

MOHR CIRCLE FOR TRACTIONS

I Main Topics

A Stresses vs. tractions

B Mohr circle for tractions

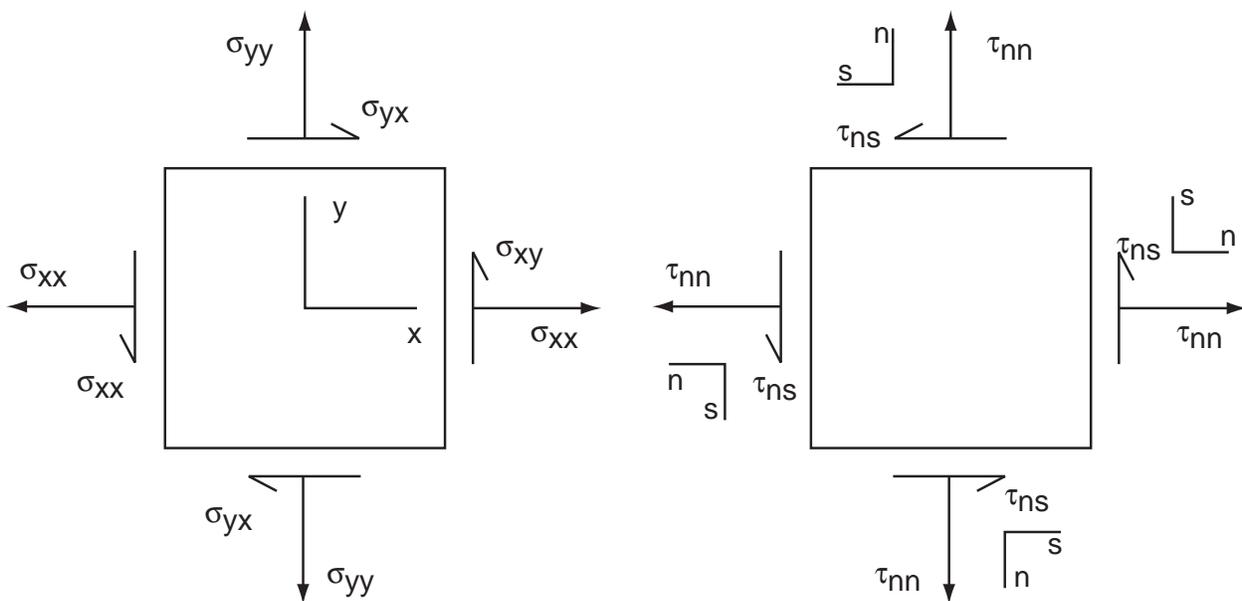
II Stresses vs. tractions

A Similarities between stresses and tractions

- 1 Same dimensions (force per unit area)
- 2 The normal stress acting on a plane matches the normal traction

B Differences between stresses and tractions

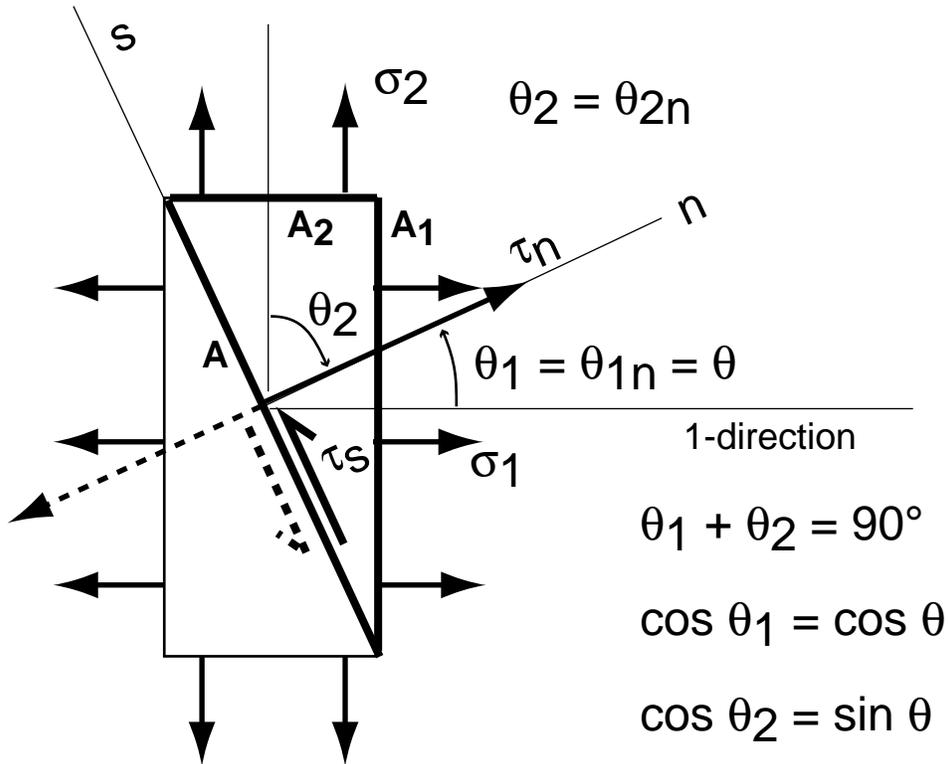
- 1 Stresses are tensor quantities and tractions are vectors.
- 2 The stress state is defined at a point using a fixed reference frame, whereas a traction is defined on a plane with a reference frame that floats with the plane.
- 3 Shear stress components on perpendicular planes have the same sign, whereas shear tractions on perpendicular planes have opposite signs.



Positive stresses at a point

Positive tractions on perpendicular planes

III Mohr circle for tractions



$$A1 \quad \tau_n = \sigma_1 \cos^2 \theta + \sigma_2 \sin^2 \theta$$

$$A2 \quad \tau_s = (\sigma_2 - \sigma_1) \sin \theta \cos \theta$$

Now $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$, $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$, and $\sin \theta \cos \theta = \frac{\sin 2\theta}{2}$, so

$$A3 \quad \tau_n = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 - \sigma_2}{2} \cos 2\theta$$

$$A4 \quad \tau_s = \frac{-(\sigma_1 - \sigma_2)}{2} \sin 2\theta$$

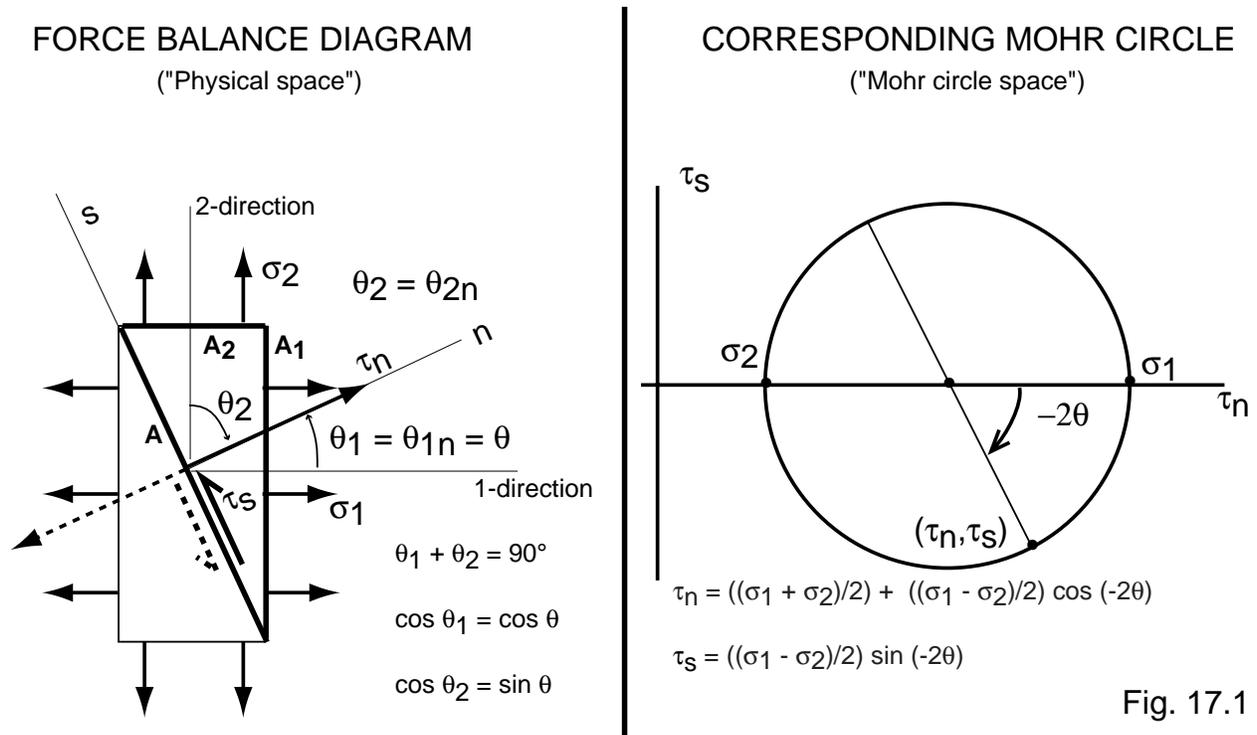
If $\frac{\sigma_1 + \sigma_2}{2} = c$ and $\frac{\sigma_1 - \sigma_2}{2} = r$, then

$$A5 \quad \tau_n = c + r \cos 2\theta = c + r \cos(-2\theta)$$

$$A6 \quad \tau_s = -r \sin 2\theta = r \sin(-2\theta)$$

These are the equations of a Mohr circle for traction that relates the tractions on planes of different orientation at a point. Its center "c" is the mean normal stress (traction), and the absolute value of "r" is the magnitude of the maximum shear traction (the circle radius). The term σ_1 is the most tensile stress.

Suppose in "physical space" the angle from the normal to plane A to the normal to plane B is $+\theta_{AB}$ (measured about the z-axis using a right-hand rule). The tractions on these planes are represented by points on a Mohr circle diagram. The circumferential angle from the point for plane A to the point for plane B is the double angle $-2\theta_{AB}$. As θ goes counterclockwise from 0 to 180° , -2θ goes from 0 to -360° ; one complete clockwise circuit is made around the Mohr circle.



B Key points

- 1 θ is the angle between the normal to the plane σ_1 acts on and the normal to the plane of interest.
- 2 The maximum shear stress is at $\theta = \pm 45^\circ$ from σ_1 ($-2\theta = \mp 90^\circ$)
- 3 The normal stress maximum is at $\theta = 0^\circ$ from σ_1 ($-2\theta = 0^\circ$)
- 4 The normal stress minimum is at $\theta = 90^\circ$ from σ_1 ($-2\theta = -180^\circ$)

C Conventions

- 1 The equations of lectures 16 and 17 are derived using a tensor “on-in” convention for defining stresses, where tensile stresses and tensile tractions are positive. Many geologists and civil engineers, however, use a different convention in which compressive normal stresses are positive and left-lateral shear stresses are positive (see Figure 17.2). In this alternative convention the leading minus sign in equation A4 is dropped. In order to minimize confusion, we will stick with one convention, the tensor convention, rather than deal with two.

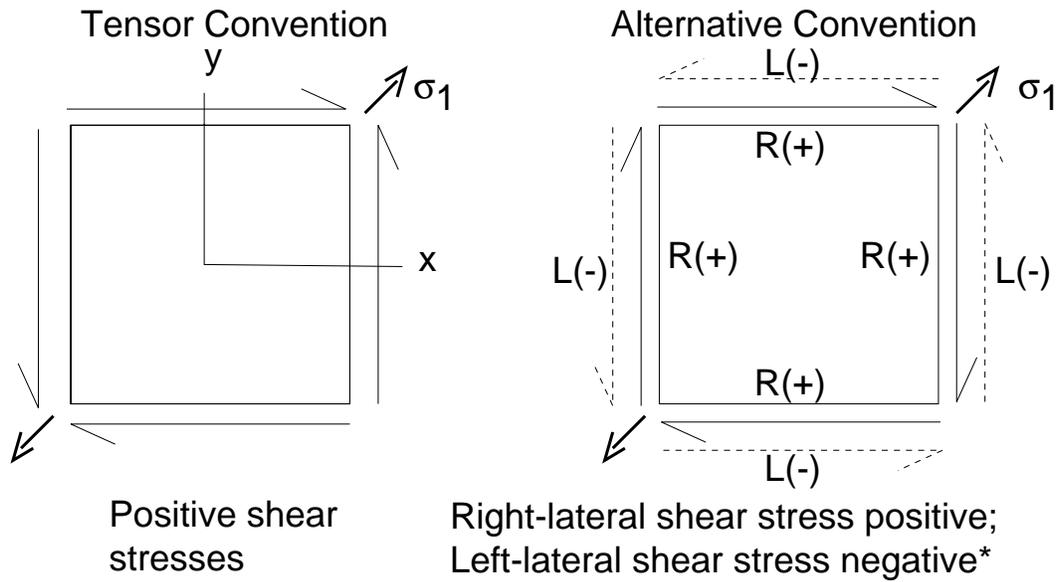
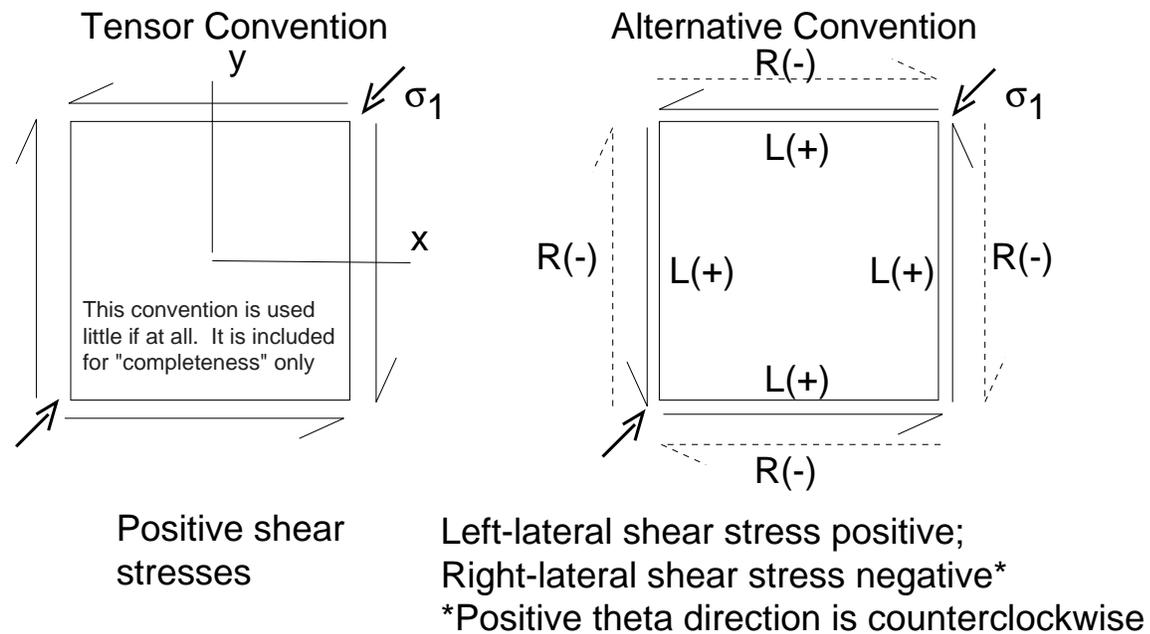
- 2 Advantages of the alternative convention
 - a In the earth the normal stresses generally are compressive, so having compressive stresses be positive is appealing.
 - b Angles on a Mohr circle can be turned in the same direction as in “physical space”
 - c The convention for defining sense of shear (right-lateral or left-lateral) is handy for describing one class of common faults (strike-slip faults).

- 3 Advantages of the tensor convention
 - a Maintains contact with the essential underlying physics far more readily than the alternative convention.
 - b It encourages consideration of the reference frames the stresses are being transformed between.
 - c It is far more useful in dealing with 3-D problems.
 - d Tied to powerful mathematical methods that are otherwise “hidden”.

Just remember to turn the angles on the Mohr diagram in the opposite sense as in “physical space.”

Fig. 17.2

SHEAR STRESS CONVENTIONS

Tension Positive**Compression Positive**

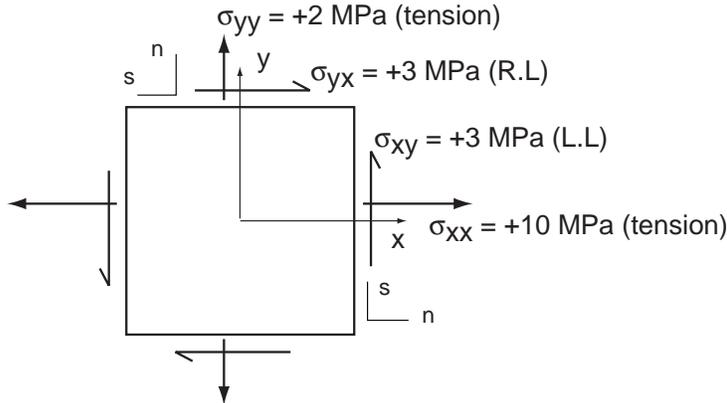
Example 1 using Mohr circle to find principal stresses

A) Draw a box in a reference frame and **clearly** label the stresses on its sides; **this is a critically important step.**

Suppose $\sigma_{xx} = +10$ MPa (tension), $\sigma_{xy} = +3$ MPa (left lateral shear), $\sigma_{yy} = +2$ MPa (tension), and $\sigma_{yx} = +3$ MPa (right lateral shear).

B) Determine the stresses and tractions on the faces of the box.

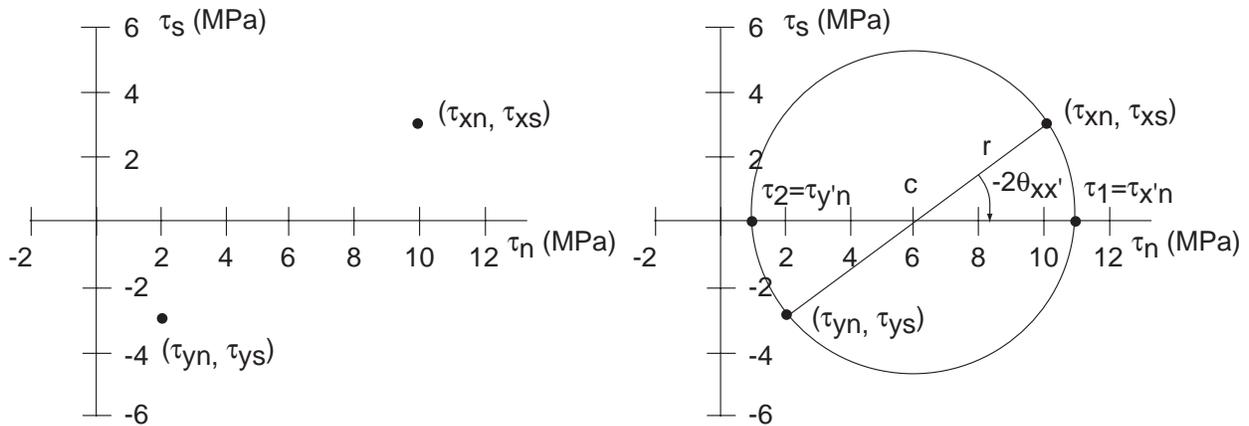
Here, we use the tensor "on-in" convention.



	Stresses		Tractions
σ_{xx}	+10 MPa	τ_{xn}	+10 MPa
σ_{xy}	+3 MPa	τ_{xs}	+3 MPa
σ_{yx}	+3 MPa	τ_{ys}	-3 MPa
σ_{yy}	+2 MPa	τ_{yn}	+2 MPa

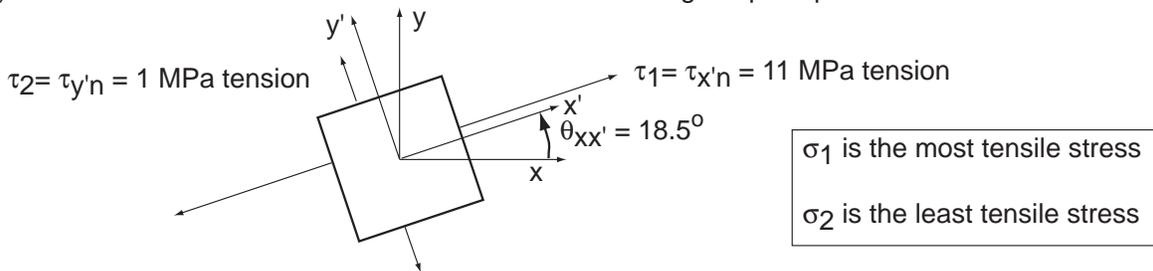
Note that the magnitude of the normal stresses and normal tractions are equal. So $\tau_1 = \sigma_1$ below.

C) Plot and label the points on a set of labelled τ_n, τ_s axes. Then draw the Mohr circle through the points by finding the center (c) and radius (r) of the circle. Now label the principal magnitudes τ_1 and τ_2 ($\tau_1 > \tau_2$); they come from the intersection of the circle with the normal stress (τ_n) axis. Then assign reference axes to the principal directions; below I chose x' for the τ_1 -direction. Label the negative **double** angle between the traction pair that act on a plane with a *known* normal direction (here, x or y) and the traction pair that act on a plane with an unknown direction (e.g., x').



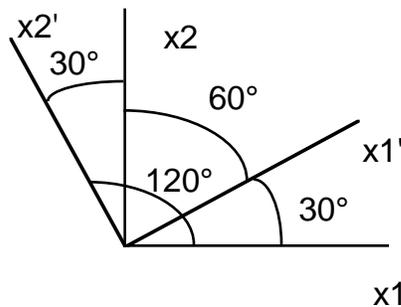
Here, $-2\theta_{xx'} = -37^\circ$ (clockwise), so $\theta_{xx'} = +18.5^\circ$ (counterclockwise)

D) Draw and **label** a new reference frame and box showing the principal stresses.



Mohr Circle Exercises

These problems can be solved with either a Mohr circle or the stress tensor equations. They are set up assuming a tensor convention for stresses (tensile stresses positive), so a compressive stress of magnitude 8 MPa would correspond to a tensile stress of -8 MPa.



- Suppose the most compressive stress is horizontal, trends east-west, and has a magnitude of -4MPa, where a negative normal stress is compressive. The least compressive stress is horizontal, trends north-south (this has to be the case if the most compressive stress trends east-west!) and has a magnitude of -2MPa. Find the normal stress and shear stress that act on a vertical fault with a strike of N30°E.
- Two locked vertical faults each support a left-lateral shear stress of 8MPa. One fault strikes N30°E, and this fault has a normal stress of -14 Mpa, where a negative normal stress is compressive. The second fault has a normal stress of -2 MPa. Assume that the stress field around the faults is uniform. (a) Draw the Mohr circle. (b) Determine the magnitude and the orientations of the principal stresses. (c) Determine the strike of the second fault.
- The pressure at the bottom of a swimming pool is ρgh , where ρ is the density of the water, g is gravitational acceleration, and h is the depth of the water. The water is still. (a) What is the pressure (in Pa) at the bottom of a 10m-deep pool? (b) If we call the pressure (normal stress) on the pool bottom $\sigma_1 = \sigma_{11}$, and the pressure (stress) that acts horizontally $\sigma_3 = \sigma_{33}$, then $\sigma_{11} = \sigma_{33}$. Plot the Mohr circle that describes the state of stress at the bottom of our 10m-deep pool (this is an example of hydrostatic stress). (c) What is the maximum shear stress at the bottom of the pool? (d) What must the level of the intermediate principal stress σ_2 be at the bottom of the pool?
- Two orthogonal planes have the following stresses acting on them: plane 1 ($\sigma_n = -9$ MPa, $\tau = -3$ MPa); plane 2 ($\sigma_n = -1$ MPa, $\tau = 3$ MPa), where a negative normal stress is compressive and a negative shear stress follows from the tensor convention. (a) Draw the Mohr circle. (b) Calculate the greatest and least compressive stresses, and the maximum shear stress. It should help to look at your Mohr circle while doing the calculations. Remember that the mean normal stress is $(\sigma_1 + \sigma_2)/2$, and the radius of the Mohr circle is $(\sigma_1 - \sigma_2)/2$. (c) Accurately and neatly draw a square showing σ_1 and σ_2 acting on the sides of the square, and show the orientation of planes 1 and 2 within the square.
- Find the τ_n and τ_s on a plane whose normal is 45° counterclockwise to the σ_1 direction, where $\sigma_1 = -8$ Mpa, and $\sigma_2 = -3$ MPa ; negative normal stresses are compressive.

- 6 Find the magnitudes and orientations of σ_1 and σ_2 from known normal stress and shear stress on two perpendicular planes P and Q. The known stresses acting on P and Q are:

Plane	Normal stress	Shear stress
P	-2.2 kbar	-1.1 kbar
Q	-1.2 kbar	1.1 kbar

- 7 Find the magnitudes and orientations of σ_1 and σ_2 from known normal stress and shear stress on two perpendicular planes P and Q. The known stresses acting on P and Q are:

Plane	Normal stress	Shear stress
P	-30kbar (a compression)	-8 kbar
Q	0 kbar	8 kbar

Answer

	σ_1	σ_2
Magnitude	-34kbar (a compression)	2 kbar (a tension)
Orientation	14° clockwise from normal to plane P	14° clockwise from normal to plane Q

- 8 Find the magnitudes and orientations of σ_1 and σ_2 from known normal stress and shear stress on two perpendicular planes P and Q. The known stresses acting on P and Q are:

Plane	Normal stress	Shear stress
P	-28kbar (a compression)	- 5 kbar
Q	-2 kbar (a compression)	5 kbar

- 9 Consider the stresses at the surface of the earth in Kansas. Suppose the most compressive horizontal stress trends N30°E and has a magnitude of 20 Pa (i.e., the most compressive horizontal stress is -20 Pa). Ignoring the weight of the atmosphere, what is the vertical normal stress at the earth's surface? Is this vertical normal stress a principal stress? Why or why not? What would be the normal and shear stresses acting on a plane that strikes N60W and dips 18.5° NE? What would be the stress on a plane that strikes 120° and dips 26.5° SW? It might help to note that $18.5^\circ \approx 0.5 \cdot \tan^{-1}(3/4)$ and $26.5^\circ \approx 0.5 \cdot \tan^{-1}(4/3)$

- 10 A rock is being prepared for a compression test, where one principal stress is -30 MPa and the other is -10 MPa, where negative stresses are compressive. Suppose the shear failure criterion for the rock is given by the following equation: $|\tau_s| = |\tau_n|(\tan 30^\circ)$. Plot the failure envelope and the Mohr circle corresponding to the state of the stress in the rock. Can the rock sustain the indicated principal stresses without failing in shear? If the rock will fail in shear, give the orientation of the shear failure plane(s) and the normal and shear stress on the failure plane. If the rock will not fail in shear, give the normal and shear stress on a plane whose normal makes a 60° angle with the σ_1 axis.

- 11 A rock is being prepared for a compression test, with the principal stresses to be applied being -40 MPa (the greatest compression) and -20 MPa (the least compression). Suppose the shear failure criterion for the rock is given by the following equation: $|\tau_s| = |\sigma_n|(\tan 30^\circ)$. What must be the fluid pressure in the rock if it is to fail in shear, assuming that the fluid pressure reduces the effective normal stresses by an amount equal to the fluid pressure?