27. Folds (I)

I  Main Topics
   A  What is a fold?
   B  Curvature of a plane curve
   C  Curvature of a surface

http://upload.wikimedia.org/wikipedia/commons/a/ae/Caledonian_orogeny_fold_in_King_Oscar_Fjord.jpg
27. Folds (I)

Anticline, New Jersey


Syncline, Rainbow Basin, California


27. Folds (I)

Folds, New South Wales, Australia


Folds in granite, Sierra Nevada, California

27. Folds (I)

Energy Resources and an Anticline

http://www.wou.edu/ias/physci/Energy/graphics/OilAnticline.jpg

27. Folds (I)

Three-dimensional Fold, Salt Dome, Zagros Mountains

http://upload.wikimedia.org/wikipedia/commons/2/2c/ZagrosMtns_SaltDome_ISS012-E-18774.jpg
27. Folds (I)

Complex Folds

II What is a fold?

A Definition: a surface (in a rock body) that has undergone a change in its curvature (at least locally)

B All kinds of rocks can be folded, even granites

C Consider a folded piece of paper...
27. Folds (I)

III Curvature of a plane curve

D Tangents

Consider a curve \( r(t) \), where \( r \) is a vector function that gives points on the curve, and \( t \) is any parameter.

1. Tangent vector: \( r' = \frac{dr}{dt} \)

2. Unit tangent vector: \( T = \frac{r'}{|r'|} \)

3. Tangent gives the slope

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D Tangents (cont.)

5. Example 1: parabola

\[ y = x^2 \rightarrow r(x) = xi + x^2 j \]

\[ r'(x) = \frac{dr}{dx} = \frac{d(x^2 + x^2 j)}{dx} = \hat{i} + 2x \hat{j} \]

\[ T(x) = \frac{r'}{|r'|} = \frac{\hat{i} + 2x \hat{j}}{\sqrt{1 + (2x)^2}} = \frac{\hat{i} + 2x \hat{j}}{\sqrt{1 + 4x^2}} \]

At \( x = 1 \), \( T = \frac{\hat{i} + 2 \hat{j}}{\sqrt{5}} \)
27. Folds (I)

D Tangents (cont.)

4 Example 2: unit circle

\[ \vec{r}(\theta) = \cos \theta \hat{i} + \sin \theta \hat{j} \]

\[ \vec{r}' = \frac{d\vec{r}}{d\theta} = \frac{d(\cos \theta \hat{i} + \sin \theta \hat{j})}{d\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j} \]

\[ \vec{T} = \frac{\vec{r}'}{|\vec{r}'|} = \frac{-\sin \theta \hat{i} + \cos \theta \hat{j}}{\sqrt{(-\sin \theta)^2 + (\cos \theta)^2}} = -\sin \theta \hat{i} + \cos \theta \hat{j} \]

Note that \( \vec{T} \cdot \vec{r} = 0 \) here

27. Folds (I)

III Curvature of a plane curve

A Tangents (cont.)

6 If origin is on curve and reference axis is tangent to curve, then local slope = 0
27. Folds (I)

III Curvature of a plane curve

B Curvature = deviation from a straight line

1. Curvature is the first derivative (i.e., rate of change) of the unit tangent (i.e., slope) with respect to distance (s) along the curve

2. Curvature vector is normal to tangent vector

\[ \lim_{s \to 0} \Delta \phi = \tan(\Delta \phi) = \left| \Delta \vec{T} \right| / \left| \vec{T}_1 \right| = \left| \Delta \vec{T} \right| / 1 = \left| \Delta \vec{T} \right| \]

27. Folds (I)

III Curvature of a plane curve

B Curvature (cont.)

3. \[ K(s) = \left| T'(s) \right| = \frac{dT}{ds} \]

4. \[ T(s) = \frac{dr}{ds} / \left| \frac{dr}{ds} \right| \]

In a local tangential reference frame, \( dr \) is in the direction of \( ds \), \( |dr| = ds \), and \( |dr/ds| = 1 \)

5. \[ K(s) = \left| T'(s) \right| = \left| \frac{dT}{ds} \right| = \left| \frac{d}{ds} \left( \frac{dr}{ds} \right) \right| = \left| \frac{d^2r}{ds^2} \right| = |r''(s)| \]
27. Folds (I)

B Curvature of a plane curve (cont.)

6 Curvature vector \( \mathbf{K} \)

\[ \mathbf{K}(t) = \frac{d\mathbf{T}}{dt} \]

7 Curvature magnitude

\[ K(t) = |\mathbf{K}| = \left| \frac{d\mathbf{T}}{dt} \right| \]

Example: circle of radius \( \rho \)

\[ \mathbf{r}(\theta) = \rho \cos \theta \mathbf{i} + \rho \sin \theta \mathbf{j} \]

\[ \frac{d\mathbf{r}}{d\theta} = \rho \left( \cos \theta \mathbf{i} + \sin \theta \mathbf{j} \right) = \rho \cos \theta \mathbf{i} + \rho \sin \theta \mathbf{j} \]

\[ \mathbf{T} = \frac{\mathbf{r}'}{|\mathbf{r}'|} = \frac{-\rho \sin \theta \mathbf{i} + \rho \cos \theta \mathbf{j}}{\sqrt{(-\rho \sin \theta)^2 + (\rho \cos \theta)^2}} = -\sin \theta \mathbf{i} + \cos \theta \mathbf{j} \]

\[ \mathbf{K}(\theta) = \left( -\cos \theta \mathbf{i} - \sin \theta \mathbf{j} \right) / \rho = \frac{-\mathbf{r}'}{\rho} = \frac{-\mathbf{r}}{\rho^2} \]

\[ |\mathbf{K}| = \left| \frac{-\mathbf{r}}{\rho^2} \right| = \frac{1}{\rho} \]
27. Folds (I)

B Curvature of a plane curve (cont.)

9 One can assign a sign to the curvature
   a  Positive = concave
       (curve opens up)
   b  Negative = convex
       (curve opens down)

27. Folds (I)

IV Curvature of a surface
   A Consider a local x,y,z “tangential”
      reference frame, where the x and y
      axes are tangent to the surface and z
      is perpendicular to a folded surface that
      was originally planar
IV Curvature of a surface

B The first partial derivatives \(\frac{\partial z}{\partial x}\) and \(\frac{\partial z}{\partial y}\) are the slopes of the curves formed by intersecting the surface with xz-plane (black curve) and the yz-plane (white curve), respectively. At the local origin, these derivatives equal zero.

\[
\begin{bmatrix}
\frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\
\frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2}
\end{bmatrix}
\]

C The second partial derivatives \(\frac{\partial^2 z}{\partial x^2}\), \(\frac{\partial^2 z}{\partial x \partial y}\), \(\frac{\partial^2 z}{\partial y \partial x}\), and \(\frac{\partial^2 z}{\partial y^2}\) can be arranged in a symmetric matrix (a Hessian matrix).
27. Folds (I)

IV Curvature of a surface

D The principal values of the symmetric Hessian matrix are the greatest and least normal curvatures.

E The principal directions $X$ of the symmetric Hessian matrix are the directions of the principal curvatures; these directions are perpendicular.

$$H = \begin{bmatrix} \frac{\partial^2 z}{\partial x^2} & \frac{\partial^2 z}{\partial x \partial y} \\ \frac{\partial^2 z}{\partial y \partial x} & \frac{\partial^2 z}{\partial y^2} \end{bmatrix} \rightarrow \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$$

$[H][X] = k[X]$ X gives directions in which slope increases or decreases most rapidly.

27. Folds (I)

Curvature-based Three-dimensional Fold Classification Scheme

- $k_1 > 0$: Basin
- $k_1 = 0$: Plane
- $k_1 < 0$: Not Possible

- $k_2 > 0$: Synform (cylindrical)
- $k_2 = 0$: Not Possible
- $k_2 < 0$: Dome

$k_1$: principal curvature
$k_2$: normal curvature