Exercise 1: Stress analysis
Consider a hypothetical state of stress near the surface of the Earth in a volcanically active region. We wish to find the orientation of dikes that might erupt in fissure eruptions and how a pre-existing set of vertical fractures might slip as faults. For our reference frame, \( x = \text{east}, y = \text{north}, z = \text{up} \).

\[
\begin{bmatrix}
\sigma_{xx} &= -1.5 \\
\sigma_{yy} &= 1.7 \\
\sigma_{zz} &= 1.7 \\
\sigma_{xy} &= \sigma_{yx} &= 1.7 \\
\sigma_{xz} &= \sigma_{zx} \\
\sigma_{yz} &= \sigma_{zy} &= -1.5
\end{bmatrix} \text{ MPa}
\]

1 MPa (mega-pascal) \( \approx \) 10 atmospheres

Part 1: Dike prediction (48 pts)

a) Draw a neatly labeled box showing the stress components in the matrix above acting on all sides of the box. Draw positively directed stress arrows. (8 pts)

b) Draw a second neatly labeled box showing the tractions above acting on the sides of the box. Draw positively directed traction arrows. (8 pts)

c) Solve for the magnitude and orientation of the principal stresses by using a Mohr circle for tractions (see example in lecture 17, p. 9-10). On the Mohr diagram, assign a reference frame (\( x'-y' \)) to the principal directions, with the \( x' \)-axis along the direction of the most tensile (or least compressive) stress. (2+2+4+2 = 8 pts)

d) Solve for the magnitude and orientation of the principal stresses by the eigenvector/eigenvalue method - long hand (see lecture 19, p.16-18) (8 pts)

e) Solve for the magnitude and orientation of the principal stresses by using the eigenvector /eigenvalue method – Matlab (see lecture 19, p. 20). Include a copy of your printout. (4 pts)

f) Draw a neatly labeled box showing the principal stresses acting on the sides of the box. Draw positively directed stress arrows. Draw the \( x'-y' \) axes inside the box, with the axes parallel and perpendicular to the sides of the box. (8 pts)

g) Neatly plot the predicted strike of the dikes in map view, and make sure your map has a title, a north arrow, and a scale. Explain why you expect the dikes to have the orientations you predict. (1+1+2 = 4 pts)

Part 2: Prediction of faulting along pre-existing vertical fractures strike 59°. (16 pts)

a) Determine the normal and shear tractions acting on planes parallel to the fractures. The method you use is up to you. Explain your work. (5 pts)

b) Draw a neatly labeled picture in map view showing the fractures and the normal and shear tractions acting on the plane of the fractures. Include a title, a north arrow, and a scale. (1+2+1 = 4 pts)

c) Assuming the fractures slip as strike-slip faults, explain why you predict that the fractures would slip left-laterally or right-laterally. (4 pts)

d) Based on your answers to (b), explain why you think the fractures will or will not slip as faults. If the compressive tractions acting on the faults are high relative to the magnitude of the shear tractions that would drive slip, the frictional resistance to sliding might be so great that the fractures are locked and cannot slip. (3 pts)
Exercise 2: Strain analysis (30 pts total)
This exercise is intended to give you some practice with principal strains and the use of eigenvectors to determine them. For geologic context, imagine that you are dealing with oolites (spherical limestone nodules) that have been deformed in simple shear, with displacements parallel to the horizontal axis, and you want to quantify the strain. You will need to complete some lines in a Matlab code based on material in the PowerPoint notes for lab 10 and describe the strain. Print off the two plots that the completed code generates to complete this part of the lab, and attach them to these pages.

Part 1: Code modification (12 pts)
a) Complete the following 12 lines in the code gg303_2011_lab_10_b.m_10_2011: lines 6, 8, 10, 12, 14, 16, 18, 20, 57, 64, 66, 68. These lines are marked by ******* . Leave a semicolon off the end of these lines so the values print out. See Matlab's online help for the command 'inv' to take the inverse of a matrix.

Part 2: Figure 1 (the strain ellipse) (12 pts)
a) On Figure 1, label the directions of principal strain axes. (2 pts)
b) On Figure 1, label the eigenvectors of the F matrix. (2 pts)
c) Describe below why the eigenvectors you labeled do not rotate in the deformation of the unit circle to the strain ellipse. (2 pts)
d) Based on measurements of the axes of the strain ellipse and determine what the magnitudes of the principal strains are. (2 pts)
e) Based on measurements of the axes of the strain ellipse and determine what the magnitudes of the principal stretches are. (2 pts)

f) State the compass bearings of the principal axes of strain (assume north is to the top of the page). (2 pts)

Part 3: Figure 1 (the reciprocal strain ellipse) (6 pts)
a) On Figure 2, label the retro-deformed principal strain axes. (2 pts)
b) On Figure 2, label the acute rotation angle (2 pts)
c) State below how this angle relates to the rotation matrix R. (2 pts)