9. SPHERICAL PROJECTIONS (II)

I Main Topics
   A Angles between lines and planes
   B Fold axes of cylindrical folds
   C Equal-angle and equal-area projections
   D Appendices
II. Angles between lines and planes

A. Angle between lines
1. The acute angle between the lines
2. Measured in the plane (great circle) containing the lines.
3. Measured along the cyclographic trace of the unique great circle representing the plane containing the two lines.
4. Can be found using the dot product or cross product of the poles

B. The angle between two planes
1. The acute angle between the planes (or the poles to the planes)
2. Measured in the plane (great circle) perpendicular to the intersection of the planes
3. Measured along the cyclographic trace of the unique great circle representing the plane containing the poles to the two planes.
4. Can be found using the dot product or cross product of the poles
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III Fold axes of cylindrical folds

A Cylindrical fold
1 A folded surface
2 A surface that can be swept out by a line moving parallel to itself
3 A “two-dimensional” fold

B Fold axis
1 The direction of a line parallel to a cylindrical fold
2 The direction of the intersection of planes tangent to a cylindrical fold
3 The direction of the cross product of normals to bedding (not shown here)
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III Fold axes of cylindrical folds

B \( \beta \) diagram
1. Fold axis is along the line of intersection of planes tangent to bedding
2. Direct method

C \( \pi \) diagram
1. Fold axis is perpendicular to the plane containing the poles to beds
2. Preferred over \( \beta \) diagram if many beds are considered
3. For two poles, the \( \pi \) diagram is akin to finding the cross product between the bedding plane poles.
4. Can be extended to many poles
IV Types of spherical projections

A  Equal angle projection
1. Shapes of plane shapes on the surface of the sphere are preserved, but their relative areas are altered.
2. The angles between planes equals the angle between their cyclographic traces
3. Plotted by hand with a Wulff net (see next page)

B  Equal area projection
1. The relative areas of plane shapes on the surface of the sphere are preserved, but their shapes are altered.
2. Good for representing the density of poles
3. Plotted by hand with a Schmidt net (see next page)
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Wulff net

Schmidt net

From http://en.wikipedia.org
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(From Hobbs, Means, and Williams, 1976, An Outline of Structural Geology)

<table>
<thead>
<tr>
<th>Property</th>
<th>Equal angle projection</th>
<th>Equal area projection</th>
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</thead>
<tbody>
<tr>
<td>Net type</td>
<td>Wulff net</td>
<td>Schmidt net</td>
</tr>
<tr>
<td>Projection preserves ...</td>
<td>Angles</td>
<td>Areas</td>
</tr>
<tr>
<td>Projection does not preserve ...</td>
<td>Areas</td>
<td>Angles</td>
</tr>
<tr>
<td>A line projects as a ...</td>
<td>Point</td>
<td>Point</td>
</tr>
<tr>
<td>Great circle projection</td>
<td>Circle</td>
<td>Fourth-order quadric</td>
</tr>
<tr>
<td>Small circle projection</td>
<td>Circle</td>
<td>Fourth-order quadric</td>
</tr>
<tr>
<td>Distance from center of primitive circle to</td>
<td>$R \tan \left( \frac{\pi}{4} - \frac{\text{dip}}{2} \right)$</td>
<td>$R \sqrt{2} \sin \left( \frac{\pi}{4} - \frac{\text{dip}}{2} \right)$</td>
</tr>
<tr>
<td>cyclographic trace measured in direction of dip</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance from center of primitive circle to</td>
<td>$R \tan \left( \frac{\text{dip}}{2} \right)$</td>
<td>$R \sqrt{2} \sin \left( \frac{\text{dip}}{2} \right)$</td>
</tr>
<tr>
<td>pole of plane measured in the direction opposite</td>
<td></td>
<td></td>
</tr>
<tr>
<td>to that of the dip</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distance from center of primitive circle to point</td>
<td>$R \tan \left( \frac{\pi}{4} - \frac{\text{plunge}}{2} \right)$</td>
<td>$R \sqrt{2} \sin \left( \frac{\pi}{4} - \frac{\text{plunge}}{2} \right)$</td>
</tr>
<tr>
<td>that represents a plunging line</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Favored use</td>
<td>Measuring angular relations</td>
<td>Contouring orientation data</td>
</tr>
</tbody>
</table>
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V Appendices

A Computer programs for spherical projections (free)

B Stereographic projection of a circle

C Stereographic projection of points on the upper hemisphere
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A Computer programs for spherical projections (free)

1 S.J. Martel’s Matlab code for spherical projections

2 "Stereonet" by R. Allmendinger at Cornell University (for the Mac)
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9. SPHERICAL PROJECTIONS (II)

Equal-Angle Projection of a Small Circle (II)  

Fig. 9.3L

- \( r_L = \tan \left[ \frac{90^\circ - (\phi + \Delta)}{2} \right] \)  
- \( r_L = R \tan \left[ \frac{90^\circ - (\phi + \Delta)}{2} \right] \)  
- \( \phi + \Delta = 90^\circ - 2(\tan^{-1} \left( \frac{r_L}{R} \right)) \)

- \( r_M = \tan \left[ \frac{90^\circ - (\phi - \Delta)}{2} \right] \)  
- \( r_M = R \tan \left[ \frac{90^\circ - (\phi - \Delta)}{2} \right] \)  
- \( \phi - \Delta = 90^\circ - 2(\tan^{-1} \left( \frac{r_M}{R} \right)) \)

- \( r_C = \frac{r_L + r_M}{2} \)
- radius of proj. small circle = \( \frac{r_L \cdot r_M}{2} \)

OV = borehole  
OL = pole to bedding  
OM = pole to bedding  

View down of primitive circle  

"L"  
"L"'  
"L"  
"M"  
"M"'  
"M"  
"V"  
"V"'  
"V"  
C' is midway between L' and M' and is the center of small circle L'M'

This line marks the trend of the borehole  

V' is where the borehole plots  
L' has the trend of V' but plunges at \( \phi - \Delta \)  
M' has the trend of V' but plunges at \( \phi + \Delta \)  
The center is at C.  

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Points Outside a Primitive Circle in Equal-angle Projections  Fig 9.4L

View down onto stereonet

The point of intersection of three circles is outside the primitive circle. What would this mean?

Point O is the center of the primitive circle

Primitve Circle

Let's return to how the projection is done to answer the question

Cross section view along OA

An upward pointing line projects outside the primitive circle! So our "outside" point A' is really A'\text{up}.

Whereas A'\text{up}-A'\text{down} is a diameter, then angle A'\text{down}-Z-A'\text{up} must be a right angle.

Also, OZ is perpendicular to A'\text{up}.

Projection Plane

View down onto stereonet

To plot the downward-pointing pole corresponding to A'\text{up} we turn the equal-angle projection method "on its side":

1. Draw a line from A'\text{up} through O
2. Draw line OP perpendicular to line OA'\text{up}
3. Draw line OA'\text{down} perpendicular to OA'\text{up}.

Points A'\text{down}, O, and A'\text{up} lie on one line.