Parametric Representation Of Animal Trajectories

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University of Hawaii, February 17, 2004

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Motivation
Advection-diffusion-reaction models (ADRM) are widely used in models of the large-scale movement and distribution of tuna populations. Data from mark and recapture experiments are used to parameterize these population models, but it has proved difficult to use tracking data in population models. Positions estimates from sophisticated and expensive electronic tags are often difficult to interpret because of large errors. The state-space Kalman filter is widely used in time-series analysis and real-time navigation problems. This model can be re-cast in terms of ADRMs to provide a parametric representation of animal tracks for use in ADRMs and to reduce the errors in position estimates from electronic tags.

1. Random Walks and Diffusion

\[
\begin{align*}
&x_t = \lambda t + \epsilon_t \quad \text{for } t \in [0, T], \\
&x_0 \sim N(0, \lambda^2 T)
\end{align*}
\]

In a biased random walk, an animal moves a fixed distance, \( \lambda \), during each step, \( \epsilon \). The bias, \( \epsilon \), is the difference between the probabilities of moving to the right or to the left:

\[
\epsilon \equiv P(X_{t+1} = x_0 + \lambda) - P(X_{t+1} = x_0 - \lambda)
\]

Diffusion models define the local rate of change of population density as a function of gradients in population density and parameters of directed, \( u \) and non-directed movement, \( D \).

\[
\frac{\partial N}{\partial t} = - \frac{\partial N}{\partial x} D \frac{\partial^2 N}{\partial x^2}
\]

The parameters \( u \) and \( D \) of an ADRM can be recast in the context of a biased random walk.

\[
\lim_{\lambda \to 0} \frac{\lambda}{\tau} = u \quad \lim_{\lambda \to 0} \frac{\lambda}{\tau} = D
\]

Finally, \( u \) and \( D \) are closely related to the Normal probability distribution.

\[
p(t, x) = \frac{1}{\sqrt{2\pi Dt}} e^{-\frac{(t-x)^2}{2Dt}}
\]

2. State Space Kalman Filter

The state-space Kalman filter statistical model consists of three components: a description of the transition of the state of the system from one time step to the next, a description of the process of measuring the state of the system, and a maximum likelihood estimation procedure.

3. Transition

\[
\begin{align*}
\alpha_{i+1} & = \alpha_i + c_i + \eta_i, \quad i = 1, \ldots, T \\
c_i & = (u \Delta t, v \Delta t), \quad Q_i = \begin{pmatrix} 2D\Delta t & 0 \\ 0 & 2D\Delta t \end{pmatrix}
\end{align*}
\]

The tagged fish moves from position \( \alpha_i \) to position \( \alpha_{i+1} \), during the period \( \Delta t = t_i - t_{i-1} \), in the direction specified by the vector \( c_i \) and with some uncertainty \( \eta_i \), a \( N(0, Q_i) \) random variable.

4. Measurement

\[
y_i = z(\alpha_i) + d_i + \epsilon_i, \quad i = 1, \ldots, T
\]

\[
d_i = \begin{pmatrix} b_x \\ b_y \end{pmatrix}, \quad H_i = \begin{pmatrix} \sigma^2_x & 0 \\ 0 & \sigma^2_y \end{pmatrix}
\]

The “measured” position of the fish, \( y_i \), depends on its true position, \( \alpha_i \), a systematic measurement bias, \( d_i \), and a random \( N(0, H_i) \) error, \( \epsilon_i \). The bias and error terms need not be symmetrical and may vary in time.

5. Parameter Estimation

- **Movement Model Parameters:** \( u, v, D \)
- **Observation Model Parameters:** \( b_x, b_y, \sigma_x, \sigma_y \)
  - Uniform: \( \sigma^2_x = \sigma^2_y \)
  - Equinox Singularity: \( \sigma^2_x = \sigma^2_y \left( \cos^2(\theta / 2) \right) \)
  - Daily Deviations: \( \sigma^2_x = \sigma^2_y \Delta t \)
- **Most Probable Track:**
  - \( a_1 = E(\alpha_1 | y_1, y_2, \ldots, y_T) \)
  - \( a_{i+1} = E(\alpha_i | y_1, y_2, \ldots, y_T) \)
- **Maximum Likelihood:**

\[
\ln L = T \ln 2\pi + 0.5 \sum_{i=1}^{T} \ln |F_i| + 0.5 \sum_{t=1}^{T} w^T F_i^{-1} w_t + \sum_{t=1}^{T} \xi^T \xi_t
\]

6. Moored Archival Tag

The “nominal track” from the unprocessed light-based position estimates is shown in gray. The Kalman filter is able to reduce all of the variability in these data so that the “most probable tracks” can be plotted as a point.

7. Autocorrelated bias

Estimates of latitude based on light intensity time series often exhibit large, systematic, autocorrelated errors. These errors appear to be independent of tag manufacturer and algorithm. If no correction is applied to the data, estimated tracks will appear to exhibit north-south migrations associated with the equinoxes. The Kalman filter model can remove these errors in some cases. In other cases, such as the Coral Sea bigeye tuna tracks, special corrections were introduced in the Kalman filter model to remove the apparent north-south migration.

8. Bigeye Tuna — Hawaii

Bigeye tuna tagged near Cairns, Australia, with an archival tag. The “nominal track” from the unprocessed light-based position estimates is shown in gray. Most probable tracks shown as blue and green lines. There appears to be an association with the Queensland Plateau in the Coral Sea.

9. Bigeye Tuna — Coral Sea

The state-space Kalman filter statistical model consists of three components: a description of the transition of the state of the system from one time step to the next, a description of the process of measuring the state of the system, and a maximum likelihood estimation procedure.

10. Atlantic Loggerhead Turtle

A loggerhead turtle tagged with an Argos transmitter near the Azores. In spite of the very high accuracy of some Argos position estimates, there are large numbers of observations with much lower accuracy. The Kalman filter model is able to estimate the errors associated with the low-accuracy fixes.

11. Next steps:

- use the parametric representation of the tracks to identify “change points” in the track where the movement parameters change indicating different behavior
- re-examine the geolocation algorithms to identify the source of the autocorrelated latitude error
- incorporate sea surface temperature into the state space model to provide an objective means of using temperature to refine latitude estimates

References


Acknowledgments

Many thanks to my collaborators for giving generously of their ideas and data that made development of this model possible. This work was sponsored in part by the University of Hawaii’s Pelagic Fisheries Research Program under Cooperative Agreement number NA67RJ0154 from the National Oceanic and Atmospheric Administration.

\[\text{Cambridge University Press.}\]