14. Folds

I Main Topics
   A Local geometry of a plane curve (cylindrical fold)
   B Local geometry of a curved surface (3D fold)
   C Numerical evaluation of curvature (geometry)
   D Kinematics of folding
   E Fold terminology and classification (geometry)

http://upload.wikimedia.org/wikipedia/commons/a/ae/Caledonian_orogeny_fold_in_King_Oscar_Fjord.jpg
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Anticline, New Jersey

Syncline, Rainbow Basin, California


Folds, New South Wales, Australia

Folds in granite, Sierra Nevada, California

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Local geometry of a plane curve (cylindrical fold) in a tangential reference frame

A Express the plane curve as a power series:

1. \[ y = C_0 x^0 + C_1 x^1 + C_2 x^2 + C_3 x^3 + \ldots \]

At \( x = 0 \), \( y = 0 \), so all the coefficients for terms with non-positive exponents must be zero

2. \[ y = C_1 x^1 + C_2 x^2 + C_3 x^3 + \ldots \]
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II Local geometry of a plane curve (cylindrical fold) in a tangential reference frame

\[ y = C_1 x + C_2 x^2 + C_3 x^3 + \ldots \]

Now examine \( y' \)

\[ y' = C_1 x^0 + 2C_2 x^1 + 3C_3 x^2 + \ldots = 0 \]

At \( x = 0, y' = 0 \), so \( C_1 = 0 \), so

\[ y = C_2 x^2 + C_3 x^3 + \ldots = 0 \]

As \( x \to 0 \), higher-order terms vanish

\[ \lim_{x \to 0} y = C_2 x^2 \]

\[ \lim_{x \to 0} k = |y(s)''| = |y(x)''| = 2C_2 \]

So all plane curves are locally second-order (parabolic).

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III Local geometry of a curved surface in a tangential reference frame

A Plane curves are formed by intersecting a curved surface with a plane containing the surface normal

B These plane curves \( z = z(x,y) \) are locally all of second-order, so any continuous surface is locally 2nd order. The general form of such a surface in a tangential frame is

\[ z = Ax^2 + Bxy + Cy^2 \]

This is the equation of a paraboloid: all surfaces are locally either elliptic or hyperbolic paraboloids

C Example: curve (normal section) in the arbitrary plane \( y = mx \)

\[ y = \lim_{x \to 0, y \to 0} z = Ax^2 + Bx(mx) + C(mx)^2 = (A + Bm + Cm^2)x^2 \]

Sum of constants

At \( x = 0, y = 0 \), \( z = 0, \frac{dz}{dx} = 0, \frac{dz}{dy} = 0 \)

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III Local geometry of a curved surface ... (cont.)

D Dilemma

1. Evaluating curvatures of a surface \( z_L = z_L(x_L, y_L) \), where \( z_L \) is normal to the surface, is easy.

2. The “global” reference frame, \( z_G = z_G(x_G, y_G) \), in which data are collected are usually misaligned with the tangential local reference frame.

3. Alignment is generally difficult.

E “resolution”

1. At certain places the local and global reference frames are easily aligned though: at the summits or bottoms of folds.

2. We will evaluate the curvatures there, leaving the more general problem to “later”.
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III Local geometry of a curved surface ... (cont.)
F Example (analytical)

\[ z_G = 4x_G y_G \]

1. First plot and evaluate \( z_G \) near (0,0)

\[
\text{>> [X,Y] = meshgrid([-2:0.1:2]);}
\text{>> Z=4*X.*Y;}
\text{>> surf(X,Y,Z);}
\text{>> xlabel('x'); ylabel('y');}
\text{>> zlabel('z'); title('z = 4xy')}\]

This is a saddle

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F Example (analytical) (cont.)

\[ z_G = 4x_G y_G \]

2. Now evaluate the first derivatives

a. \( \frac{\partial z_G}{\partial x_G} = 4y_G \)

b. \( \frac{\partial z_G}{\partial y_G} = 4x_G \)

c. Both derivatives equal zero at (0,0)

3. The local tangential and global reference frames are aligned at (0,0)

\[
\text{>> hold on}
\text{>> plot3(X(21,:),Y(21,:),Z(21,:),'r')}
\text{>> plot3(X(:,21),Y(:,21),Z(:,21),'y')}\]

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F Example (analytical) (cont.)

\[ z_G = 4x_G y_G \]

4 Now evaluate the second derivatives

\( a \ \frac{\partial^2 z_G}{\partial x_G^2} = 0 \)

\( b \ \frac{\partial^2 z_G}{\partial x_G \partial y_G} = 4 \)

\( c \ \frac{\partial^2 z_G}{\partial y_G \partial x_G} = 4 \)

\( d \ \frac{\partial^2 z_G}{\partial y_G^2} = 0 \)

5 Now form the Hessian matrix

\[
H = \begin{bmatrix}
0 & 4 \\
4 & 0
\end{bmatrix}
\]

6 Find its eigenvectors and eigenvalues

\[
\text{>> } \text{H=[0 4;4 0]}
\]

\[
\text{H =}
\begin{bmatrix}
0 & -4 \\
-4 & 0
\end{bmatrix}
\]

\[
\text{>> [v,k]=eig(H)}
\]

\[
v =
\begin{bmatrix}
-0.7071 & 0.7071 \\
0.7071 & 0.7071
\end{bmatrix}
\]

\[
k =
\begin{bmatrix}
-4 & 0 \\
0 & 4
\end{bmatrix}
\]

Saddle geometry much more clear in principal reference frame.
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IV  Evaluation of curvature from discrete data (geometry)
A  Three (non-colinear) points define a plane – and a circle.
B  Locate three discrete non-colinear points along a curve (e.g., L, M, N)
C  Draw the perpendicular bisectors to line segments LM and MN
D  Intersect perpendicular bisectors at the center of curvature C.
E  The radius of curvature ($\rho$) equals the distance from C to L, M, or N.
F  The curvature is reciprocal of the radius of curvature ($k = 1/\rho$)
G  Local geometry of a curve also is circular!

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IV  Kinematics of folding (strain)
A  Curvature of a plane curve
   \[ k = \frac{d\phi}{ds}, \] where
   \[ \phi = \text{orientation of tangent \( t \) to curve} \]
   \[ s = \text{distance along curve} \]
B  Curvature of a circular arc
   \[ \phi = \theta + 90^\circ, \text{ so } d\phi = d\theta \]
   \[ s = \rho \theta, \text{ so } ds = \rho d\theta \]
   \[ k = \frac{d\phi}{ds} = \frac{d\theta}{ds} = \frac{1}{\rho} \]
   Large curvature = small radius
   Small curvature = large radius
C  Curvature can be assigned a sign
   + = concave up
   - = concave down

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V Kinematics of folding (cont.)

D Layer-parallel normal strain ($\varepsilon_{\theta\theta}$) for cylindrical folds

1. Mid-plane of layer ($y = 0$) maintains length $L_0$

2. Layer maintains thickness $t$ during folding

$$
\varepsilon_{\theta\theta} = \frac{\Delta L}{L_0} = \frac{L_1 - L_0}{L_0} = \frac{(\rho + y)\theta - \rho \theta}{\rho \theta} = \frac{y}{\rho} = y k
$$

$$
\varepsilon_{\theta\theta}(y = \frac{t}{2}) = \frac{+tk}{2} \quad \text{(elongation)}
$$

$$
\varepsilon_{\theta\theta}(y = -\frac{t}{2}) = \frac{-tk}{2} \quad \text{(contraction)}
$$

Note: If convex curvature is considered negative, then all the equations here should minus signs on the right side.

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V Kinematics of folding (cont.)

E Layer-parallel normal strain for three-dimensional folds

1. Gauss’ Theorem: If the product of the principal curvatures (i.e., the Gaussian curvature $K = k_1 k_2$) a deformed surface remains unstrained

2. For geologic folds, the Gaussian curvature invariably changes during folding, so layer-parallel strains will occur on the surfaces as well as interiors of folded layers

$k_1 > 0$ Basin

$k_1 = 0$ Not Possible

$k_1 < 0$ Not Possible

$k_2 > 0$ Synform (cylindrical)$^*$

$k_2 = 0$ Plane$^*$

$k_2 < 0$ Saddle

Antiform (cylindrical)$^*$

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VI Fold terminology and classification

A **Hinge point:** point of local maximum curvature.

B **Hinge line:** connects hinge points along a given layer.

C **Axial surface:** locus of hinge points in all the folded layers.

D **Limb:** surface of low curvature.

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14. Folds (cont.)

D **Cylindrical fold:** a surface swept out by moving a straight line parallel to itself

1 **Fold axis:** line that can generate a cylindrical fold

2 **Parallel fold:** top and bottom of layers are parallel and layer thickness is preserved*

3 **Non-parallel fold:** top and bottom of layers are not parallel; layer thickness is not preserved*

* Assumption: bottom and top of layer were originally parallel
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VI Fold terminology and classification (cont.)

E Fleuty’s Classification

1 Based on orientation of axial surface and fold axis
2 First modifier (e.g., "upright") describes orientation of axial surface
3 Second modifier (e.g., "horizontal") describes orientation of fold axis

 Fold terminology and classification (cont.)

F Inter-limb angle

<table>
<thead>
<tr>
<th>Interlimb angle</th>
<th>Classification</th>
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<tbody>
<tr>
<td>180° - 120°</td>
<td>Gentle</td>
</tr>
<tr>
<td>120° - 70°</td>
<td>Open</td>
</tr>
<tr>
<td>70° - 30°</td>
<td>Close</td>
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