13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

Main Topics (see chapters 14 and 18 of Means, 1976)

A Fundamental principles of continuum mechanics
B Position vectors and coordinate transformation equations
C Displacement vectors and displacement equations
D Deformation
E Homogeneous and inhomogeneous strain

Transition From Particle Mechanics to Continuum Mechanics

Newton’s Pendulum
http://www.lhup.edu/~dsimanek/scenario/collision-r.jpg

Sheep Mountain Anticline, Wyoming
http://www.geology.wisc.edu/~maher/air/air07.htm
II Fundamental principles of continuum mechanics

A Number of particles is sufficiently large that the concept of bulk material behavior is meaningful

B Relates natural world to the realm of mathematics

C Densities of mass, momentum, and energy exist (no "holes")
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

II  Fundamental principles of continuum mechanics

D  Examples of continuous properties
   1  Density \( \rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} \)

So certain derivatives have to exist

10/3/12

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

II  Fundamental principles of continuum mechanics

D  Examples of continuous properties (cont.)
   2  Hydraulic conductivity ("permeability")

E  Scale matters

Note that the concept of derivatives becomes difficult at certain scales

10/3/12
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

II Fundamental principles of continuum mechanics
F Variability
   1 Heterogeneity: material property depends on position


Hand sample of gneiss

10/3/12
GG303
7
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

III Position vectors and coordinate transformation equations
A \( \mathbf{X} = \) initial (undeformed) position vector
B \( \mathbf{X}' = \) final (current, or deformed) position vector (at time \( \Delta t \))
C Coordinate transformation equations
1 \( \mathbf{X}' = f(\mathbf{X}) \)
   Lagrangian: final position a function of initial position
2 \( \mathbf{X} = g(\mathbf{X}') \)
   Eulerian: initial position a function of final position

IV Displacement vector (\( \mathbf{U} \))
A \( \mathbf{U} = \mathbf{X}' - \mathbf{X} \)
1 \( x \)-component: \( u_x \) or \( u \)
2 \( y \)-component: \( u_y \) or \( v \)
3 \( z \)-component: \( u_z \) or \( w \)
B Lagrangian \( \mathbf{U}(\mathbf{X}) \):
   displacement in terms of initial position
C Eulerian \( \mathbf{U}(\mathbf{X}') \):
   displacement in terms of final position
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

V Deformation: rigid body motion + change in size and/or shape

A Rigid body translation

1. No change in the length of line connecting any points
2. All points displaced by an equal vector (equal amount and direction); no displacement of points relative to one another
3. \[ X' = [U] + [X] \]
   matrix addition (U is a constant)

B Rigid body rotation

1. No change in the length of line connecting any points
2. All points rotated by an equal amount about a common axis; no angular displacement of points relative to one another
3. \[ X' = [a][X] \]
   matrix multiplication; rows in [a] are dir. cosines!
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

V Deformation: rigid body motion + change in size and/or shape

C Change in size and/or shape (distortional strain)
1 At least some line segments connecting points in a body change lengths (i.e., the relative positions of points changes)
2 $\ddot{u}$ is not a constant throughout the body (i.e., $\ddot{u}$ varies)

13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

C Change in size and/or shape (distortional strain) – cont.
3 Change in linear dimension
   A Extension (or elongation): $\varepsilon$
   $\varepsilon = \frac{\Delta L}{L_0} = \frac{L_1 - L_0}{L_0}$
   B Stretch: $S$
   $S = \frac{L_1}{L_0} = \frac{L_1 - L_0 + L_0}{L_0} = 1 + \varepsilon$
   C Quadratic elongation: $\lambda$
   $\lambda = \left(\frac{L_1}{L_0}\right)^2 = S^2$
   D All are dimensionless

Elongation

$\varepsilon = \frac{L_1 - L_0}{L_0}$
$S = \frac{L_1}{L_0}$
$\lambda = \frac{L_1}{L_0}^2$
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

C Change in size and /or shape (distortional strain) – cont.

3 Shear strain: $\gamma$
   a Describe change in right in angle between originally perpendicular lines
   b $\gamma = \tan \psi$
      For small $\psi$, $\tan \psi \approx \psi$
   c Dimensionless

Shear Strain

$\gamma = \tan \Psi$

D Change in volume (dilational strain)

1 Dilation ($\Delta$)
   \[ \Delta = \frac{\Delta V}{V_0} = \frac{V_1 - V_0}{V_0} \]
   2 Dimensionless

\[ \Delta = (V_1 - V_0) / V_0 \]
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

D Change in volume (dilational strain) – cont.

3 Example

\[ \Delta = \frac{V - V_0}{V_0} \]

\[ V_0 = \begin{bmatrix} a_0 & 0 & 0 \\ 0 & b_0 & 0 \\ 0 & 0 & c_0 \end{bmatrix} \]

\[ V_1 = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix} \]

\[ \Delta V_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

\[ \Delta V_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

\[ \Delta V = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \]

\[ V_1 = \begin{bmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{bmatrix} \]

For small strains (\( \varepsilon << 1 \))

VI Homogeneous and inhomogenous strain

Example of homogeneous strain in one dimension

\[ u = u(x) = x' = 2x - x \]

Lagrangian

\[ \varepsilon = \frac{\Delta L}{\Delta x} = \frac{\left( (x' - x_1) - (x_2 - x_1) \right)}{(x_2 - x_1)} \]

\[ = \frac{\left( (x' - x_2) - (x_1' - x_1) \right)}{(x_2 - x_1)} \]

\[ = \frac{\left( x_1 - u_1 \right)}{(x_2 - x_1)} = \Delta u/\Delta x \]
13. BASIC CONCEPTS OF KINEMATICS AND DEFORMATION

VI Homogeneous and inhomogenous strain

Example of inhomogeneous strain in one dimension

\[ x' = x^2 \]

\[ u = u(x) = x' - x = x^2 - x \]

\[ \varepsilon = \lim_{\Delta x \to 0} \frac{\Delta L}{L_0} = \lim_{\Delta x \to 0} \frac{\Delta u}{\Delta x} = \frac{du}{dx} = 2x - 1 \]