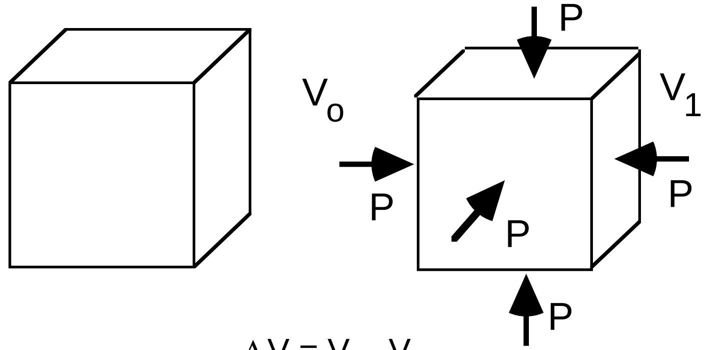
Elastic moduli describe how a material will deform elastically under a given load or stress. Elastic moduli have dimensions of stress; they are the ratio between an applied or induced stress and the associated strain. Strain is a dimensionless term that describes the deformation of a material.

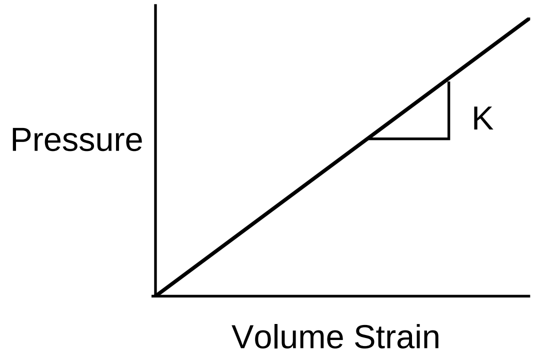
There are a few common ways strain is described. Longitudinal strain (ε) refers to the change in length of an element of material divided by its original length: $\varepsilon = \Delta L/L_o$. Volume strain ($\Delta$) refers to the change in area of an element of material divided by its original area: $\Delta = \Delta V/V_o$.
Consider the uniaxial loading arrangement below:



Young's modulus (E) relates the normal stress σ to the longitudinal strain ε as follows: $E = \sigma/e$.

Now consider the three-dimensional loading arrangement below, where the load on each side of the element is the pressure P





$\Delta V = V_o - V_1$

Volume strain = $\Delta V / V_o = \Delta$

The bulk modulus (K) relates the pressure (P) to the volume strain Δ as follows: $K = P/\Delta$. The reciprocal of the bulk modulus is the compressibility ß. A highly compressible material has a low bulk modulus.

As one might guess, K and E are related. For isotropic materials:
$E = 3K(1-2\nu)$, where ν = Poisson's ratio, and $K = E/\{3(1-2\nu)\}$

The Mechanical Energy in Flowing Water

Water flows from high potential energy to low potential energy.

How is the mechanical energy partitioned in the water?

Consider the mechanical energy contained by a small mass of water m. The water occupies a volume V, and the density of the water is ρ. We consider steady state flow of an incompressible fluid (its density ρ is constant) and neglect the loss of energy due to loss of heat. Fluid flow driven by thermal and chemical gradients are also not treated.

Total energy = kinetic energy + elevation potential energy + pressure potential energy (1)

$$E_{total} = 1/2\, mv^2 + mgh + E_{pressure}. \quad (2)$$

Note that the dimension of pressure (Force/area) is the same as energy/volume (i.e. Force x distance/volume). It turns out that pressure is a measure of internal energy in a volume of fluid. Dividing (2) by the volume of the water gives the energy density in the water:

$$E_{total}/V = 1/2\rho v^2 + \rho gh + P. \quad (3)$$

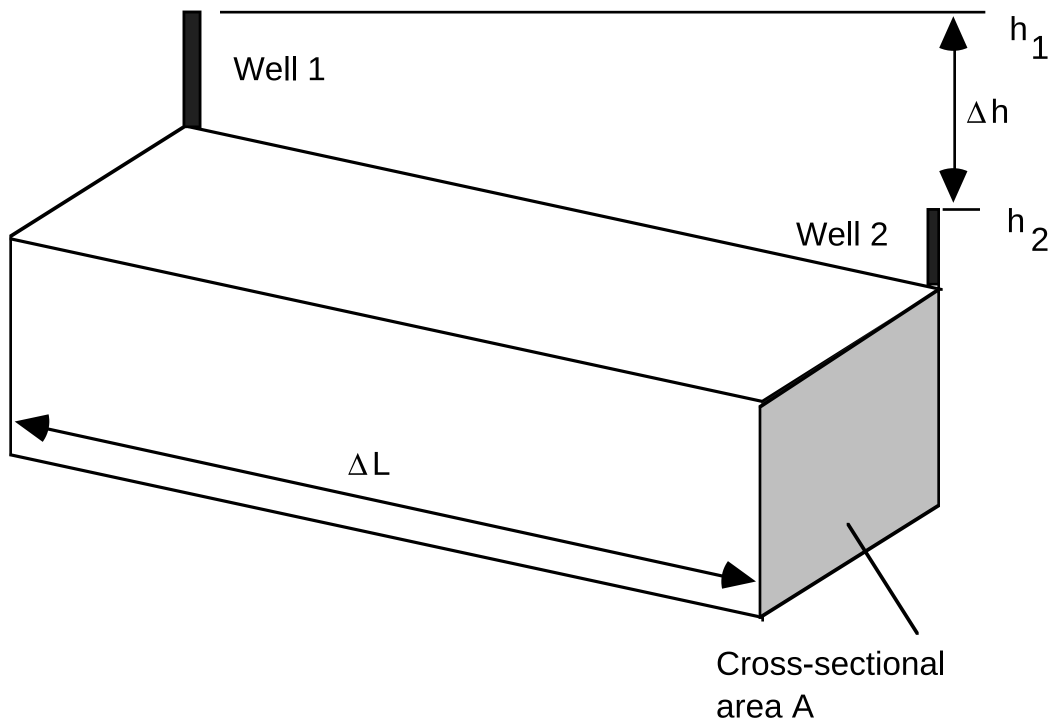
Dividing both sides of equation (3) through by the product ρg, which in many cases is a constant, yields:

$$E_{total}/V\rho g = v^2/2g + h + P/gh. \quad (4)$$

Total head = Velocity head + elevation head + pressure head

The sum of all these factors is called the hydraulic head and has dimensions of length. The hydraulic head can easily be measured in the field: it is the standing elevation that water rises to in a well. Usually the kinetic energy term is negligible for ground water flow, so the hydraulic head is effectively the elevation head plus the pressure head. Water flow from high head to low head. Note that this is very different from the water flowing from high pressure to low pressure; if water did that, it would flow from the bottom of a swimming pool to the top!

DARCY'S LAW



Q	=	-K	A	$(\Delta h/\Delta L)$
Discharge	=	(-hydraulic conductivity)	(Area)	(head gradient)
(m^3/sec)	=	(m/sec)	(m^2)	(m/m)

The head gradient is the change in head divided by the length of the flow tube. The minus sign indicates that flow is in the direction of decreasing head (i.e. flow is from high potential energy to low potential energy). In cases of unconfined aquifers where the flow direction is nearly horizontal, the expression $\Delta h/\Delta L$ is effectively the slope of the ground water table.

The hydraulic conductivity is really a function of both the porous medium and the fluid that flows through it:

$$K = K_i (\rho g/\mu)$$

where K is the hydraulic conductivity, K_i is the intrinsic permeability *of the porous medium*, ρ is the fluid density, g is gravitational acceleration, and μ is the dynamic viscosity of the fluid. Low-density, viscous (i.e. "thick") fluids will flow slower than high-density, "thin" fluids. Intrinsic permeability is measured Darcies. 1 Darcy = 9.87×10^{-9} cm^2. For H_20 at 15.6°C, an intrinsic permeability of 1 Darcy is equivalent to a hydraulic conductivity of 8.61×10^{-4} cm/sec.