

INTERSECTIONS OF PLANES

Lab 4

Read each exercise completely before you start it so that you understand the problem-solving approach you are asked to execute. This will help keep the big picture clear.

Exercise 1: Apparent dip (57 points total)

Graphical and trigonometric solutions

- 1) For plane ABC on the next page, graphically determine its strike and dip. Then find the apparent dip of plane ABC as seen in vertical cross section plane P2, which strikes $N60^\circ W$. Check the apparent dip you obtain graphically against the apparent dip you obtain trigonometrically using the expression in the course notes. Enter your results in the table. Show your trigonometric calculations in the box below the table. **(12 points total)**

Strike of ABC (graphical) (3pts)	True dip of ABC (graphical) (3 pts)	Apparent dip of ABC (Graphical) (3 pts)	Apparent dip of ABC (Trig.) (3 pts)

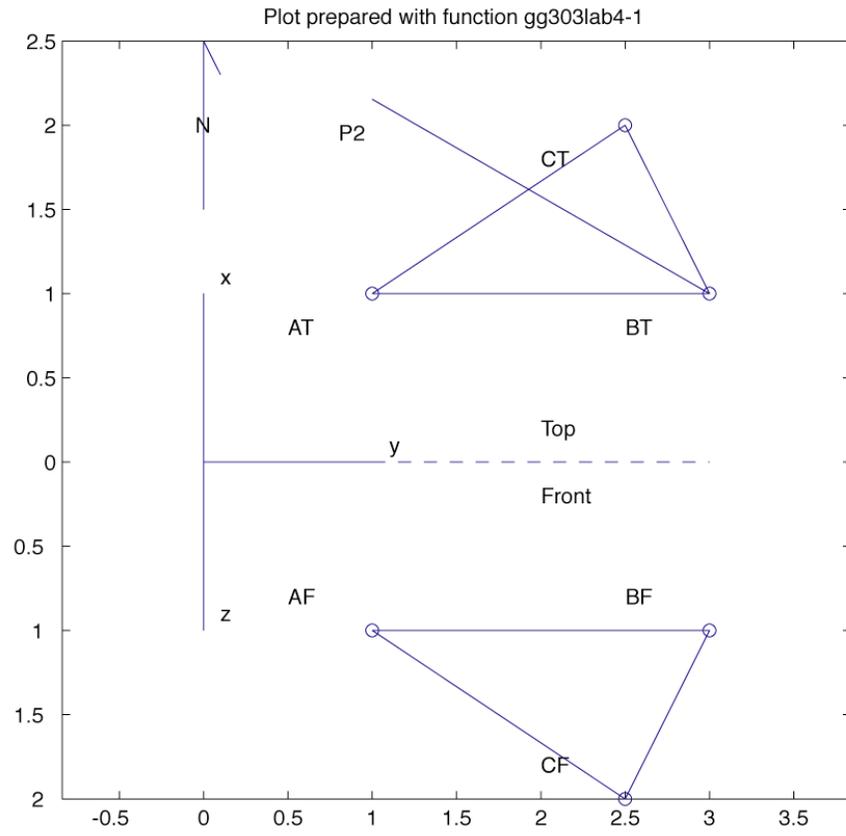
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Solution from cross-products of poles

You will now find the orientation of the line of intersection between plane ABC and the cross section plane from the cross product of the poles to the planes using two different techniques (see page 2). These solutions are to be done on Matlab. One-line Matlab commands can address each question. Include a printout of your work showing your answers. Annotate the printout as you see fit.

Poles from strike and dip

- 2) From the strike and dip of cross section plane P2 find the trend and plunge of its pole, first in degrees, and then in radians. The trend and plunge, measured in radians, are the two angular coordinates in a spherical reference frame; the third spherical coordinate, the distance from the origin, has a value of 1 for a unit length vector. Then use the Matlab function sph2cart to find the direction cosines (Cartesian coordinates of a pole of unit length) from the spherical coordinates of the pole. **(18 points total; 1 point per box)**



Plane ABC (plane 1)

Strike (°)	Dip (°)	Pole trend (°)	Pole plunge (°)

Pole n1 trend (radians)	Pole n1plunge (radians)	$n1_x = \alpha1$	$n1_y = \beta1$	$n1_z = \gamma1$

Plane P2

Strike (°)	Dip (°)	Pole trend (°)	Pole plunge (°)

Pole n2 trend (radians)	Pole n2 plunge (radians)	$n2_x = \alpha2$	$n2_y = \beta2$	$n2_z = \gamma2$

At this point one could proceed directly to question 11, and you are welcome to do that if you want to maintain the flow of the exercise (**if you do that though, you still need to answer questions 3-10**). However, one might want to pause to check the intermediate results to make sure a mistake hasn't been made already that might only become apparent at the end of the exercise. So a powerful, direct alternative method for obtaining the direction cosines for the poles is introduced in questions 3-10.

Poles directly from coordinates of points in plane (just for plane ABC)

3) Find the x,y,z coordinates of points A, B, and C in terms of the dimensionless units on the axes. Give the coordinates to the nearest 0.1 units. NOTE THAT POSITIVE Z POINTS DOWN, so the coordinates given in the attached figure for the front view should have increasing positive values with distance below the top plane. **(9 points total; 1 point/box)**

	x	y	z
Point A			
Point B			
Point C			

4) Enter the x,y,z coordinates of points A, B, and C (i.e., vectors **OA**, **OB**, and **OC**, respectively) into Matlab as 1-row, 3-column arrays. For example: $Q = [3,4,5]$. **(3 points total)**

5) Using Matlab and vector subtraction, find vectors **AC** and **AB**, where **AC** “points” from point A to point C, and **AB** “points” from point A to point B. **(2 points)**

	x	y	z
Vector AC			
Vector AB			

6) Using Matlab, find the x, y, and z components of **N1**, where **N1** is the cross product **AC** x **AB**, and write its x,y,z components in the table below. **(1 point)**

	x	y	z
Vector N1			

7) Using Matlab’s cart2sph function, find the trend and plunge of **n1** in radians. Note that cart2sph yields the trend and plunge only for x = north, y = east, z = down. **(2 points)**

n1 trend (radians)	n1 plunge (radians)

8) Using Matlab’s cart2sph function, find the trend and plunge of **n1** in degrees. Note that cart2sph yields the trend and plunge only for x = north, y = east, z = down. **(2 points)**

n1 trend (degrees)	n1 plunge (degrees)

9) Check: is this result consistent with the graphical solution for the strike and dip of the plane ABC for question 2, yes or no? **(1 point)**

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For questions 10-12, fill out the table below.

10) Using Matlab and the direction cosine values for the planes addressed in question 2, find $\mathbf{V2} = \mathbf{n1} \times \mathbf{n2}$, where $\mathbf{n2}$ is the unit normal to Plane P2. (1 point)

	$V2_x$	$V2_y$	$V2_z$
Vector $\mathbf{V2}$			

11) Check to see if the cross product points up (γ_2 is negative) or down (γ_2 is positive). If the pole points up, reverse the signs on α_2 , β_2 , and γ_2 to get the direction cosines for a pole that points down (i.e., $\mathbf{V2}^* = -\mathbf{V2}$). If the point points down, then write the direction cosines in the table above in the table below, keeping the signs the same (i.e., $\mathbf{V2}^* = \mathbf{V2}$).

	$V2^*_x$	$V2^*_y$	$V2^*_z$
Vector $\mathbf{V2}^*$			

12) Using your answer to question (11) and Matlab's cart2sph function, find the trend and plunge of $\mathbf{V2}^*$ in radians (2 points)

	Trend (radians)	Plunge (radians)
Vector $\mathbf{V2}^*$		

13) Write the trend and plunge of $\mathbf{V2}^*$ in degrees (2 points)

	Trend (degrees)	Plunge (degrees)
Vector $\mathbf{V2}^*$		

14) Explain what the answer to question (13) is in terms of the original problem as described in question 1: what is $\mathbf{V2}^*$? Write your answers below. (2 points)

You have prepared a graphical solution, and you checked your solution by two independent algebraic methods. The graphical solution lets you “see” what is going on visually. You can save your algebraic solutions on a computer, however, and do another problem rapidly just by changing the input values of question 5. If you have many problems to do, the computer will be faster.

Exercise 2: Equation of a plane: $\mathbf{n} \cdot \mathbf{v} = d$ (10 points total)

In this exercise you will find the normal form of the equation of plane ABC (see page 1). Start by checking to make sure that your solution for \mathbf{n}_1 (the unit normal) is correct.

1) Take the dot product of \mathbf{AC} and \mathbf{n}_1 and write it below. (1 point)

2) Take the dot product of \mathbf{AB} and \mathbf{n}_1 and write it below. (1 point)

3) Check: \mathbf{n}_1 is supposed to be perpendicular to the ABC plane. Describe below whether or not your answers to (1) and (2) support this, and say why. (2 points)

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4) Find $d_1 = \mathbf{n}_1 \cdot \mathbf{v}$, where $\mathbf{v} = \mathbf{OA}$, the vector from the origin to point A. (1 point)

5) Find $d_2 = \mathbf{n}_1 \cdot \mathbf{v}$, where $\mathbf{v} = \mathbf{OB}$, the vector from the origin to point B. (1 point)

6) Find $d_3 = \mathbf{n}_1 \cdot \mathbf{v}$, where $\mathbf{v} = \mathbf{OC}$, the vector from the origin to point C. (1 point)

7) What is the distance from the origin to plane ABC? (1 point)

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8) Check: is this consistent with the points as plotted in the figure? (1 point)

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9) Write the equation for plane ABC in normal form ($a_{n_1x}x + a_{n_1y}y + a_{n_1z}z = d$), filling in all the terms that you know (a_{n_1x} , a_{n_1y} , a_{n_1z} , and d). (1 point)

Exercise 3: Point-of-intersection problem (85 pts total)

A scientific team plans to test whether earthquakes along the Snake Eyes Fault can be triggered by injecting water down boreholes into the fault. The team needs you to tell them where the borehole will intersect the fault. Here is the geometric data for one borehole option:

Pt.	Location of drill pad			Borehole trend/Borehole plunge	
	Northing (x)	Easting (y)	Depth (z)	Borehole trend	Borehole plunge
D	1	3	0	300°	60°

The fault plane has been located at three spots, Points A,B,C from Exercise 1.

Pt.	Northing (x)	Easting (y)	Depth below surface (z)
A	1	3	0
B	0000m east	1000m north	el. = 500m
C	1000m east	1000m north	el. = 800m

To solve this problem, you will need to put together pieces like those of the first two exercises.

You might also benefit from reviewing the line intersection problem of lab 1.

Graphical Approach

- 1) In your cross section of exercise 1m plot the borehole and label where it intersects the fault. **(2 points)**.
- 2) In the cross section, measure the distance along the borehole from the drill pad to the fault. Report the distance in “meters in the field”, not “cm on the piece of paper.” **(2 points)**.

Distance along the borehole from the drill pad to the fault (meters)	1.62m
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Algebraic Approach (Solution of simultaneous linear equations, section V B)

Much of the rest of exercise requires using Matlab. Include a printout of your Matlab work.

- 3) Find the strike and dip of the **vertical** plane (Plane 2) that contains the borehole, then find the trend and plunge of the pole to the plane, and write the values in the table below. **(8 points)**.
- 4) Find the strike and dip of an **inclined** plane (Plane 3) that contains the borehole and that dips at an angle equal to the borehole plunge, then find the trend and plunge of the pole to the plane, and write the values in the table below. **(8 points)**.

	Strike (3pts/box)	Dip (3pts/box)	Pole trend (1 pt/box)	Pole plunge (1 pt/box)
Plane 2 (vertical)				
Plane 3 (inclined)				

Write the answers for questions 5-7 in the table below.

For the following questions use a reference frame where x=north, y=east, z=down.

Make sure your answers are consistent with this frame.

- 5) Using Matlab, find the direction cosines of the poles to each plane. **(9 points)**.
- 6) Write the x,y,z coordinates of a point in plane P1. In Plane P2. In Plane P3. You must convert eastings, northings, and elevations to the z-down reference frame here. Positive elevations thus will have negative z-values. This is vital to do correctly. **(9 points)**.
- 7) Find the distance (d) from the origin to each plane along the direction of the appropriate unit normal ($\mathbf{n} \cdot \mathbf{V} = d$). Write the answers in the right-hand column of the table. **(3 points)**

	α (1 pt/box)	β (1 pt/box)	γ (1 pt/box)	x,y,z (1 pt/box)	d (1 pt/box)
Plane 1 (fault)				x: y: z:	
Plane 2 (vertical)				x y z	
Plane 3 (inclined)				x y z	

- 8) Check: do the normals for Plane 1, Plane 2, and Plane 3 seem to point in the right direction based on a qualitative check of α , β , and γ ? For example, if α , β , and γ are all positive, then the pole trends northeast. State your reasoning below. **(3 points total)**

The remaining questions deal with the location where the borehole intersects the fault.

- 9) Fill out the matrices below for the equation $AX = B$, for which you know values. Circle the boxes that you need to solve for to find where the borehole intersects the fault. Then use Matlab to solve for the coordinates of where the borehole would intersect the fault, and fill in those coordinates in the appropriate matrix below (Hint: you might find the Matlab help pages for “Arithmetic Operators” and “inv” to be most helpful). (1 point per box; 1 point for circling the correct matrix; 16 points total).

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A				X		B

- 10) Using Matlab, determine the length of the borehole based on the coordinates of its drillpad and the coordinates of where it would intersect the fault. Compare the answer here with the answer from question 2 of Exercise 3. (2 points).

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At this point, take a few minutes to write down in your own words the steps you used in your procedure. This should help draw all the pieces together to help you understand what you did, and not have this lab seem like a long series of unrelated and incomprehensible steps.

Note: although you were not asked to do this here, you could have calculated the direction cosines for the normals to the planes directly from the coordinates of three points in each plane, just as you did in Exercise 1.