DYNAMICS OF MAGMA STORAGE
IN THE SUMMIT RESERVOIR OF KILAUEA VOLCANO, HAWAII

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ABSTRACT

Gravity, leveling, tilt, and trilateration data recently obtained at Kilauea Volcano indicate that magma is accommodated within the summit reservoir by a combination of elastic magma compression, elastic chamber expansion, and gradual inelastic edifice widening in a south-southeast direction.

Elastic (reversible) magma storage is modeled using data from brief summit deflation episodes and assuming a spherical geometry for the magma reservoir. Changes in pressure in the source produce the deformation; changes in mass in the source produces the gravity changes. The ratio of residual gravity change (corrected for the free-air effect) to elevation change is dependent on both the aggregate magma bulk modulus (which constrains bulk compression of the reservoir contents) and the edifice shear modulus (which constrains distortion of the surrounding edifice).

The aggregate compressibility of Kilauea's reservoir is significantly increased by volumetric changes of the CO₂ gas phase by bulk compression and solution effects with pressure. This effect is greater for events with a shallow source depth because the bulk modulus of gas is proportional to pressure. The average reservoir CO₂ weight fraction may be near 0.0005 to 0.0009. CO₂ may be more concentrated (0.0030 weight fraction CO₂) at shallow levels.

Magmastatic pressure variation implied by changing elevation of the surface of the lava column filling the Mauna Ulu and Pu'u O'o is at least 0.43 Pa/m³ of associated summit uplift volume. Uplift volumes for these events were relatively small. In contrast, major deflations give a lower ratio of less than 0.22 Pa/m³ determined from vent elevation and magma density versus deflation data. This variation in the rate of pressure change with deformation indicates differing effective source volume, ranging from 1.6 km³ to more than 13 km³. An explanation is that an initially small source may evolve to a much larger
volume with time and strain, encompassing a significant portion of the total magma reservoir region, as the boundary between plastic and elastic behavior migrates outward.

Trilateration data show that Kilauea's edifice widens inelastically in time intervals of months to years, primarily in a southeast direction. Over these long time periods, gravity and elevation changes indicate surface subsidence accompanied by no mass change, or even mass addition. This implies that horizontal extension of Kilauea's edifice allows magma injection without vertical uplift.
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<tr>
<td>γ</td>
<td>universal gravitational constant</td>
<td>6.67x10^{-11} Nm²kg⁻²</td>
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<tr>
<td>μ</td>
<td>crustal shear modulus</td>
<td>GPa</td>
</tr>
<tr>
<td>ν</td>
<td>edifice Poisson's ratio</td>
<td>dimensionless</td>
</tr>
<tr>
<td>ρ</td>
<td>magma density</td>
<td>2600 kg/m³, Fujii and Kushiro, 1977</td>
</tr>
<tr>
<td>Δτ</td>
<td>tilt change</td>
<td>μradian</td>
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<tr>
<td>ω</td>
<td>mass of 1 mole of CO₂</td>
<td>0.044 kg/mole</td>
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<tr>
<td>b</td>
<td>CO₂ solubility constant</td>
<td>5.9x10^{-12}Pa⁻¹</td>
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<tr>
<td>Δg</td>
<td>gravity change</td>
<td>μGal</td>
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<tr>
<td>Δg*</td>
<td>free-air corrected gravity change</td>
<td>μGal</td>
</tr>
<tr>
<td>Δh</td>
<td>height change</td>
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<td>K</td>
<td>magma bulk modulus</td>
<td>11.5 GPa, Murase et al., 1977</td>
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<td>ΔM</td>
<td>reservoir magma mass change</td>
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<td>ΔM*</td>
<td>lava flow or dike injection mass</td>
<td>kg</td>
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<tr>
<td>m</td>
<td>exsolved CO₂ in magma</td>
<td>weight fraction</td>
</tr>
<tr>
<td>N</td>
<td>total CO₂ in magma</td>
<td>weight fraction</td>
</tr>
<tr>
<td>n</td>
<td>limit of dissolved CO₂ in magma</td>
<td>weight fraction</td>
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<tr>
<td>ΔP</td>
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<tr>
<td>R</td>
<td>gas constant</td>
<td>8.314 m³Pa/mole°K</td>
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<td>T</td>
<td>reservoir magma temperature</td>
<td>1420-1470°K</td>
</tr>
<tr>
<td>Vr</td>
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<td>m³</td>
</tr>
<tr>
<td>ΔVr</td>
<td>source volume change</td>
<td>m³</td>
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<tr>
<td>Ve</td>
<td>edifice volume change</td>
<td>m³</td>
</tr>
<tr>
<td>V₀</td>
<td>exsolved CO₂ in reservoir</td>
<td>m³</td>
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<tr>
<td>X</td>
<td>horizontal distance to source</td>
<td>m</td>
</tr>
<tr>
<td>Z</td>
<td>depth to source</td>
<td>m</td>
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PREFACE

My dissertation fieldwork began in the crater of Mount St. Helens, Washington in March 1982. This work continued for almost two years; however, the difficult logistics of a remote mountainous environment, generally harsh weather, and lack of interesting data forced me to look for a new volcano to study. Hawaiian volcanoes are ideally suited for field work in general and gravity surveys in particular because of the rapid pace and large volume of their activity, and the shallow (3 km) depth of magma reservoirs. Their eruptions are relatively safe and generally destroy only a few benchmarks set on top of the edifice. The Hawaiian Volcano Observatory is located attractively on the crater rim and the summit area is circled by a road. A new program of fieldwork in Hawaii was begun in January 1984 with the goal of better modeling the dynamics of magma storage in the summit reservoirs of Hawaiian volcanoes. Work included finding and processing previously collected gravity data, maintaining a schedule of weekly (and more frequent near the time of Pu'u O'o eruptive episodes) high-quality gravity observations at the summit area of Kilauea, and obtaining baseline gravity observations at Mauna Loa before the eruption that was anticipated by Decker and others (1983). A compilation of the Hawaiian gravity data may be obtained from the Hawaiian Volcano Observatory.

Some of the first applications of gravity methods to modeling the dynamics of a volcano are based on two previous surveys between November 1975 and October 1977 (Jachens and Eaton, 1980; Dzurisin et al., 1980). These papers demonstrate two important points: careful gravity and geodetic measurements are capable of detecting mass changes of magma within Kilauea's summit reservoir, and the relation between uplift and mass change is variable and does not follow a simple pattern.

My studies have focused on understanding the physical properties of the volcano that govern the relation between mass change and uplift. The model that I have developed
for Kilauea now includes consideration of the mechanical properties of the volcano edifice and reservoir, gas content of the magma, and proposed structural models for Kilauea's magmatic system. Some initial results are published as a chapter of a compilation on volcanism in Hawaii by the U.S. Geological Survey [Johnson, 1987]. A copy of this paper is included here in the appendix.

This dissertation not only advances the evolution of gravity as a volcanological tool, but introduces a new strategy, or way of thinking, in volcanology. Gravity studies, as a monitoring technique, enable more accurate estimates of the magma budget of Hawaiian volcanoes. Analyses that consider the theoretical and empirical relation between magmatic events at depth and crustal response tap a rich source of previously unexplored data. By treating together material properties and volcanic processes, we may derive a more complete understanding of the forces shaping the timing and nature of volcanic activity. This may lead to improved predictions of future behavior of monitored volcanoes.
I. INTRODUCTION

Active Hawaiian shield volcanoes contain a fluid filled summit reservoir. Magma, buoyantly rising from a mantle source [Eaton and Murata, 1960], accumulates within the reservoir where its further ascent by buoyancy alone is prevented by the low density of the overlying crust [Ryan, 1987]. The reservoir itself may consist of a plexus of dikes and sills separated by screens of solid rock [Fiske and Kinoshita, 1969; Davis et al., 1974]. Surrounding the fluid-filled region is likely a zone of high-temperature material, possibly containing pockets of partial melt. This volume of fluid and high-temperature (hence plastic) material associated with Kilauea's magma storage region is large enough to produce a detectable aseismic zone [Koyanagi et al., 1976] and seismic low-velocity zone [Thurber, 1984]. Source locations from inversion of vertical surface displacement data [Fiske and Kinoshita, 1969; Dvorak et al., 1983] are generally within the seismic anomaly zone. Migration of the apex of inflation or deflation with time is thought to reflect shifts of magma transfer between discrete areas (presumably individual dikes or sills) [Fiske and Kinoshita, 1969; Dvorak et al., 1983]. The seismic and deformation data define the reservoir region from 2 km down to 6 km below the south rim of Kilauea Caldera.

Shallow magma transfer to and from Kilauea's central reservoir is accompanied by crustal deformation, as the surrounding solid edifice is inflated or deflated to accommodate volume changes of magma [Mogi, 1958; Eaton, 1962]. Magma accommodation may also be by bulk compression or decompression of the entire chamber magma and gas content [Sanderson et al., 1983; Eggers, 1983; Johnson, 1987]. These processes solve the "room problem" associated with magma transfer. The balance between the role of each process, crustal deformation and magma compression, hinges on the relative elastic properties of crust and magma [Johnson, 1987]. Whichever component is more deformable takes the
lead role in making space for new magma. Subsurface magma transfer also varies the internal magma pressure and external crustal stress distribution [Mogi, 1958]; the magnitude of the effect depends on the elastic moduli of the edifice and magma reservoir.

This dissertation is motivated by an interest in determining variations in pressure and volume of the reservoir, magma CO₂ content, and effective volume of the portion of the reservoir that serves as the source of surface deformation. Previous studies of Kilauea geodetic data have been directed at resolving the geometry and location of the sources. In contrast, this study focuses on the mechanical properties of the magma reservoir and surrounding edifice; properties responsible for the key observations of uplift or subsidence and gravity change. Gravity monitoring is used as a means of detecting mass changes in the summit magma reservoir. This may be verified in some cases of deflation by comparison with the volume of lava and intrusives reaching the surface and rift zone. Geodetic monitoring of surface displacement measures crustal strain associated with magma transfer.

At least the upper portions of Kilauea's rift zones are believed to contain a fluid core such that magma may pass through without deforming the surrounding country rock. For example, consider seismicity and deformation associated with the January 1983 eruption. Magma drained from the reservoir underneath a subsiding summit and related seismicity. On its passage to the eruption site in the middle of the east rift zone it produced only limited deformation and seismicity indicating an open conduit. The forceful magma intrusion near the eruption site however was marked by measurable uplift and intense earthquake activity.

Analysis is restricted to periods of rapid summit deflation accompanying rift zone eruptions and intrusions. These events are brief and have a locus of eruption or intrusion sufficiently distant from the summit reservoir and monitoring sites such that the specific effects of magma withdrawal and summit subsidence are isolated. To avoid modeling the complex effect of anomalous horizontal extension produced by a major earthquake [Lipman
et al., 1985] and near-surface dike intrusion and eruption above the summit deflation source [Tilling et al., 1976], the November 1975 deflation [Jachens and Eaton, 1980] is not considered here. Analysis of gravity and geodetic data from several brief summit deflations associated with the eruption at the Pu'u O'o vent have been previously published [Johnson, 1987]. This study supplements these earlier data with observations from later episodes, as well as data for major Kilauea summit subsidences in August 1981 and January 1983.
II. A MODEL FOR THE DYNAMICS OF KILAUEA VOLCANO

Analysis of crustal deformation and gravity changes is based here on a point source model approximating a spherical source. Such a model is used because: (1) it generally gives satisfactory fits to short-term vertical displacement data [Dvorak et al., 1983]; (2) there is no seismic or geodetic evidence for shallow dike injection above the Kilauea central reservoir during the selected deflation intervals; and (3) use of a more complex model is not justified by the available data. Davis [1986] attributes the general success of the point source model to the dynamics of the reservoir region that behaves as a fluid-filled cavity because of deformation of high-temperature plastic and strain-fractured solid elements. With time the pattern of deformation from a source of any geometry may appear more like that of a larger spherical source because of outward migration of the plastic deformation zone.

Gravity monitoring will detect accumulation or loss of mass (presumably magma) from reservoirs within the crust if corrections are applied for (1) vertical movement of the observation point in the Earth's gravity field (the free-air effect), and (2) deformation-induced density change of the crust surrounding a reservoir. A free-air gradient of -0.3273 µGal/mm is used here. This value was determined by observing the gravity change with a 930 mm height change at three sites within the Kilauea Caldera. Adjusted data residuals are indicated as $g^*$. A positive Bouguer gravity anomaly of +320 milligals in the summit region of Kilauea [Kinoshita et al., 1963] is probably the source of this abnormally high gradient [Hammer, 1970]. The density-change correction of (2) was assumed to be zero. This is theoretically valid for a hydrostatically inflated sphere, or point source of dilatation [Rundle, 1978; Walsh and Rice, 1979] and for multiple point sources [Sasai, 1986]. Note that there is a gravity correction (2) for other, nonaxisymmetric source geometries, such as
vertical and horizontal cracks in which the component double forces are not equal [Savage, 1984; Sasai, 1986].

The inflation of a spherical source in a homogeneous isotropic elastic half-space (which simulates a reservoir source in an edifice) causes vertical displacement of a point on the surface (modified from Mogi [1958])

\[ \Delta h = \frac{\Delta V_e \cdot Z}{2 \pi (X^2 + Z^2)^{3/2}} \]  

(1)

where \( Z \) is the source depth, \( X \) the radial distance of the point from the source epicenter, and \( \Delta V_e \) is integrated volume of uplift. This \( V_e \) is a measure of the source strength and is given by

\[ \Delta V_e = \frac{3 \Delta P \cdot V_r \cdot (1-\nu)}{2 \mu} \]  

(2)

where \( \Delta P \) is the pressure change, \( V_r \) is the volume of the source, and \( \nu \) and \( \mu \) are the Poisson's ratio and shear modulus of the edifice.

The gravity data considered here have poor areal coverage so there is insufficient information to solve for both the location and the magnitude of mass change at depth from gravity changes alone. Therefore, to solve for mass change, \( \Delta M \), a source location is derived from the inversion of surface displacement measurements. This is a hidden step in practice as the functional form of the equations for gravity change and uplift are identical, such that the geometrical parameters cancel when the two equations are combined. In this way, combination of equations 1 and the familiar equation for the gravity effect of a point mass [Dobrin, 1960, p. 374] gives

\[ \frac{\Delta M}{\Delta V_e} = \frac{\Delta g^*}{2 \pi \gamma \cdot \Delta h} \]  

(3)
which relates magma mass accumulation or loss and deformation to gravity change and uplift. The value of $\gamma$ is $6.67 \times 10^{-11}$ Nm$^2$kg$^{-2}$.

A good spatial correlation between gravity and elevation change has been found at Kilauea [Jachens and Eaton, 1980; Dzurisin et al., 1980]. This supports the validity of the modeling procedure utilized here. Poorer correlation would be observed if the mass change and dilatation source were not coincident, if the source were to behave much differently than a sphere of uniform dilatation, or if there were a significant lateral variation in crustal elastic properties. The data analysed here are too sparse to evaluate the spatial integrity of the gravity to height change correlation. The correlation is assumed to be good such that relative changes may be considered proportional to absolute changes.

The deformation source may coincide with the entire reservoir region or, more likely, it may be only a small portion of the whole reservoir. Davis [1986] envisioned the magma chamber as a tangle of dikes and sills within a matrix of solid rock. Although the solid matrix may have a short-term ability to support shear strain, eventual failure and creep effects lead to an outward migration of the fluid-elastic boundary, that is, the hydrostatically equilibrated source region, $V_r$, becomes progressively larger. $V_r$ is that sub-volume of the entire reservoir which behaves as a fluid (or plastic) over the time interval of the deformation event. $V_r$, in addition to a melt phase, most likely contains subsolidus rock, its aggregate bulk modulus however is assumed equal to that of the melt. This assumption is valid because the effect of the melt phase is to reduce the aggregate bulk modulus to near that of melt [Brown and Korringa, 1975; Ryan, 1980]. External to the fluid source region, behavior is elastic.

Magma arriving in Kilauea's central reservoir from the mantle is estimated by Greenland et al. [1985] to contain 0.0032 CO$_2$ by weight. This value was derived by comparing measurements of the sum of CO$_2$ (in noneruptive summit fumaroles, eruptive plumes, and trapped in lavas) with the total lava output. Gerlach and Graeber [1985] give a
value of 0.0065 by weight, derived from volcanic gas analysis of sustained summit lava lake activity and residual concentrations in fountain spatter.

A separate CO₂ gas phase may be found in pockets of magma within the 3-km-deep reservoir because CO₂ has a low solubility at the corresponding pressure. Harris [1981] measured the concentration of CO₂ in vesicle-free glass separates from submarine tholeiitic basalts quenched at pressures of as much as 50 MPa. He found the limiting weight fraction, \( n \), of CO₂ that may be dissolved in a basalt melt as a function of pressure \( P \) is

\[
n = 5 \times 10^{-6} + bP
\]

(4)

where the solubility constant \( b \) is 5.9\( \times \)10\(^{-12} \) for pressure in units of Pa. Exsolved CO₂ gas, \( m \), in the case of a saturated melt is given as the total CO₂, \( N \), minus the portion that remains in solution; there is no gas phase if \( N \) is less than the saturation limit. These relations are given by

\[
m = \begin{cases} 
N - bP & (N > bP) \\
0 & (N < bP)
\end{cases}
\]

(5)

where the relatively small constant in the solubility equation has been neglected.

The ideal gas law is satisfactory for calculating the volume of exsolved gas at the pressure found in 3-km-deep reservoirs. Mass fraction figures for CO₂ are multiplied by the source volume \( V_r \) and magma density \( \rho \) to obtain the total volume of gas in the source

\[
V_g = m \left( \frac{\rho V_r}{P} \right) \frac{RT}{\omega} = \frac{\rho V_r}{\omega} \left[ \frac{N}{P} - b \right]
\]

(6)

The mass of 1 mole of CO₂, \( \omega \), is 0.044 kg. \( R \) is the gas constant, equal to 8.314 m\(^3\)Pa/mole\(^o\)K, and \( T \) is the temperature, here assumed to be equivalent to magmatic temperatures at reservoir conditions of 1420-1470\(^o\)K for Kilauea [M. Garcia, personal commun.]. The volume of gas changes with pressure as a result of bulk compression and solution effects. This relation is found by taking the derivative of equation 6.
\[
\frac{\partial V_g}{\partial P} = -\frac{\rho V_r N R T}{P^2 \omega} \tag{7}
\]

Three mechanisms that change the volume of the source region are added together; the total change is
\[
\Delta V_r = \frac{\Delta M}{\rho} - \frac{\Delta P V_r}{K} - \frac{\Delta P \rho V_r N R T}{P^2 \omega} \tag{8}
\]

Each term on the right hand side of (8) represents a particular physical contribution to the volume change, \(\Delta V_r\), of the reservoir: the first term stems from magma injection or withdrawal and may be detected gravimetrically; the second term stems from bulk compression (or expansion) with pressure of the magma and finally the third term is due to gas phase volume change with pressure (including change with volume compression or expansion and solution in the melt). A value of 11.5 GPa, determined for gas-free Kilauea 1921 olivine tholeiite [Murase et al., 1977], is appropriate for \(K\); the density of Kilauea basaltic melt has been measured near 2600 kg/m\(^3\) [Fujii and Kushiro, 1977].

As the source expands or contracts, the ground surface is displaced with magnitude given by [Johnson, 1987]
\[
\Delta V_e = 2 (1-\nu) \Delta V_r \tag{9}
\]

Equation 8 is rearranged to solve for \(\Delta M\), and each term divided by \(\Delta V_e\), or the equivalent as given by equations 2 or 9
\[
\frac{\Delta M}{\Delta V_e} = \frac{\rho}{2 (1-\nu)} + \frac{2 \rho \mu}{3 (1-\nu)} \left[ \frac{1}{K} + \frac{\rho N R T}{P^2 \omega} \right] \tag{10}
\]

Equation 10 shows that, for a given mass change at depth, a relatively strong edifice (e.g. large \(\mu\)) implies more magma and gas compression and, hence, less uplift volume \(\Delta V_e\). Conversely, stiff magma with little CO\(_2\) (e.g. large \(K\) and small \(N\)) compresses less and forces the edifice to deform more, thus producing greater uplift volume. The role of CO\(_2\) in
accommodating volume change of magma, thereby limiting crustal deformation, is greater at low pressure.

The intrinsic shear modulus of Hawaiian basalt was found by pulse transmission to average 25 GPa with a range of 4.7 to 40.5 GPa (Manghnani and Wollard, 1968). Ryan [1987] found a shear modulus of 26 GPa for a sample of olivine tholeiite basalt. The effective shear modulus of Hawaiian volcanic edifices should be less than the above laboratory values because these volcanoes are built of many vesicular, commonly rubbly lava flows with abundant void space [Ryan et al., 1983]. Detailed studies of seismic wave velocities provide some information about the rigidity of large parcels of crust. A shear modulus of approximately 9 GPa is suggested by the $P$-wave velocity of near 3500 m/sec in the upper few kilometers of Kilauea [Crosson and Koyanagi, 1979], given a density of 2300 kg/m$^3$ [Kinoshita et al., 1963], and a Poisson's ratio of 0.25. Again, this specific value may not be appropriate for applications in crustal deformation modeling. First, it is a dynamic value; crustal response to a static stress imposed over periods of days to years may be different. Secondly, there are large lateral and vertical variations in crustal strength within the edifice. Seismic velocities and, hence, rigidity determined from $P$-wave arrival data from earthquakes and explosions increase with depth and vary laterally (Thurber, 1987; Hill and Zucca, 1987) reflecting the compressive reduction of porosity with increasing depth and the contrast in strength between solidified intrusive zones and country rock.

Laboratory values of $\mu$ are considered to be an order of magnitude larger than values for large fractured rock masses loaded in-situ [Rubin and Pollard, 1987]. A $\mu$ of less than 3 GPa is proposed by Davis et al. [1973, 1974] and Davis [1986] to explain the absence of a piezomagnetic effect during periods of observed crustal deformation at Kilauea. They surmised that anelastic failure of crustal material within the summit of
Kilauea, such as by earthquakes during periods of intense deformation, produces such a low value for $\mu$ and prohibits significant storage of elastic shear stresses.

Sufficient gas may be present in Kilauea's reservoir to be an important factor in determining source compressibility. A previous estimate of the shear modulus of Kilauea's edifice by Johnson [1987] was determined for summit subsidences associated with Pu'u O'o eruptions with an assumption of no gas present in Kilauea's reservoir. This assumption was based on an interpretation by Greenland et al. [1985] that CO$_2$ is lost from the magma as soon as it reaches shallow crustal levels. The value for $\mu$ of 23 GPa by Johnson [1987] is similar to the laboratory estimates and is probably too high considering the evidence given above that structural defects should result in an effective shear modulus of the volcano that is less than the intrinsic shear modulus of hand samples. If a lower value for crustal $\mu$ is used then the effective $K$ calculated is lower than the gas-free $K$ by Murase et al. [1977], which indicates the presence of a separate compressible gas phase.
III. DATA

The key datum needed for analysis of the mechanical properties of Kilauea's summit is the ratio of reservoir mass loss to surface subsidence. Without the gravity data, mass loss may be approximated using the volume of newly injected rift zone dikes and lava flows. Mass loss will be worked out in both ways so as to give an independent check of the interpretation of the gravity observations. The mass of intruded magma and erupted lava are found below to be in reasonable agreement with the gravimetrically-determined mass of magma drained from the summit reservoir. A density of 2600 kg/m³ is assumed for molten basalt intruded in rift zone dikes [Fujii and Kushiro, 1977]. The density of solidified lava flows has been estimated at 2300 kg/m³ [Wolfe et al., 1987].

Gravity observations

Gravity surveys before 1984 consist of at least 3 closed loops on separate days with a single gravimeter. Surveys done after January 1984 involved two gravimeters run over two or more closed loops on the same day, usually within a few hours. Gravity readings are corrected for tidal effects [Longman, 1959] using a compliance factor of 1.16. Calibration functions with linear and periodic terms were determined from calibration ranges and applied to the data. A least-squares procedure was used to simultaneously solve polynomials approximating time-dependent changes in the reading level of the gravimeters (gravimeter drift), offsets of the reading level (tares) as needed, and relative gravity, g, at each surveyed station. A second-order (or sometimes higher) polynomial was selected if permitted by a sufficient number of redundant observations. The effects of anomalous earth tides, ocean tides, and variations in atmospheric pressure, presuming that they are more or less uniform over the relatively small network areas, will appear as a gradual change in reading level at all stations and will be removed by the drift correction. No
Figure 1.—Location map of monitoring networks in summit area of Kilauea volcano, showing gravity benchmarks as dots, tilt stations as triangles, and leveling routes as dashed lines. Principal gravity sites labeled; continuously-recording electronic tiltmeter indicated as UWE. Lines show major fault scarps and craters.
attempt was made to correct the gravity data for water-table changes because base and monitoring sites at Kilauea are located within a short distance of each other, and in areas of similar rainfall intensity, so gravity variations caused by water-table changes are probably similar at all sites and do not affect the relative gravity difference.

**Magma Transfer during the August 1981 southwest rift zone intrusion**

A magmatic event began August 10, 1981 and lasted two days. It drained magma from the summit reservoir into the southwest rift zone [Pollard et al., 1983; Dvorak et al., 1986; Klein et al., 1987]. Approximately 45x10^6 m^3 of summit subsidence, reflecting magma withdrawal, has been estimated by Dvorak et al. [1986] from height changes between level surveys conducted June 1 and August 13. Tilt changes at UWE that indicate inflation between the first leveling survey and onset of deflation suggest that the actual collapse volume may be near 56x10^6 m^3. The pattern of vertical deformation in the southwest rift zone measured during this period has been interpreted by Pollard et al. [1983] to indicate shallow dike intrusion. The dike is estimated [Dvorak et al., 1986] to be within 250 meters of the surface, with a width of about 1 meter and a height of 3 kilometers. The injection volume is 75x10^6 m^3 if the dike extends with these dimensions for the full 25 kilometer length of the associated seismic swarm [Klein et al., 1987].

This volume together with the density of the injected magma of 2600 kg/m^3 gives an estimated $\Delta M^*/\Delta V_e$ ratio of 3480 kg/m^3 for this August 1981 summit subsidence. In comparison, the mass loss/subsidence volume ratio estimated from the residual gravity and height change data gives 3050±1645 kg/m^3. This value is derived using equation (3) and the $\Delta g*/\Delta h$ relation of 0.128±0.069 µGal/mm given by a least-squares fit to the equally-weighted data in figure 2A.
Figure 2.--Changes in observed gravity $\Delta g$ plotted (with x symbols) against corresponding elevation changes $\Delta h$. Open circles plot gravity residuals $\Delta g^*$ after free-air correction for the effect of the vertical movement of the observation point. Observations are for Kilauea summit stations where both data types obtained. Height changes determined by optical leveling to third order standards [HVO, unpublished data]; uncertainties in the gravity changes are less than ±15 µGal. A, Data for August 1981 southwest rift zone intrusion. Gravity changes between June 24-26 and September 24-October 4 surveys; leveling surveys in early-June and mid-August [HVO, unpublished data]. B, Data for January 1983 east rift zone eruption. Initial gravity survey October 27 and 28, and December 2, 1982; post-deflation survey February 15-22, 1983. Leveling data from Wolfe et al. [1987].
Magma transfer during the January 1983 east rift zone intrusion and eruption

A significant summit subsidence event occurred on January 2, 1983 and lasted for 6 days. A dike was intruded in the middle east rift zone area and erupted to the surface [Wolfe et al., 1987; Dvorak et al., 1986]. The volume of summit subsidence from inversion of tilt data (obtained using spirit-level technique measuring differential tilt between three nearby benchmarks) is 72x10^6 m^3 [Dvorak and Okamura, 1987]; from level data it is 61x10^6 m^3 [Dvorak et al., 1986]. Again tilt changes at the continuously-recording meter at UWE between the time of the geodetic surveys and the deflation indicate that the actual subsidence volume may have been greater than that recorded by the geodetic surveys by a factor of 1.37 [HVO, unpublished data]. So, \( \Delta V_e \) is estimated at -83x10^6 m^3 (level data) to -98x10^6 m^3 (spirit-level tilt data). Dvorak et al. [1986] modeled horizontal displacements observed in the vicinity of the affected segment of the east rift zone and estimated an intrusive volume of 98x10^6 m^3 for the dike. Wolfe et al. [1987] estimate a dike volume of 120x10^6 m^3 based on observed surface extension of one distance monitor across the dike trace and extent of the intrusive earthquake swarm. With a dike length equal to that of the eruptive fissure the volume is 60x10^6 m^3. An additional 14x10^6 m^3 of lava reached the surface [Wolfe et al., 1987] which must be added to the intrusive volume in calculating \( \Delta M^* \).

Using an average of the two subsidence volume estimates for \( \Delta V_e \) (1.37x(61+72)x 10^6/2=92x10^6), and with \( \Delta M^* \) as given by the sum of the intruded [Dvorak et al., 1986] and extruded volumes times their respective density (98x10^6x2600+ 14x10^6x2300), \( \Delta M^*/\Delta V_e \) is an estimated 3170 kg/m^3. The unweighted residual gravity versus height change spanning the major January 1983 deflation of Kilauea's summit is \( \Delta g^*/\Delta h=0.110\pm0.018 \mu \text{Gal/mm} \) [fig. 2B]. The \( \Delta M/\Delta V_e \) ratio implied by this and equation (3) is 2625\pm430 kg/m^3.
**Magma Transfer during 1983-1986 Puʻu Oʻo eruption**

The Puʻu Oʻo vent is located in the middle of Kilauea's east rift zone and began its activity following the January 1983 summit subsidence and rift zone intrusion. During subsequent activity the summit of Kilauea showed a pattern of cyclic inflation and deflation superimposed on a long-term trend of collapse [Johnson, 1987; Wolfe et al., 1987]. Summit tilt as recorded at UWE (see fig. 1 for location) is given in figure 3; on this graph increasing values of tilt generally accompany inflation. Each episode of vigorous lava effusion is marked by rapid deflationary tilt. Summit inflation during the repose intervals and rapid deflation during lava effusion at the vent, is widely regarded as evidence for significant magma transfer from the summit reservoir to the rift zone at the time of each of the eruptive events [Dvorak and Okamura, 1985; Johnson, 1987; Wolfe et al., 1987].

Estimating elevation changes, volume of summit subsidence, and source depth during each event of the Puʻu Oʻo eruption series is virtually impossible for the first 30 eruptive episodes after the major subsidence, as no attempt was made to bracket deformation cycles with geodetic surveys. Such an effort was considered at the time to be not worthwhile as the magnitude of cyclic deformation was relatively slight and the period averaged only three weeks. Spurred by preliminary results from this gravity investigation, bracketing geodetic surveys started with episode 32 in April 1985.

For episodes 2-31 of Puʻu Oʻo activity tilt vectors from three continuously recording sites in the Kilauea summit area [HVO, unpublished data] point to a source of cyclic deformation below the south rim of Kilauea Caldera. The recording tilt data alone cannot be inverted because they are too few, spatially poorly distributed, and have a high noise content. Leveling data from December 1983 to July 1984 are given in figure 17.26B of Wolfe et al. [1987]. The center of elevation change was also at the southern margin of the caldera. Inversion of these leveling data yields a source location 3.5±0.24 km below
Figure 3.--East-west tilt at UWE, November 1982-July 1986 continuously recording, 1-m-base, mercury-capacitance meter. Tilt relative to an arbitrary baseline; positive on record is east up. Rapid decreases in tilt reflect summit deflation associated with the numbered eruptive episodes at Pu'u O'o vent.
Figure 4.--Map and cross section views of deformation source locations determined from point-source solutions to Kilauea level or tilt data. Location corresponds to lower left corner of label; horizontal and vertical uncertainty less than ±300 m for all solutions. Alphabetical plot labels are keyed to survey intervals. A, Map view; principal fault scarps and craters shown as lines. B, North-south cross-sectional view; no vertical exaggeration.

SYMBOLS:
A. Mauna Ulu deflations
B. August 1981 intrusion
C. January 1983 eruption
E. July 1984–Feb. 1985
F. Episode 32, April 1985
G. Episode 38, Oct. 1985
H. Episode 39, Nov. 1985
I. Episode 40, Dec. 1985
the south caldera rim (fig. 4). Inversion of vertical changes between the July 1984 survey and one done on February 5, 1985 [HVO, unpublished data], gives a source one kilometer north of the earlier interval and a shallower depth of 2.5±0.17 km (fig. 4).

Previous analysis by Wolfe et al. [1987] of early Pu' u O'o eruption and summit subsidence volume used tiltmeter data and empirical volume-to-tilt correlations from data collected before the Pu'u O'o epoch [Dzurisin et al., 1984; Dvorak et al., 1986]. One drawback to use of these factors is that they represent an average; the actual tiltmeter response to a specific deformation event greatly depends upon the source location. Tiltmeter response is found by taking the derivative of equation 1 with respect to $X$

$$\frac{\Delta V_e}{\Delta \tau} = \frac{2 \pi (X^2+Z^2)^{5/2}}{3 X Z}$$

Figure 2 of Dzurisin et al. [1984] gives the range of $\Delta V_e/\Delta \tau$ expected for various source locations between 2 and 4 km horizontal, and 2 to 4 km vertical from a tiltmeter. Factors range from 0.09 to 0.75 million cubic meters per µrad (in an azimuth directly radial to the source). Since sources have been previously observed at the entire range of distances and depths covered by Dzurisin et al. [1984], determining the source hypocenter is certainly a necessary prerequisite to extrapolating edifice volume change from tilt change. So this method cannot be applied to the summit deflations of the pre-April 1985 Pu'u O'o eruptions because of the lack of information on the location of the cyclic source.

Some long-term leveling data from 1984, interpolated by tilt change, was used by Johnson [1987] to model cyclic Pu' u O'o deflation data. Use of this technique has been reconsidered and rejected here because of uncertainty about what the long-term tilt and leveling data actually reflect. Because of the lengthy interval between geodetic surveys in 1984-April 1985, measured changes are contaminated by the effects of long-term processes. Gradual and sustained southeast directed rifting, summit collapse, and gravity increase at the free-air rate characterizes the long-term behavior of Kilauea during this time.
[Johnson, 1987]. So different is this pattern from that expected for simple brief inflation or deflation that it is prudent to avoid long-term data for short-term modeling.

The northerly migration of the short-term source continued after February 1985. By April 1985 the cyclic eruption related displacements focused near the east side of Halemaumau Crater, while long-term collapse continued to the south. Surface displacement surveys that spanned individual short periods of deformation were begun at this time to isolate the short-term source (table 1, fig. 4). The first of these surveys involved leveling a profile from P-1 to HVO34 (fig. 1) just before and after the episode 32 deflation of April 21, 1985. A point source model given by equation 1 [Mogi, 1958] was used to estimate $\Delta V_e$, which differs from the value given by Johnson [1987] because data for a short spur route near Halemaumau crater was not considered previously. A monitoring program utilizing the spirit-level tilt technique was carried out between October 1985 and January 1986 [HVO, unpublished data]. Measurements were made before and after eruptive deflations at three to five sites, and supplemented by data from the two-component water-tube tiltmeter at Uwekahuna. Source parameters were estimated with equation 11. The short-term spirit-level tilt monitoring at Kilauea was discontinued after January 1986. Hence, source parameters for episodes 41 to 48 are unknown, but may be inferred from a dominantly east-west direction of cyclic tilt at UWE to be from a northerly source, like that of the later 1985 events.

Values for $\Delta M^* / \Delta V_e$ for the four well monitored deflations are given in table 1. Ratios for episodes 38-40 have been adjusted by up to 30 percent since the geodetic surveys were done over a longer interval than the actual deflations and, hence, included some inflationary change as indicated by the UWE tilt. Collapse volumes are quite small in comparison to the quantity of erupted lava, a relation previously noted by Wolfe et al. [1987] for earlier Pu’u O’o deflations.
An important question is where in the volcanic system is the site of transient changes in magma storage during Pu'u O'o activity: the summit reservoir or rift zone interior? Analysis of the gravity data that follows will reveal that a substantial portion of the magma accumulated within the summit reservoir during repose and entered the rift zone conduit system only at the time of rapid deflation.

**TABLE 1**

*Lava production and deflation volume, April 1985-January 1986*

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>Episode</th>
<th>Deflation Volume (10^6 m³)</th>
<th>aLava Volume (10^6 m³)</th>
<th>b(\Delta M^*/\Delta V_e) (kg/m³)</th>
</tr>
</thead>
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<tr>
<td>Apr. 21, 1985-Apr. 22, 1985</td>
<td>32</td>
<td>-4.3</td>
<td>16.3</td>
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<tr>
<td>Nov. 8, 1985-Nov. 14, 1985</td>
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<td>13.7</td>
<td>14830</td>
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<tr>
<td>Dec. 27, 1985-Jan. 3, 1986</td>
<td>40</td>
<td>-1.9</td>
<td>11.6</td>
<td>9830</td>
</tr>
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</table>

aHVO unpublished data; volumes not corrected for density.

bAssumed lava density 2300 kg/m³.

Observed gravity at HVO34 relative to P1 is plotted in figure 5 for the series of observations made from January 1984 to July 1986. Gravity differences plotted are not adjusted for elevation change. The long-term gravity trend seen in figure 5 is an increase. Much of it may be explained by the free-air effect of progressive subsidence centered near the location of HVO34. Long-term subsidence is indicated by leveling data [Johnson, 1987; Wolfe et al., 1987] and decreasing UWE tilt (fig. 3). Separate gravity signatures to short-term and long-term deformation is apparent by plotting together the progression of gravity and tilt values (fig. 6). Figure 6 shows only values corresponding to high and low points of the deformation cycle.
The critical observation here is the relationship between gravity and tilt change for specific inflation and deflation intervals (table 2). Changes are plotted together in figure 7; a weighted least squares fit to the data gives $\Delta g/\Delta \tau$ values of $-0.13 \pm 0.10 \, \mu\text{Gal}/\mu\text{rad}$ for all deflation intervals and $0.01 \pm 0.12 \, \mu\text{Gal}/\mu\text{rad}$ for all inflation intervals. In general the fits show that the observed changes at HV034 average near zero despite up to 20 $\mu\text{rad}$ of UWE tilt. This pattern is not unrealistic, as the observed gravity changes reflect a composite of free-air gravity increase due to subsidence and gravity decrease ($\Delta g^*$) due to removal of magma mass. For these data the two contributions equally offset.

A large portion of the data scatter seen in figure 7 is due to uncertainty in the gravity observations. This is particularly noticeable in the later data, when the standard error of both the original observations (fig. 5) and nominal changes (table 2 and fig. 7) increased. To avoid bias by the poorly-determined values, the data were weighted by $1/\sigma^2$ while solving $\Delta g/\Delta \tau$. There is no evidence for a systematic temporal change in the gravity response to deformation cycles; solution for $\Delta g/\Delta \tau$ for sub-groups of data give roughly the same result as the whole. As the data scatter is large relative to the average gravity change, differences between any specific inflation or deflation period cannot be considered significant.

Only for episode 32 was the height change of HV034 directly measured by leveling before and after deflation. The -80 mm change accompanied -17 $\mu\text{rad}$ of UWE tilt, giving a $\Delta h/\Delta \tau$ of 4.7 mm/$\mu\text{rad}$. Values determined during inversion of the spirit-level tilt observations for episodes 38-40 give lower changes, averaging 40 mm, in accordance with the smaller observed UWE tilt change. Fortunately, the shortage of height change data is not critical here as $\Delta g/\Delta \tau$ is close to zero and, therefore, $\Delta g/\Delta h$ is also near zero. Using 4.7 mm/$\mu\text{rad}$ from episode 32 as representative of the uplift rate gives $\Delta g/\Delta h$ of $-0.028 \pm 0.021 \, \mu\text{Gal}/\text{mm}$ for the deflations and $0.002 \pm 0.025 \, \mu\text{Gal}/\text{mm}$ for the inflations.
### TABLE 2

Relative gravity change at HVO34 and tilt change at UWE, January 1984-July 1986

[Times given are Hawaii standard time; figures for L give number of closed loops between HVO34 and P1 for each gravity survey. Errors given as standard error]

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<th>Second survey</th>
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**1985**

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TABLE 2 (Continued) Relative gravity change at HVO34 and tilt change at UWE, January 1984-July 1986

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<td>2</td>
<td>Feb. 18</td>
<td>0645</td>
<td>2.3</td>
<td>2</td>
<td>-2 ±20.0d</td>
</tr>
<tr>
<td>42---</td>
<td>Feb. 18</td>
<td>0645</td>
<td>2.3</td>
<td>2</td>
<td>Feb. 23</td>
<td>1530</td>
<td>1.1</td>
<td>2</td>
<td>10 ±9.8</td>
</tr>
<tr>
<td>44---</td>
<td>Apr. 12</td>
<td>0645</td>
<td>1.1</td>
<td>2</td>
<td>Apr. 15</td>
<td>0915</td>
<td>1.3</td>
<td>2</td>
<td>-9 ±13.8</td>
</tr>
<tr>
<td>repose</td>
<td>May 8</td>
<td>1615</td>
<td>1.0</td>
<td>2</td>
<td>June 1</td>
<td>1630</td>
<td>1.2</td>
<td>2</td>
<td>-3 ±9.2</td>
</tr>
<tr>
<td>46---</td>
<td>June 1</td>
<td>1630</td>
<td>1.2</td>
<td>2</td>
<td>June 2</td>
<td>1515</td>
<td>1.1</td>
<td>2</td>
<td>-10 ±9.0</td>
</tr>
<tr>
<td>repose</td>
<td>June 2</td>
<td>1515</td>
<td>1.1</td>
<td>2</td>
<td>June 24</td>
<td>0615</td>
<td>1.3</td>
<td>2</td>
<td>16 ±20.0d</td>
</tr>
<tr>
<td>47---</td>
<td>June 24</td>
<td>0615</td>
<td>1.3</td>
<td>2</td>
<td>June 27</td>
<td>0600</td>
<td>1.0</td>
<td>2</td>
<td>-6 ±20.0d</td>
</tr>
<tr>
<td>repose</td>
<td>June 27</td>
<td>0600</td>
<td>1.0</td>
<td>2</td>
<td>July 17</td>
<td>0545</td>
<td>1.0</td>
<td>2</td>
<td>-11 ±20.0d</td>
</tr>
<tr>
<td>48---</td>
<td>July 17</td>
<td>0545</td>
<td>1.0</td>
<td>2</td>
<td>July 19</td>
<td>0545</td>
<td>1.2</td>
<td>2</td>
<td>7 ±20.0d</td>
</tr>
</tbody>
</table>

aSurvey done shortly after deflation onset.
bSurvey done shortly before deflation complete.
cGravimeter G-721 only
dInsufficient data for error calculation; estimated ±20µGal
Figure 5.---Observed gravity \( g \) at HVO34 differenced with P1 plotted versus time, January 1984-July 1986. Standard errors, computed from the covariance matrix and data residuals during the inversion process, are less than \( \pm 10 \) \( \mu \)Gal for the double-looped surveys and \( \pm 6 \) \( \mu \)Gal for surveys having three or more loops. Surveys made before July 1984 have lower standard error compared to later surveys because of additional stations, double the number of readings at each station per loop, and sometimes more than the usual two loops. These factors increase data redundancy and, hence, improve instrument drift and gravity estimates. Later surveys had less redundancy to reduce the time required for each survey, at the expense of data precision. Some gravity surveys done in 1986 were done with only one gravimeter owing to malfunction of the second meter; data for these surveys have estimated standard errors of \( \pm 15 \) \( \mu \)Gal.
Figure 6.--Observed gravity at HVO34 versus east-west tilt at UWE, January 1984-July 1986. Only data immediately before deflation (solid symbols) and after (open circle) plotted.
Figure 7.--Changes in observed gravity $\Delta g$ at site HVO34 plotted against corresponding east-west tilt change at site UWE. Data intervals correspond to periods of gradual summit inflation, which give positive tilt change, and rapid deflation indicated by negative tilt change.
These figures were then adjusted for the free-air effect to obtain $\Delta g^*/\Delta h$ values (table 3). Then equation 3 was used to derive values for $\Delta M/\Delta V_e$ given in table 3.

**Magma transfer during 1969 Mauna Ulu eruption**

The character of the first stage of eruption in 1969 of Mauna Ulu [Swanson et al., 1979] is similar to that of Pu'u O'o. In particular, both featured alternating periods of magma accumulation when the summit reservoir gradually inflated, and brief summit deflation accompanying eruption when previously stored magma supplied a vigorous spray of lava. One difference noted by Wolfe et al. [1987] is an almost two-fold difference in volume of erupted lava per unit deflationary tilt at UWE. Deflations are reconsidered here in terms of the specific subsidence volume given by inversion of tilt data from a network of tilt sites because the apparent difference may partly relate to a change in the source location between the two periods, and hence a change in the sensitivity of the tiltmeter to deformation.

The lava mass versus summit subsidence volume is determined for the 12 fountaining episodes that characterized the first phase of Mauna Ulu activity in 1969. Overall uplift accompanied this activity as indicated by inflationary tilt change at UWE [fig. 2 in Swanson et al., 1979]. Net uplift, measured by leveling surveys between March 1969 and February 1970 [HVO, unpublished data], was greatest in the southern margin of Kilauea Caldera. However, tilting during individual subsidence events accompanying Mauna Ulu eruptions focused at a center located at the east rim of Halemaumau Crater, north of the long-term inflation source. Spirit-level tilt observations were made at four to five sites on Kilauea just before and just after episodes 5, 7, 9, and 12 [HVO, unpublished data]. Inversion of three of these data sets (no stable solution could be found for episode 5 data) gave an average $0.14 \times 10^6$ m$^3$/µrad of summit subsidence UWE tilt measured on a N60W azimuth. The ratio of north tilt to east tilt at UWE [see fig. 2, Swanson et al., 1979]
remained roughly the same for each deflation event of the first stage of the Mauna Ulu eruption, suggesting that the volume-to-tilt relation for the three events is representative of all events.

Lava volumes and tilt changes for episodes 2-12 of 1969 Mauna Ulu activity are given by Dzurisin et al. [1984] as 0.36x10⁶ m³/µrad N60W tilt. A slightly higher lava density of 2400 kg/m³ is appropriate here to account for higher density of the portion of the Mauna Ulu lavas that filled Alae Crater [Swanson et al., 1979]. Combination of the lava versus tilt ratio and subsidence to tilt ratio gives an estimate for $\Delta M^*/\Delta V_e$ of 6000 kg/m³. This is roughly 50 percent less than the average value found for the sample of 4 Pu'u O'o eruptions considered above, but significantly larger than the August 1981 and January 1983 events.

**TABLE 3**

*Reservoir mass change and deflation volume*

<table>
<thead>
<tr>
<th>Time Interval</th>
<th>$\Delta g^*/\Delta h$ (µGal/mm)</th>
<th>$\Delta M/\Delta V_e$ (kg/m³)</th>
<th>$\Delta M^*/\Delta V_e$ (kg/m³)</th>
<th>$Z$ (km)</th>
<th>$\Delta V_e$ (10⁶ m³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mauna Ulu deflations</td>
<td></td>
<td>3050±1645</td>
<td>6000a</td>
<td>2.7a</td>
<td>-2.6a</td>
</tr>
<tr>
<td>Aug. 1981 intrusion</td>
<td>0.128±0.069</td>
<td>3050±1645</td>
<td>6000a</td>
<td>2.7a</td>
<td>-2.6a</td>
</tr>
<tr>
<td>Jan. 1983 eruption</td>
<td>0.110±0.018</td>
<td>2625±430</td>
<td>3480</td>
<td>3.5</td>
<td>-56</td>
</tr>
<tr>
<td>Pu'u O'o inflations</td>
<td>0.329±0.025</td>
<td>7855±595</td>
<td>3170</td>
<td>4.0</td>
<td>-83 to -98</td>
</tr>
<tr>
<td>Pu'u O'o deflations</td>
<td>0.300±0.021</td>
<td>7165±500</td>
<td>11180b</td>
<td>2.5b</td>
<td>-2.6b</td>
</tr>
</tbody>
</table>

^aAverage value for Mauna Ulu eruptive episodes 7, 9, and 12.

^bAverage value for Pu'u O'o eruptive episodes 32 and 38-40 given separately in Table 1.

Table 3 is a compilation of all $\Delta M^*/\Delta V_e$ and $\Delta M/\Delta V_e$ estimates, along with additional parameters for source depth and deflation volume which is used for modeling. The major observation is that the Pu'u O'o and Mauna Ulu eruptions were accompanied by limited subsidence relative to reservoir magma withdrawal. In comparison, collapse during
the August 1981 and January 1983 events was larger in terms of both absolute and relative volumes. An important constraint is that the source depths of both the 1981 and 1983 subsidences was greater than those for the smaller events. A possible exception is the 1984-April 1985 Pu'u O'o events for which the short-term source depth is unknown.

Evidence of summit reservoir pressure variations

Uplift from a Mogi [1958] point source model as given by equation (2) is proportional to source pressure change as well as volume; these parameters cannot be separated by analysis of the surface displacements alone [McTigue, 1987]. The height of the surface of lava lakes and eruptive vents, however, give independent information that may be used to model reservoir pressure.

Epp et al. [1983] show that the amount of summit collapse at Kilauea indicated by tilt change at UWE during periods of rapid deflation is inversely proportional to the elevation of the eruption site. Deflation reflects a pressure drop within the summit reservoir if the height and therefore weight of the magma column is reduced in proportion to the vent elevation. Pressure drop in the central reservoir is greater for eruptions at lower elevation in distant parts of the rift zone several hundred meters below the summit elevation and thus allowing a larger reduction of the magmastatic head (or weight) of the magma column.

Deriving \( \Delta P/\Delta V_e \) from vent height and magma density versus deflation data here implies: (1) each deflation begins with roughly the same initial summit source pressure; and (2) source pressure drops to that of the magmastatic pressure of a lava column with height equal to the elevation of the vent site. The three largest summit deflations (1955, 1960, and 1961) are associated with low elevation (46-399 m, 30 m, and 396-792 m) flank eruptions [Epp et al., 1983, fig. 1]. The 1955 event followed months of summit inflation [Dzurisin et al., 1984], and summit eruptions actually preceded the 1960 and 1961 cycles. Hence requirement (1) above is satisfied for the three low elevation discharges, as well as the
preceding summit eruptions. Another event in 1977 began with very little summit inflation in the preceding 6 months [Dzurisin et al., 1980], suggesting that summit pressure may have been low at the time of the eruption onset. The volume of subsidence in fact falls slightly short of the general correlation given by Epp et al. [1983, fig. 1]. Assumption (2) is probably never satisfied, particularly for eruptions on the distant portions of the east rift zone (several 10's of km from the summit) because flow resistance in the conduit system may stop magma motion before pressure equilibrium is reached. In addition, the infrequently intruded and, hence, relatively cool lower east rift zone is an especially poor environment for eruption longevity [Hardee, 1987]. Dike-shaped conduits that feed surface vents are prone to closure by rapid cooling against wallrock and are inefficient for magma transport. Therefore, the source pressure drop is most likely less than implied by the vent height and the \( \Delta P/\Delta V_e \) ratio so calculated represents a maximum value.

Decker et al. [1983] infer a pressure change of 0.085 MPa/\( \mu \)rad of N60W UWE tilt from the Epp et al. [1983] vent elevation versus tilt correlation; implicit is a magma column density of 2550 kg/m\(^3\). Adjusting values to an east tilt at UWE (assuming that N60W is a radial vector to the average source epicenter) gives \( \Delta P/\Delta \tau \) of 0.098 MPa/\( \mu \)rad. With an average \( \Delta V_e/\Delta \tau \) factor of 0.45x10\(^6\) m\(^3\)/\( \mu \)rad for east-west tilt [Dvorak and Okamura, 1985], \( \Delta P/\Delta V_e \) is 0.22 Pa/m\(^3\).

A separate calculation of the pressure change and subsidence volume is made for the largest subsidence event which, because of its size, strongly influences the overall relation given above. The summit eruption in December 1959 was at a 1100 m elevation vent, and followed by the discharge at sea level in January 1960. The reservoir pressure drop, \( \Delta P \), corresponding to a 1100 m drop in the magma column (of density 2600 kg/m\(^3\)) thus is a maximum of 28 MPa. For that deflation in January 1960 Eaton et al. (1962, fig. 12) found \( \Delta V_e=150x10^6 \) m\(^3\), determined by visual fit of a point-source model to observed tilts in the Kilauea summit region. Therefore \( \Delta P/\Delta V_e \) is 0.19 Pa/m\(^3\).
One half of the 18 deflations considered by Epp et al. [1983] produced deflationary tilts larger than 45 µrad (N60W tilt at UWE) with the largest deflation in January 1960 totaling 310 µrad. Therefore the pressure estimate given above is appropriate for intermediate-sized deformations.

During the 1972-1974 Mauna Ulu activity Tilling [1987] observed a positive correlation between lava-lake surface elevation and the level of summit inflation, as measured by tilt. He interpreted this to imply that Kilauea's magmatic system was fully engorged and open so that as the reservoir expanded the lava lake expanded too. It can also be considered in terms of a pressure link. A pressure increase within the summit reservoir would raise the elevation of the lava lake and, hence, the magmastatic head. Eruptions and intrusions at rift zone sites near Mauna Ulu tapped magma from the Mauna Ulu system and reduced pressure locally. In response the elevation of the lava lake dropped and the magma flow from the summit increased. Flow continued until the pressure gradient between summit reservoir and rift zone returned to an equilibrium value, leaving the summit partly deflated. Therefore as a first approximation summit pressure variations may be estimated from changes in the Mauna Ulu lava lake height.

Two of the larger 1973 summit deflations and lava lake draining episodes and intervening reinflation described by Tilling [1987] are re-examined here. Summit subsidence accompanied a Mauna Ulu lake-draining event which also included brief lava effusion at Pauohi and Hiiaka craters on May 5, 1973. The lake emptied of lava within two hours leaving a 200 meter deep crater floored by rubble, a change of more than 160 m (it is unknown how far the column retracted below the rubble cover). Lava begin to refill the crater of Mauna Ulu on May 8, and 150 m was added between May 8 and May 25. Filling of the Mauna Ulu lava lake and summit inflation continued to June 9 when a second deflation event occurred, this time non-eruptive and not as severe as the first.
Unfortunately only the gradual inflation interval between the deflations is well characterized by geodetic data [fig. 16.41C of Tilling et al., 1987]. The source locations for the two rapid summit deflations are unknown; hence, volumes of deflations cannot be calculated with much certainty. Inversion of the inflationary tilt data using a point source model gives an uplift volume of $2.3 \times 10^6$ m$^3$. The source is below the northeast rim of Halemaumau Crater at a depth of about 2.8 km. The summit pressure change estimated from the lava lake filling with a density of 2600 kg/m$^3$ and summit uplift between May 7 and May 25, 1973 is $AP/AV_c=1.66$ Pa/m$^3$. If the source position for the two periods of rapid deflation centered near the position of the intervening inflation, then the UWE tilt changes given by Tilling et al. [1987] suggest collapse volumes of $2.5 \times 10^6$ and $1 \times 10^6$ m$^3$ for the May and June events, respectively. The May 5 deflation event gives a minimum (as the total lava level change is unknown) pressure change of 1.63 MPa/m$^3$. Pressure change is an estimated 2.39 Pa/m$^3$ for the June 9 event.

Between 1984 and 1986, repetitive periods of summit inflation at Kilauea were associated with an increase in the level of the lava column filling the vertical eruptive vent within the Pu'u O'o cone [Wolfe et al., 1987; HVO, unpublished data]. At the end of an eruptive episode, lava retracted and disappeared below a depth of 50 m (lava could generally not be seen at depths greater than this) for a period of a few days to three weeks. The period of conduit refilling and summit reinflation culminated in one or two days of spectacular fountaining at Pu'u O'o. Following episode 34, lava was sighted 3 days after the eruption at a depth of 100 m [HVO, unpublished data]. A 50 to 100 m rise in the height of a column of magma (density 2600 kg/m$^3$) produces a hydrostatic pressure increase of 1.3 to 2.5 MPa. This is a minimum pressure increase for the summit reservoir because the lava column may descend to greater depths within the conduit, and because additional pressure is required to overcome the yield strength threshold of the rift zone magmatic
system before resuming flow. Inflation volumes of near $3 \times 10^6 \text{ m}^3$ give a rate of reservoir pressure change with uplift of at least 0.43 Pa/m$^3$ and possibly more than 0.83 Pa/m$^3$.

Table 4 summarizes the reservoir pressure estimates given here. The smaller Mauna Ulu and Pu'u O'o events have a high rate of pressure change compared to the rate given by Decker et al. [1983] which is sensitive to larger-sized events.

**TABLE 4**

*Pressure constraints for Mauna Ulu and Pu'u O'o activity*

<table>
<thead>
<tr>
<th>Observations</th>
<th>$\Delta V_e$ (10$^6 \text{ m}^3$)</th>
<th>$\Delta P/\Delta V_e$ (Pa/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1960 deflation</td>
<td>-150</td>
<td>less than 0.19</td>
</tr>
<tr>
<td>Flank eruptions$^a$</td>
<td>0 to -150</td>
<td>less than 0.22</td>
</tr>
<tr>
<td>Mauna Ulu lava lake</td>
<td>2.3</td>
<td>1.66</td>
</tr>
<tr>
<td>Pu'u O'o conduit filling</td>
<td>3</td>
<td>greater than 0.43 (possibly greater than 0.83)</td>
</tr>
</tbody>
</table>

$^a$Flank eruption events listed by Epp et al, 1983.
An inverse relationship between deflation size, $\Delta V_e$, and $\Delta M/\Delta V_e$ is apparent in comparing summit subsidences for Pu'u O'o eruptions with those of August 1981 and January 1983 (Table 3). The main parameters in equation 10 that may affect this are CO$_2$ gas content, pressure, and crustal shear modulus, $\mu$. The role of CO$_2$ expansion changes in proportion to its abundance and, for a given gas content, with pressure. Variation of $\mu$ may be due to a possible strain-dependent transition.

Variation of $\Delta M/\Delta V_e$ is probably not due to a change in the abundance, $N$, of comagmatic CO$_2$ gas. The arrival of fresh mantle magma to Kilauea's reservoir, which is the source of CO$_2$, is thought by Dzurisin et al. [1984] to have been relatively constant during the past two decades, varying between $5 \times 10^6$ m$^3$ and $11 \times 10^6$ m$^3$ per month. Degassing of the summit reservoir since measurements began in 1979 is also a reasonably steady process [Casadevall et al., 1987]. The variation of CO$_2$ necessary to produce the magnitude of change of $\Delta M/\Delta V_e$ observed between January 1983 and the later Pu'u O'o events in 1984 is unrealistic.

Preferential leakage of exsolved gas from the reservoir during larger events is not likely to change $\Delta M/\Delta V_e$. Degassing at a rate two orders of magnitude greater than measured by Greenland et al. [1985] in December 1983 and February 1984 is required to explain the excess of collapse in 1981 and 1984 compared to the Pu'u O'o events. There is no data on the specific rate of CO$_2$ emission at Kilauea at the time of the 1981 and 1983 events. During January and February 1983, however, geochemical data for Kilauea summit fumaroles [HVO, unpublished data] do not indicate a surge of CO$_2$ emission, in particular from the ratio of carbon to sulfur or from emission rates of SO$_2$ [Casadevall et al., 1987].
A mechanism of subsidence resulting from anomalous widening of the summit region [Johnson, 1987] cannot be applied here (to the short-term data) because measured horizontal changes during the events considered here are adequately explained by a point source. Nor is there evidence from seismic observations to suggest dike injection or rifting in the summit area. Ratios of $\Delta g^*/\Delta h$ for the 1981 and 1983 subsidence events are similar to the November 1975 collapse [Jachens and Eaton, 1980], so the model for deflation developed here may also at least partly apply to the 1975 event.

Possible variation of $\mu$ with strain and time

Lower ratios of $\Delta M/\Delta V_e$ for the larger subsidences may result from a reduction of the effective crustal shear modulus $\mu$ accounting for creep effects with time or anelastic failure at high stress. Diminishing $\mu$ lowers both $\Delta M/\Delta V_e$ (equation 10) and $\Delta P/\Delta V_e$ (equation 2). The changes of $\Delta M/\Delta V_e$ and $\Delta P/\Delta V_e$ observed for small and larger events are consistent with a model for varying $\mu$. Some additional observations (below), however, do not support this model. Furthermore, variation in the effective source volume $V_t$ may also change $\Delta P/\Delta V_e$ in equation 2.

The short deflation, and considerably longer inflation intervals of the Pu'u O'o eruption produce similar $\Delta g/\Delta \tau$ (read: $\Delta g^*/\Delta V_e$) (table 3). Therefore a time variation of $\mu$ (in equation 10) is not indicated.

Time and strain-dependence of $\mu$ (as well the remaining variables in equation 10) are also absent in gravity and tilt observations at a summit site of Mauna Loa volcano for the three-week deflation during the 1984 eruption [Lockwood et al., 1985]. Except for the first day affected by the initial shallow dike injection, the ratio of gravity to tilt changes remained constant.
Source depth, pressure, and CO₂

One likely candidate for differences in \( \Delta M/\Delta V_e \) (Table 3) is the internal reservoir pressure (\( 1/P^2 \) in equation 10) which controls volume changes of exsolved CO₂ that partially fills the source volume \( V_r \). This term reflects a combination of the volume of the exsolved gas phase (inversely proportional to \( P \)) and its bulk modulus (essentially \( P \)). As pressure \( P \) increases with depth, exsolved CO₂ volume changes with pressure change will be most pronounced for shallow reservoir depths. The Mauna Ulu and Pu'u O'o (at least after April 1985) deflations had source depths of 2.2-2.8 km while the 1981 and 1983 events were 3.5-4.0 km (fig. 4). Indeed the more pronounced volume increase of CO₂ in shallow, low-pressure levels of the reservoir during Mauna Ulu and Pu'u O'o related summit deflations may be responsible for the relatively small amount of subsidence relative to mass loss.

Source volume

The volume of the source \( V_r \) does not affect \( \Delta M/\Delta V_e \) in equation 10. It however affects the source pressure change \( \Delta P \) as shown by equation 2. Assuming \( \mu \) is constant, observed differences in \( \Delta P/\Delta V_e \) indicate a relatively big source volume for the larger subsidence events as compared to smaller Mauna Ulu and Pu'u O'o ones (Table 4). Variation of \( V_r \) in this context does not relate to volume changes by simple inflation and deflation, but to migration of the boundary between elastic and fluid behavior. This is essentially a transition to visco-elastic behavior after a certain yield stress is exceeded in the high-temperature, high-strain zone surrounding the deformation source [Davis, 1986]. A visco-elastic material under constant stress will increase in deformation (or creep) with time and under constant strain, stress will decrease with time (relax). Hence such a behavior implies that \( V_r \) is a function of stress and time.
The spatial pattern of vertical surface displacement is virtually identical for a spherical source of any size [McTigue, 1987] and so \( V_r \) is generally not determined from geodetic data. Some systematic trends in the history and distribution of deformation at Kilauea, however, support the idea of a changing source size. Some leveling or tilt data span brief time intervals, and identify discrete source hypocenters [Fiske and Kinoshita 1969; Dvorak et al., 1983; figure 4]. These may correspond to small, distinct source volumes, such as an individual pocket of melt. In contrast, essentially the same source hypocenter and deformation pattern have been observed for different, but larger events and time intervals. Examples are the 1924 subsidence [Ryan et al., 1983; Mogi, 1958], the November 1975 deflation [Lipman et al., 1985, fig. 17C] and two periods of gradual deflation during January 1976-April 1979 and January 1984-July 1986 [HVO, unpublished data]. During these time intervals the effective source volume may coincide with the entire plastic region and extensions reaching into both the southwest and east rift zones. The location and geometry of this source volume remains fixed and governs the repeat pattern of surface displacements. Collapse during these periods is greatest at the south margin of Kilauea caldera. Some subsidence is found in the upper reaches of the rift zones and well to the south of the deflation center. This pattern is unlikely to be due to a simple point source of deformation; the complex distribution of change suggests an extended source. Subsidence over the upper rift zone areas may correspond to migration of the boundary between plastic and elastic behavior all the way into the aseismic rift zone cores [Ryan et al., 1983], principal conduits for magma transfer out off the summit reservoir. In other words, the effective source volume for these time intervals includes the rift zones.

Migration of the subsidence center is commonly observed during the course of rapid deflation at Kilauea [Fiske and Kinoshita, 1969; Jackson et al., 1975; Swanson et al., 1976; Dvorak and Okamura, 1987]. The usual pattern is for the northern part of the caldera to subside first followed quickly by the bulk of subsidence at the south end [Decker, 1987;
Dvorak and Okamura, 1987]. This pattern would be expected if the geometric center of the reservoir complex was to the south of the locus of magmatic activity. Spreading of the plastic zone and, hence, expansion of the effective source would be accompanied by a southward shift of the subsidence focus because of the position difference between the small initial source and the bulk of the reservoir.

Estimates of CO₂ content and source volume

Solution for the source CO₂ concentration from $\Delta M/\Delta V_e$ is by equation (10) 
\[ K=11.5 \text{ GPa}, \rho=2600 \text{ kg/m}^3, T=1470^\circ \text{K}, \omega=0.044 \text{ kg/mole}, R=8.314 \text{ m}^3\text{Pa/mole}^\circ \text{K}, v=0.25 \]. Davis [1976] gives an upper bound for $\mu$ of 3 GPa. This limiting value is constrained by the absence of a piezomagnetic anomaly associated with deformation. Reservoir pressure, $P$, is approximately equal to lithostatic pressure (crustal density of 2300 kg/m³). Therefore $P=56$ MPa for the source depth of the Pu'u O'o deflations (an estimated 2.5 km). This and the $\Delta M/\Delta V_e$ ratio of 7165 kg/m³ corresponds to a source CO₂ content of 0.0030 weight fraction. The deeper (3.5 and 4.0 km) 1981 and 1983 events have lower $\Delta M/\Delta V_e$ ratios (3050 and 2625 kg/m³) and much lower CO₂ values of 0.0009 and 0.0005 weight fraction. The CO₂ values for Pu'u O'o phases are compatible with previous estimates for Kilauea magma of 0.0065 [Gerlach and Graeber, 1985] and 0.0032 [Greenland et al., 1985] by weight determined from geochemical data. The difference in gas content between shallow and deep sources suggests that upward migrating gas may collect within pockets of magma at the upper portion of the magma reservoir.

The source volume may be estimated from $\Delta P/\Delta V_e$ and equation (2) (v=0.25, $\mu=3$ GPa). For the small Pu'u O'o inflation cycles a maximum source volume of 6 km³ is implied by the $\Delta P/\Delta V_e$ value of at least 0.43 Pa/m³. A source volume of only 1.6 km³ is associated with the May 1973 Mauna Ulu lava lake filling event ($\Delta P/\Delta V_e=1.66 \text{ Pa/m}^3$). In
contrast, the significantly larger January 1960 deflation (ΔP/ΔV ≠ no more than 0.20 Pa/m³) had a source volume of greater than 13 km³.

If the 1981 and 1983 events also had large source volumes, then the CO₂ value given above for them would be more or less the averaged gas content of the reservoir. The apparently exceptionally gas-rich pocket of melt at shallow depth, source of the Mauna Ulu and Pu'u O'o deformation events, is therefore anomalous.

**Long-term changes**

Decker [1987] suggested that long-term deformation may not correlate with reservoir pressure change. This may be seen from analysis of Kilauea's eruptive history. Eruptions within the summit caldera took place in November 1967, August 1971, July and September 1974, November 1975, and April and September 1982 [Dzurisin et al., 1984]. These eruptions took place at a wide range of inflation levels, indicated by changes in summit tilt [Dzurisin et al., 1984]. Analysis of leveling surveys [HVO, unpublished data] suggests that collapse in the eight year period between 1974 and 1982 amounts to roughly 300x10⁶ m³. However, eruption of lava to the elevation of the caldera floor in 1982 shows that reservoir pressure was just as high then as it was in 1974 despite the net collapse. This would not be possible for an elastic deflation mechanism to explain long-term subsidence because then reservoir pressure would decrease in proportion to subsidence.

The processes of rifting (and widening) of Kilauea's edifice is an ubiquitous feature of Kilauea trilateration data. Long-term summit subsidence is therefore explained by loss of horizontal support of the fluid reservoir [Johnson, 1987]. This mechanism is consistent with the observation that the gravity to height change ratio for long-term intervals is sometimes near the free-air rate; such a ratio cannot be obtained from simple deflation relating to magma extraction. The further attraction of the rifting model is that it explains long-term collapse by change in the reservoir shape rather than pressure.
After a major collapse, additional gradual subsidence may occur because of continued escape of gas bubbles that first formed during the initial event. Such a process produces a source volume contraction with little mass loss, and by itself would produce surface subsidence with no Δg*. In fact this is the pattern of change observed between January 1983 and July 1986 as well as between December 1975 and October 1977 [Dzurisin et al., 1980]. Both periods follow major summit collapses.

Unfortunately, no data exist on the emission rate of CO₂ at Kilauea’s summit before 1983. SO₂ emission measurements however have been made since 1979 [Casadevall et al., 1987]. Emissions of SO₂ from the summit area of Kilauea show that reservoir degassing is favored when internal pressure is low such as after a major deflation. Shortly after both the August 1981 and January 1983 deflations SO₂ degassing increased by a factor of 1.6 [Casadevall et al., 1987; Greenland et al., 1985]. The August 1981 event was followed by reinflation while the SO₂ release rate returned to previous levels. Ever since the rapid January 1983 deflation, however, Kilauea has remained in a slowly deflating trend and SO₂ degassing has continued at a high rate [Casadevall et al., 1987; HVO, unpublished data]. This is consistent with a pressure recovery after the August 1981 event which limited degassing, and a sustained low pressure regime after the January 1983 event which has facilitated gas release.
ELASTIC AND INELASTIC MAGMA STORAGE AT KILAUEA VOLCANO

By Daniel J. Johnson

ABSTRACT

Gravity, leveling, and trilateration data recently obtained at Kilauea Volcano indicate that magma is accommodated within the summit reservoir by a combination of elastic magma compression, elastic chamber expansion, and gradual inelastic edifice widening in a south-southeast direction.

Elastic (reversible) magma storage is modeled using data from 15 brief summit-deflation episodes in 1984–85 and assuming a spherical geometry for the magma reservoir. Changes in pressure in the reservoir produce the deformation; changes in mass in the reservoir produce the gravity changes. The ratio of residual gravity change (corrected for the free-air effect) to elevation change, measured near the apex of maximum subsidence, averaged $0.28 \pm 0.025 \ \mu\text{Gal/mm}$. If magma density is assumed to be 2.75 $\text{g/cm}^3$, the volume of magma withdrawn from the reservoir averaged $2.4 \pm 0.21$ times the associated volume of subsidence. This volume difference is dependent on both the magma bulk modulus (which constrains bulk compression of the reservoir contents) and the edifice shear modulus (which constrains distortion of the surrounding edifice). If deformation is assumed to be from a point dilatational source in an elastic half-space, the observed volume difference implies that the shear modulus of the edifice is $2.0 \pm 0.24$ times the bulk modulus of the magma.

Trilateration data show that Kilauea's edifice widens inelastically in time intervals of months to years, primarily in a northwest direction. Over these long time periods, gravity and elevation changes indicate both surface subsidence and mass addition. These observations imply that horizontal extension of Kilauea's edifice decreases lateral support of the fluid magma reservoir, thus lowering the pressure in the reservoir and allowing more magma to be injected without vertical uplift.
Increased summit collapse occurred during the November 1975 earthquake, when new reservoir space created by inelastic seaward displacement of the volcano’s south flank was not immediately filled by new magma from depth. For 16 months after the earthquake, additional reservoir capacity created by continued edifice dilation was filled with new magma. From January 1984 to July 1985, Kilauea’s edifice subsided by 200 mm and widened by as much as 250 mm. No net subsurface mass changes were measured during this period, implying that the subsidence was entirely the result of horizontal extension.

A complete magma-budget estimate for Kilauea must therefore consider both elastic and inelastic magma storage. This requires a synthesis of leveling, trilateration, and gravity data.

INTRODUCTION

Kilauea Volcano grows by the accumulation of mantle-derived magma that is initially stored in a shallow summit magma reservoir (Eaton and Murata, 1960). The location of this storage region, indicated by an aseismic zone (Koyanagi and others, 1976) and by the inversion of measured surface displacements (Fiske and Kinoshita, 1969; Dvorak and others, 1983), is in a zone below the south rim of the caldera about 3 km in diameter and from 2 to 6 km below the surface. Slow uplift of the volcano’s summit over several weeks to a few months is caused by the gradual addition of new magma from a deep source region. Occasional rapid surface subsidence, over periods of hours to days, occurs as magma is withdrawn from the reservoir, either migrating upward or laterally through a conduit system into one of two rift zones (Ryan and others, 1981). Though a part of the mantle-derived magma supply may eventually reach the surface, some magma remains in the summit reservoir and rift systems (Dzurisin and others, 1984).

Previous estimates of magma-volume changes in the subsurface reservoir have been based on geodetic measurements (Duffield and others, 1982; Dvorak and others, 1983; Dzurisin and others, 1984). Because the elastic properties of both magma and volcanic edifice are poorly known, subsurface volume changes cannot be determined from geodetic data alone. Rather, the volume change of the edifice, as determined by integrating vertical displacements measured at the surface, is typically used to approximate the subsurface volume change.

Subsurface volume changes can be estimated more accurately if measurements of changes in both gravity and elevation are available.
Gravity and elevation measurements obtained at Kilauea by Jachens and Eaton (1980) and Dzurisin and others (1980) demonstrated that between November 1975 and September 1977 changes in the volume of magma in the reservoir exceeded changes in the volume of the edifice. This was attributed to the effect of void spaces created during periods of magma withdrawal (Jachens and Eaton, 1980), later refilled during periods of magma accumulation (Dzurisin and others, 1980). Gravity studies by Johnsen and others (1980) at Krafla, Iceland, by Sanderson and others (1983) at Mount Etna, Italy, and by Eggers (1983) at Pacaya Volcano, Guatemala, also indicate differences between edifice and subsurface volume changes. Sanderson and others (1983) explained the differences by bulk compression of the magmatic fluid and changes in volume of exsolved gas enabling subsurface magma accumulation to exceed the volume of edifice expansion. Eggers (1983) proposed density changes of about 0.4 g/cm³ in shallow magma bodies caused by changes in the magmatic water content and vesicularity.

A series of gravity observations made in the Kilauea summit region between January 6, 1984, and July 8, 1985, have been analyzed to determine the empirical relation between magma-volume changes in the summit reservoir and edifice-volume changes as evidenced by surface uplift or subsidence. The short-term relation observed during brief episodes of elastic deflation corresponding to eruptions at the Puu Oo vent along Kilauea's middle east rift zone (Wolfe and others, chapter 17) has been used to estimate the relative elastic properties of the magma reservoir region and the surrounding volcanic edifice. The long-term relation has been used to evaluate the effect of inelastic summit widening on volume changes of the edifice and magma content of the summit reservoir. Results of this study will help improve estimates of the rate of magma supply to Kilauea.

ACKNOWLEDGMENTS

This study is part of a doctoral dissertation in progress at the Department of Geology and Geophysics of the University of Hawaii. The Hawaiian Volcano Observatory of the U.S. Geological Survey provided logistical support and funds for field expenses. I thank Eduard Berg, Robert Decker, Roger Denlinger, John Dvorak, Daniel Dzurisin, Albert Eggers, Robert Jachens, and Thomas Wright for helpful ideas, suggestions, and assistance given during all phases of this study. I am indebted to Thomas Wright for his continued interest and kind support. He showed this
in an exceptional manner by personally doing most of the gravity surveys made after July 1984. This manuscript has benefited from reviews by Edward Brown, John Dvorak, Daniel Dzurisin, Harry Glicken, Robert Jachens, and Michael Ryan.

Figure 47.1.—Location map of Kilauea summit region showing principal fault scarps and craters, gravity monitoring sites HVO23, P1, HVO27, 204YY, HVO32, and HVO34, and east-west component electronic tiltmeter at Uwekahuna (UWE). Hachures mark inner side of crater walls.

DATA

A total of 141 gravity surveys were conducted in the Kilauea summit region between January 6, 1984, and July 16, 1985. Initially, measurements were made at five monitoring sites referenced to a base station (P1) 5 km northwest of the point of maximum vertical movement (from earlier leveling surveys). The monitoring network was reduced to two sites after February 16, 1984, and to one site, HVO34, after July 12, 1984, because data from this site were found to be representative of the others. Only gravity data for HVO34, referenced to P1, are presented in this paper; locations of all measured stations are shown in figure 47.1.
Two LaCoste and Romberg model-G gravimeters, each equipped with galvanometer-assisted readout, were used throughout the study. Measurements were made over two (109 surveys) or three (32 surveys) closed loops between P1 and the network stations. The time to complete each loop, initially 4 h shortened to 1 h as the number of sites was reduced. Gravity readings were corrected for tidal effects (Longman, 1959) using a compliance factor of 1.16. The effects of instrumental drift and residual tide variations were removed by a least-squares solution of a second-order polynomial approximating time-dependent changes in the reading level of each gravimeter. Linear and sinusoidal gravimeter calibration functions were determined from standard calibration loops and applied to all the Kilauea data (table 47.1).

No attempt was made to correct the gravity data for water-table changes. As P1 and HVO34 are only 5 km apart, gravity variations caused by water-table changes are probably similar at both sites and do not affect the relative gravity differences.

Because data from each gravimeter were analyzed separately, agreement between the two gravimeters is a measure of the precision of the observations. A plot of the measurement discrepancy of the two gravimeters at HVO34 versus time is shown in figure 47.2 for surveys done with three loops (triangles) and with two loops (circles). The standard deviation of the intermeter difference for triple-looped surveys is 9.7 µGal and for double-looped surveys is 11.9 µGal. These figures correspond to standard errors of the mean of 3.4 µGal and 4.2 µGal, respectively.

Elevation changes were determined by optical leveling of a line from P1 to HVO34. The surveys were made in only one direction to third-order standards. East-west tilt variations, measured by a continuously recording, 1-m-base, mercury-capacitance tiltmeter at Uwekahuna (fig. 47.1, UWE), were used to interpolate elevation and edifice-volume changes to values corresponding in time with the gravity surveys. Resurveys of an extensive trilateration network in the summit area of Kilauea in January 1984 and July 1985 provided information on horizontal displacement.

RESULTS

Observed gravity differences (fig. 47.3C) and elevation differences (fig. 47.3A) between HVO34 and P1 and changes in east-west tilt (fig. 47.3B) at Uwekahuna have been plotted as a function of time. Episodes of the eruption at Puu Oo are sequentially
numbered (fig. 47B). Increasing tilt values correspond to inflation of the volcano. Tilt data indicate that activity was characterized by cycles of gradual inflation followed by brief deflation episodes during eruptive pulses at the Puu Oo vent (Wolfe and others, chapter 17). Considering the uncertainty of the gravity observations (3–4 μGal), no systematic short-term gravity changes can be inferred. Apparently the free-air gravity changes caused by cycles of elevation variations are compensated by gravity changes caused by fluctuations in the mass of magma in the reservoir. Superimposed on these eruptive cycles is a net deflation of the summit indicated by a tilt change of −42 μrad and an elevation change of −200 mm. During the same period, the gravity at HVO34 gradually increased by about 65 μGal. These long-term variations follow a free-air relation, implying that there was no net subsurface mass change.

**TABLE 47.1.** — Linear and sine-form calibration functions for LaCoste and Romberg gravimeters G-615 and G-721 as a function of dial reading

[Linear factor determined by resurveying 6 stations of calibration line established by Jachens and Eaton (1980) on Mauna Kea Volcano. Sine-form calibration functions determined in June 1984 by measuring 25 stations near Palm Desert, California, and comparing results to values obtained by Robert Jachens (written commun., 1984) with LaCoste and Romberg model-D gravimeters. Sine functions were checked and adjusted slightly by comparing values for G-615 and G-721 obtained at Ocean View Estates, Hawaii, in December 1984. c.u., dial reading in counter units; n.d., not detected]

<table>
<thead>
<tr>
<th>Gravimeter</th>
<th>Wavelength = 73.33 c.u.</th>
<th>Wavelength = 36.67 c.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1/2 amp (μGal)</td>
<td>Phase zero (c.u.)</td>
</tr>
<tr>
<td>G-615</td>
<td>0.01739</td>
<td>2,924.54</td>
</tr>
<tr>
<td>G-721</td>
<td>.01415</td>
<td>2,996.05</td>
</tr>
</tbody>
</table>

**INTERPRETATION**

The gravity data for HVO34 presented in figure 47.3C are corrected for earth tides. The effects of anomalous earth tides, ocean tides, and variations in atmospheric pressure are at least partly removed by the drift correction. The remaining processes that affect the gravity values include (1) accumulation or loss of magma from the reservoir, (2) vertical movement of the observation point in the Earth’s gravity field (the free-air effect), and (3) density distribution change in the edifice as a result of deformation. These processes are usually coupled at Kilauea. For example, an intrusion of magma will increase gravity because of its mass, but it will decrease gravity because of the surface uplift that it causes. The gravity effect of (3) for a spherically symmetric source of dilatation is zero, as shown by a numerical analysis (Rundle, 1978) and by an analytical treatment.
FIGURE 47.2.—Discrepancy between gravity measurements made by gravimeters G-615 and G-721 at site HVOA4 plotted as a function of time. Triangles, surveys done with three loops; circles, surveys with two loops.
Figure 47.3.—Time plots of relative elevation and gravity at HVO34, and east-west tilt at Uwekahuna. 

**A.** Relative elevation of site HVO34. 

**B.** East-west tilt at Uwekahuna; eruptive episodes at Puu Oo vent indicated by numbers. 

**C.** Relative gravity at site HVO34, uncorrected for vertical uplift. Triangles, gravity surveys done with three loops; circles, surveys done with two loops.
Table 47.2.—Nominal relative gravity change at HVO34 and tilt changes at Uwekahuna for deflation intervals corresponding to 15 eruptions at Puu Oo, 1984–85

[Times given are Hawaii standard time; figures for loops give number of closed loops between HVO34 and P1 for each gravity survey. s.e., standard error]

<table>
<thead>
<tr>
<th>Eruptive episode</th>
<th>First survey</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Second survey</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Date</td>
<td>Start time</td>
<td>Duration (h)</td>
<td>Loops</td>
<td>Date</td>
<td>Start time</td>
<td>Duration (h)</td>
<td>Loops</td>
<td>( \Delta g ) (±1 s.e.)</td>
<td>( \Delta \text{tilt} ) (µrad)</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>Jan. 20, 1984</td>
<td>1445</td>
<td>4.00</td>
<td>2</td>
<td>Jan. 22, 1984</td>
<td>0900</td>
<td>4.75</td>
<td>2</td>
<td>-4 (5.8)</td>
<td>-10.4</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Jan. 29, 1984</td>
<td>1200</td>
<td>5.00</td>
<td>2</td>
<td>Jan. 31, 1984</td>
<td>0800</td>
<td>7.50</td>
<td>3</td>
<td>4 (5.1)</td>
<td>-7.3</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Mar. 3, 1984</td>
<td>0815</td>
<td>6.00</td>
<td>3</td>
<td>Mar. 5, 1984</td>
<td>0745</td>
<td>6.50</td>
<td>3</td>
<td>2 (4.4)</td>
<td>-14.1</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>Mar. 28, 1984</td>
<td>1230</td>
<td>3.75</td>
<td>2</td>
<td>Mar. 31, 1984</td>
<td>1245</td>
<td>3.75</td>
<td>2</td>
<td>-2 (5.8)</td>
<td>-9.5</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Aug. 19, 1984</td>
<td>1130</td>
<td>2.00</td>
<td>2</td>
<td>Aug. 20, 1984</td>
<td>1845</td>
<td>2.00</td>
<td>2</td>
<td>6 (5.8)</td>
<td>-14.1</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>Sept. 18, 1984</td>
<td>1000</td>
<td>1.50</td>
<td>2</td>
<td>Sept. 20, 1984</td>
<td>0645</td>
<td>1.75</td>
<td>2</td>
<td>-16 (5.8)</td>
<td>-14.4</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>Nov. 2, 1984</td>
<td>0845</td>
<td>1.75</td>
<td>2</td>
<td>Nov. 3, 1984</td>
<td>0500</td>
<td>2.00</td>
<td>2</td>
<td>-1 (5.8)</td>
<td>-7.5</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>Nov. 18, 1984</td>
<td>1045</td>
<td>3.00</td>
<td>3</td>
<td>Nov. 20, 1984</td>
<td>1415</td>
<td>2.00</td>
<td>3</td>
<td>4 (4.4)</td>
<td>-12.5</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>Dec. 3, 1984</td>
<td>0815</td>
<td>2.75</td>
<td>3</td>
<td>Dec. 4, 1984</td>
<td>1315</td>
<td>3.00</td>
<td>3</td>
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<td>-15.6</td>
<td></td>
</tr>
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<td>29</td>
<td>Jan. 1, 1985</td>
<td>0645</td>
<td>1.50</td>
<td>2</td>
<td>Jan. 4, 1985</td>
<td>0730</td>
<td>1.50</td>
<td>2</td>
<td>6 (5.8)</td>
<td>-15.2</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>Feb. 2, 1985</td>
<td>0645</td>
<td>1.75</td>
<td>2</td>
<td>Feb. 5, 1985</td>
<td>0400</td>
<td>2.00</td>
<td>2</td>
<td>12 (5.8)</td>
<td>-21.5</td>
<td></td>
</tr>
<tr>
<td>31</td>
<td>Mar. 12, 1985</td>
<td>0930</td>
<td>2.00</td>
<td>3</td>
<td>Mar. 14, 1985</td>
<td>0615</td>
<td>1.50</td>
<td>2</td>
<td>6 (5.1)</td>
<td>-19.9</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>Apr. 21, 1985</td>
<td>1515</td>
<td>2.25</td>
<td>3</td>
<td>Apr. 22, 1985</td>
<td>1445</td>
<td>4.25</td>
<td>6</td>
<td>-2 (3.8)</td>
<td>-16.9</td>
<td></td>
</tr>
<tr>
<td>34</td>
<td>July 6, 1985</td>
<td>1130</td>
<td>1.25</td>
<td>2</td>
<td>July 7, 1985</td>
<td>1245</td>
<td>1.25</td>
<td>2</td>
<td>8 (5.8)</td>
<td>-12.2</td>
<td></td>
</tr>
</tbody>
</table>

1First gravity survey done shortly after onset of deflation.
2Second gravity survey done shortly before deflation was complete.

Table 47.4.—Point-source solutions to level data for selected time intervals in Kilauea summit-area deflation events

[Uncertainties given are 1 standard error]

<table>
<thead>
<tr>
<th>Time interval</th>
<th>Longitude (W°)</th>
<th>Latitude (N°)</th>
<th>Depth (km)</th>
<th>Edifice volume change (10⁶ m³)</th>
<th>Vertical movement of HVO23 (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dec. 28, 1983–Mar. 22, 1984</td>
<td>155.2848° (±0.06 km)</td>
<td>19.4002° (±0.13 km)</td>
<td>2.92 (±0.19)</td>
<td>-4.42 (±0.70)</td>
<td>-9 (±2)</td>
</tr>
<tr>
<td>July 23, 1984–Jan. 4, 1985</td>
<td>155.2801° (±0.58 km)</td>
<td>19.4014° (±0.62 km)</td>
<td>2.46 (±0.78)</td>
<td>-3.26 (±0.13)</td>
<td>-0.5 (±2)</td>
</tr>
<tr>
<td>Apr. 21, 1985–Apr. 22, 1985</td>
<td>155.2753° (±0.30 km)</td>
<td>19.4101° (±0.11 km)</td>
<td>2.87 (±0.37)</td>
<td>-6.09 (±2.04)</td>
<td>-15 (±7)</td>
</tr>
<tr>
<td>Dec. 28, 1983–July 8, 1985</td>
<td>155.2822° (±0.13 km)</td>
<td>19.3960° (±0.26 km)</td>
<td>3.06 (±0.26)</td>
<td>-13.68 (±3.38)</td>
<td>-24 (±6)</td>
</tr>
</tbody>
</table>
(Walsh and Rice, 1979). The gravity effect of (3) for other source geometries, such as vertical and horizontal cracks, are not zero (Savage, 1984). While other geometries (consistent with the geologic structures seen in ancient volcanoes) have been suggested by Dieterich and Decker (1975) and Ryan and others (1983), the point source approach has been used in this report because of its computational convenience. In considering the gravity data presented here, the assumption was made that the gravity effect of (3) was indeed zero.

Because changes in the magma content of the subsurface reservoir are partly accommodated by slight elastic compression of the total volume of reservoir magma, the density of the reservoir magma can change slightly through time. In this analysis, the phrase magma-volume change indicates a change in the magma content of the reservoir and is expressed using units of volume calculated at a constant, arbitrary density of 2.75 g/cm³. Although the parameter that is actually being measured by gravity is the mass change, conversion to volume change is done here to enable comparison with dense-rock equivalent volumes of lava flows and intrusions reported elsewhere. Another type of subsurface volume change discussed here is changes in the capacity of the reservoir, which refers to changes in the physical dimensions of the reservoir.

**RELATION BETWEEN EDIFICE AND SUBSURFACE VOLUME CHANGES**

The ratio of the change in magma volume of the reservoir, \( \Delta V_m \), to change in volume of the edifice, \( \Delta V_e \), may be estimated from gravity changes, \( \Delta g' \), derived from the observed gravity differences, \( \Delta g \), by correcting for vertical displacement with a free-air gradient of \(-0.3086 \mu\text{Gal/mm}\), and from elevation differences, \( \Delta h \). The analysis is based on a point dilatational source in an elastic half-space (see, for example Mogi, 1958). The volume change of the edifice was determined by integrating vertical surface displacement. Uplift at the surface is given (Mogi, 1958) by:

\[
\Delta h = \frac{3a^3 \Delta P}{4\mu_e} \cdot \frac{Z}{(X^2 + Z^2)^{3/2}},
\]

where \( a \) is the radius of the source, \( \Delta P \) is the pressure change, \( \mu_e \) is Lame's constant for the edifice, \( Z \) is depth to the source, and \( X \) is horizontal distance to the chamber. Integration of equation 1 over the Earth's surface gives an expression for the volume increase of the edifice, \( \Delta V_e \):
\[ \Delta V_e = \frac{3\pi a^3 \Delta P}{2\mu_c}. \]  

Division of equation 2 by equation 1 and rearrangement yields:

\[ \Delta V_e = \frac{2\pi (X^2 + Z^2)^{3/2} \Delta h}{Z}. \]  

The mass change of the reservoir was determined next. So that it may be compared with volumes of subsidence and of eruptive products, mass change is expressed in terms of an effective magma-volume change, \( \Delta V_m \), using a constant density \( \rho \) for the magma. The gravitational attraction of the new magma, approximated by a point mass, is given by:

\[ \Delta g = \Delta V_m \gamma \rho \frac{Z}{(X^2 + Z^2)^{3/2}}, \]  

where \( \gamma \) is the universal gravitational constant (\( 6.67 \times 10^{-11} \text{ nm}^2/\text{kg}^2 \)) and \( \rho \) is the standard magma density. If the location of the dilational source in equation 3 is assumed to be the same as the new magma in equation 4, a combination of equations 3 and 4 yields an expression relating the observed gravity and elevation change to volume changes of the edifice and magma within the reservoir:

\[ \frac{\Delta V_m}{\Delta V_e} = \frac{\Delta g'}{\Delta h 2\pi \gamma \rho}. \]

Jachens and Eaton (1980) and Dzurisin and others (1980) found a good spatial correlation between gravity changes and elevation changes at Kilauea; their results are consistent with coincident sources for the gravity changes and the dilatation. This correlation of gravity and elevation changes indicates that in equation 5 the ratio of relative changes at two or more points showing differential changes can be used as validly as the ratio of absolute changes. The present analysis uses relative changes between two points. Notice that when \( \Delta V_m = \Delta V_e \) (that is, when there is no subsurface density change), equation 5 is equivalent to the Bouguer gradient. Furthermore, when \( \Delta V_e = 1.5 \Delta V_m \) (that is, no magma compression but dilation of the edifice as predicted by the Mogi model with a Poisson’s ratio of 0.25), equation 5 plus the free-air gradient is equivalent to theoretical gravity gradients proposed by Dzurisin and others (1980, equation A-11) and Hagiwara (1977, equation 19). In this analysis, the ratio of magma accumulation in the reservoir
to surface uplift is considered to be an independent variable. Equation 5 shows that the gravity-change gradient $\Delta g'/\Delta h$ is near $0.08 \, \mu\text{Gal/mm}$ for an incompressible magma with a density of 2.75 $\text{g/cm}^3$ accommodated by edifice expansion. The gradient approaches infinity for magma compressed into an unyielding edifice.

Gravity measurements were made near the beginning and end of 15 summit-deflation events corresponding to eruptive episodes 13–17, 24–32, and 34 at Puu Oo. Because of the brevity of each deflation, these gravity changes should not be influenced by long-term changes (for example, south-flank movement or magma migration unrelated to eruption), but should reflect mass and elevation changes related to simple elastic deflation. The changes in relative gravity at HVO34 and in east-west tilt at Uwekahuna during the monitored deflations are summarized in table 47.2. No real gravity changes ($\Delta g$) significantly larger than the data uncertainty were measured during these events because the gravity data reflect a composite of free-air gravity increases owing to subsidence and gravity decreases, $\Delta g'$, owing to removal of magma.

Elevation changes of HVO34 between gravity surveys were estimated for the period from December 28, 1983, to July 16, 1985, from the calculated relation between elevation of HVO34 and tilt at Uwekahuna. A factor of 5.19 mm/µrad was derived by a least-squares fit to tilt and elevation data (fig. 47.4). Elevation changes during eruptive episode 32 were measured directly by leveling surveys conducted on April 21 and 22, 1985. Table 47.3 summarizes the estimates of $\Delta g'$ (corrected for vertical displacement with a free-air gradient of $-0.3086 \, \mu\text{Gal/mm}$), $\Delta h$, and the ratio $\Delta V_m'/\Delta V_e$ calculated using equation 5. Because virtually no real gravity changes ($\Delta g$) were measured, most of the $\Delta g'$ values shown in table 47.3 result from the free-air correction. These calculations show that the volume of magma lost from the reservoir is 1.2 to 4.5 times (average $2.4 \pm 0.21$, 1 standard error) greater than the volume of subsidence at the surface during the periods of rapid deflation.

SUBSURFACE VOLUME CHANGES DURING DEFLATION EVENTS

The empirical relation between volume changes of magma within the summit reservoir and volume changes of the volcanic edifice can be used to estimate the former from measurements of the latter. This means of estimating magma-volume changes at depth is
particularly useful for studying intrusions, in which no lava reaches the surface.

The change in volume of the edifice ($\Delta V_e$) was computed by fitting a point-source model (Dvorak and others, 1983) to measured elevation changes for periods when sufficient elevation change and a uniform pattern of surface displacement enabled a stable solution. These periods were: December 28, 1983, to March 22, 1984; July 23, 1984, to January 4, 1985; and April 21 to 22, 1985. The solutions obtained are listed in table 47.4.

Edifice-volume changes (table 47.4) were interpolated to the time of the gravity surveys by means of the continuous record of tilt change at Uwekahuna. From the estimated volume changes of the edifice and the calculated ratios of subsurface-magma to edifice volume change given in table 47.3, the changes of magma volume in the reservoir were calculated. These volume changes are listed in table 47.5, together with estimated lava-flow volumes from the Puu...
TABLE 47.3.—Changes in residual gravity ($\Delta g'$) and elevation ($\Delta h$), ratio between them, and ratio of subsurface ($\Delta V_m$) to edifice volume ($\Delta V_e$) changes for 15 Kilauea deflation episodes, 1984–85

[Note: Elevation changes interpolated from tilt changes using factor of 5.19 mm/µrad. Subsidence during eruptive episode 32 was determined by leveling surveys before and after period of deflation.]

<table>
<thead>
<tr>
<th>Eruptive episode</th>
<th>$\Delta g'$ (µGal)</th>
<th>$\Delta h$ (mm)</th>
<th>$\Delta g'/\Delta h$ (µGal/mm)</th>
<th>$\Delta V_m/\Delta V_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>-21</td>
<td>-54</td>
<td>0.38</td>
<td>3.3</td>
</tr>
<tr>
<td>14</td>
<td>-8</td>
<td>-38</td>
<td>0.20</td>
<td>1.8</td>
</tr>
<tr>
<td>15</td>
<td>-6</td>
<td>-46</td>
<td>0.13</td>
<td>1.2</td>
</tr>
<tr>
<td>16</td>
<td>-20</td>
<td>-73</td>
<td>0.28</td>
<td>2.4</td>
</tr>
<tr>
<td>17</td>
<td>-17</td>
<td>-49</td>
<td>0.35</td>
<td>3.0</td>
</tr>
<tr>
<td>24</td>
<td>-16</td>
<td>-73</td>
<td>0.23</td>
<td>2.0</td>
</tr>
<tr>
<td>25</td>
<td>-39</td>
<td>-75</td>
<td>0.52</td>
<td>4.5</td>
</tr>
<tr>
<td>26</td>
<td>-13</td>
<td>-39</td>
<td>0.33</td>
<td>2.9</td>
</tr>
<tr>
<td>27</td>
<td>-16</td>
<td>-65</td>
<td>0.25</td>
<td>2.1</td>
</tr>
<tr>
<td>28</td>
<td>-23</td>
<td>-81</td>
<td>0.28</td>
<td>2.5</td>
</tr>
<tr>
<td>29</td>
<td>-18</td>
<td>-79</td>
<td>0.23</td>
<td>2.0</td>
</tr>
<tr>
<td>30</td>
<td>-22</td>
<td>-111</td>
<td>0.20</td>
<td>1.7</td>
</tr>
<tr>
<td>31</td>
<td>-26</td>
<td>-103</td>
<td>0.25</td>
<td>2.2</td>
</tr>
<tr>
<td>32</td>
<td>-27</td>
<td>-80</td>
<td>0.33</td>
<td>2.9</td>
</tr>
<tr>
<td>34</td>
<td>-11</td>
<td>-63</td>
<td>0.18</td>
<td>1.6</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td>2.4</td>
</tr>
</tbody>
</table>

TABLE 47.5.—Edifice volume change ($\Delta V_e$), subsurface volume change ($\Delta V_m$), and lava flow volume for 12 Kilauea eruptive episodes, 1984–85

[All figures in millions of cubic meters. Edifice volume changes, from level data inversion, interpolated to time of gravity surveys by tilt variations. Subsurface mass changes, expressed as volume change at standard density of 2.75 g/cm$^3$, estimated using equation 5 and changes of edifice volume, relative gravity, and relative elevation. Measured lava-flow volumes (Wolfe and others, chapter 17; George Ulrich, written comm., 1985) not adjusted for flow porosity.]

<table>
<thead>
<tr>
<th>Eruptive episode</th>
<th>$\Delta V_e$</th>
<th>$\Delta V_m$</th>
<th>Lava volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>-3.7</td>
<td>-12.3</td>
<td>10</td>
</tr>
<tr>
<td>14</td>
<td>-2.6</td>
<td>-4.7</td>
<td>6</td>
</tr>
<tr>
<td>15</td>
<td>-3.2</td>
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<td>8</td>
</tr>
<tr>
<td>16</td>
<td>-5.1</td>
<td>-12.3</td>
<td>12</td>
</tr>
<tr>
<td>17</td>
<td>-3.4</td>
<td>-10.2</td>
<td>10</td>
</tr>
<tr>
<td>24</td>
<td>-4.0</td>
<td>-7.9</td>
<td>12</td>
</tr>
<tr>
<td>25</td>
<td>-4.0</td>
<td>-18.2</td>
<td>11</td>
</tr>
<tr>
<td>26</td>
<td>-2.1</td>
<td>-6.1</td>
<td>7</td>
</tr>
<tr>
<td>27</td>
<td>-3.5</td>
<td>-7.3</td>
<td>8</td>
</tr>
<tr>
<td>28</td>
<td>-4.4</td>
<td>-11.0</td>
<td>12</td>
</tr>
<tr>
<td>29</td>
<td>-4.3</td>
<td>-8.6</td>
<td>13</td>
</tr>
<tr>
<td>32</td>
<td>-6.1</td>
<td>-17.7</td>
<td>16</td>
</tr>
<tr>
<td>Total</td>
<td>-46.4</td>
<td>-120.0</td>
<td>125</td>
</tr>
</tbody>
</table>
Oo vent for comparison (Wolfe and others, chapter 17; George Ulrich, written commun., 1985).

Absence of significant geodetic change along the rift zones during 1984 and early 1985 indicates that the cumulative volume of lava erupted was equal to the volume of magma drained from the summit reservoir. This inference is supported by the gravity and surface-displacement data. A total of $125 \times 10^6$ m$^3$ of lava was extruded during the monitored eruptive episodes (table 47.5). Corrected for 20 percent (Swanson, 1972) to 25 percent (Wolfe and others, chapter 17) vesicularity, the total volume of lava flows is $94 \times 10^6 - 100 \times 10^6$ m$^3$. For comparison, the cumulative volume of reservoir magma lost during the deflations, based on gravity and geodetic measurements, is an estimated $120 \times 10^6$ m$^3$ (table 47.4).

Gravity and elevation data for eruptive episode 32 were obtained over a timespan of 24 h bracketing the deflation. Because independent calculations of the volume change of the edifice and of the subsurface magma were made from these data, without requiring tiltmeter interpolation, the results for this period provide a good check of the more general procedure. While the edifice subsided by about $6 \times 10^6$ m$^3$ during eruptive episode 32, nearly $18 \times 10^6$ m$^3$ (calculated to 2.75 g/cm$^3$) of magma was removed from the reservoir (table 47.5). This agrees well with the estimated $12 \times 10^6 - 13 \times 10^6$ m$^3$ (table 47.5, corrected for 20–25 percent vesicularity) of lava erupted at Puu Oo.

The general agreement between reservoir magma-volume change and lava output supports the idea of a volume-for-volume link between the summit reservoir and Puu Oo, with magma transfer from summit to rift zone taking place principally during episodes of summit deflation. In other words, during eruptions at Puu Oo, summit magma enters the rift-zone conduit system and forces out an equal volume of rift-zone magma at the Puu Oo vent.

**ELASTIC PROPERTIES OF VOLCANO AND MELT**

Data for the ratio of edifice-volume to subsurface magma-volume changes for summit deflations accompanying eruptions of Puu Oo may be used to model the relative strength of the volcanic edifice and the magma-reservoir region. For this purpose, I have derived an expression that relates the ratio of subsurface-magma and edifice-volume changes to the ratio of the rigidity of the edifice, $\mu_e$, and the compressibility of the reservoir melt, $K_m$. Assumptions are that Poisson's ratio for the edifice is 0.25, that the source region can
be approximated by a spherical volume of fluid, that the injected magma has a density of 2.75 g/cm$^3$, and that the response of the volcanic edifice and contained fluid to subsurface pressure changes is elastic. Although the source region most likely contains subsolidus rock in addition to a melt phase, the aggregate bulk modulus of the source region is assumed equal to the bulk modulus of the melt phase. This assumption is valid because the effect of the melt phase is to reduce the aggregate bulk modulus to close to that of melt (Ryan, 1980). First, the change in capacity of the spherical source region was determined. A pressure change within the reservoir displaces the reservoir boundary radially outward. The change in radius, $\Delta a$, of a spherical source region of radius $a$ is given by (Hagiwara, 1977):

$$\Delta a = \frac{a \Delta P}{4 \mu_e}.$$  \hspace{1cm} (6)

If the radius of the source is large relative to the change in radius, the volume of the newly added shell, $\Delta V_r$, is approximately:

$$\Delta V_r = \frac{\pi a^3 \Delta P}{\mu_e}.$$  \hspace{1cm} (7)

The volume of magma, $\Delta V_m$, accommodated by compression of the fluid within the spherical reservoir region of volume $4\pi a^3/3$ and within the newly added shell of magma, $\Delta V_r$, is dependent on the bulk modulus of the magma, $K_m$:

$$\Delta V_m = \frac{\Delta P}{K_m} \left(\frac{4\pi a^3}{3} + \Delta V_r\right).$$  \hspace{1cm} (8)

For simplicity, the relatively small term $\Delta V_r$ in equation 8 can be ignored. The total amount of magma injected is the sum of magma accommodated by enlargement of the reservoir (equation 7) and by compression of the material in the reservoir (equation 8):

$$\Delta V_m = \frac{4}{3} \pi a^3 \Delta P \left(\frac{1}{K_m} + \frac{3}{4\mu_e}\right).$$  \hspace{1cm} (9)

The combination of equations 9 and 2 gives:

$$\frac{\Delta V_m}{\Delta V_r} = \frac{8}{9} \frac{\mu_e}{K_m} + \frac{2}{3},$$  \hspace{1cm} (10)

which can be related directly to the gravity gradient by equation 5.
Some edifice deformation and magma compression are expected during magma injection. Equation 10 shows that, for a given value of $\Delta V_m/\Delta V_e$, a relatively strong edifice implies more magma compression. Conversely, a relatively strong magma implies more edifice deformation. Solution of equation 10, incorporating the average $\Delta V_m/\Delta V_e$ factor of 2.4±0.21 (1 standard error) from the 15 deflations listed in table 47.3, yields $\mu_e = 2.0\pm0.24 K_m$. Additional error stems from uncertainty of the magma density, here assumed to be 2.75 g/cm$^3$.

The effective shear modulus of Kilauea's edifice, $\mu_e$, is difficult to determine independently because Kilauea is built of many vesicular, commonly rubbly lava flows with abundant void space (Ryan and others, 1983). The intrinsic shear modulus of Hawaiian basalt was found by pulse transmission to average 25 GPa (Manghnani and Wollard, 1968) with a range of 4.7-40.5 GPa. Samples with greater porosity had a lower shear modulus. Presence of voids in the volcanic pile results in an effective shear modulus of the volcano that is less than the intrinsic shear modulus of hand samples.

Seismic velocities, and hence rigidity, determined from $P$-wave arrival data from earthquakes and explosions increase with depth and vary laterally (Thurber, chapter 38; Hill and Zucca, chapter 37). This reflects partly the compressive reduction of porosity with increasing depth, and the solidified intrusive zones surrounding the cores of the summit reservoir and rift zones. Any estimate of $\mu_e$ from seismic velocities would be strongly dependent on the depth and horizontal position of the source, and size of the region chosen to represent the edifice.

The bulk modulus of magma, $K_m$, in Kilauea's reservoir should be uniform because the size of the summit magma reservoir is perhaps 1,000 times the yearly magma supply rate (Swanson, 1972), so short-term fluctuations in the gas content and chemical composition of the supplied magma are distributed over a large volume and do not affect the overall bulk modulus. Laboratory determinations of the bulk modulus made from lava erupted from Kilauea, therefore, should be representative of the bulk modulus of magma in the reservoir. The value of 11.5 GPa determined for $K_m$ of molten Kilauea 1921 olivine tholeiite by Murase and others (1977) was used to calculate the shear modulus of Kilauea's edifice from the relation $\mu_e = (2.0\pm0.24) \cdot K_m$ determined above. The value for $\mu_e$ so determined, 23±3 GPa, is close to the value for the average shear modulus of Hawaiian basalt given above. Note that this value does not apply to Kilauea's edifice as a whole, but only to
the region surrounding the magma reservoir. This is because it was determined by modeling in-place melt-volume changes in the reservoir and resulting surface displacements.

Presence of a CO₂ gas phase in the magma would lower the aggregate bulk modulus of the magma. This may not be important for Kilauea, because, as Greenland and others (1985) assert, as CO₂ is lost from the magma as soon as it reaches the shallow magma reservoir. However, if some CO₂ is retained in the reservoir, then both the magma bulk modulus and the predicted edifice shear modulus would be reduced.

**RELATION BETWEEN LONG-TERM INELASTIC EDIFICE WIDENING AND SUBSURFACE VOLUME CHANGES**

The ratio of the long-term change of magma volume within Kilauea's summit reservoir to the edifice volume change between January 6, 1984, and July 16, 1985, may be determined from equation 5 and from the long-term elevation and gravity changes. A gradient of \( \Delta g/\Delta h = -0.29 \text{ \mu Gal/mm} \) was determined for the interval by a least-squares fit to the tilt and gravity data, adjusted by the elevation/tilt correlation of 5.19 mm/\( \mu \text{rad} \) given above. This is essentially the free-air gradient, so \( \Delta g' \) is zero and equation 5 shows that the ratio of \( \Delta V_m/\Delta V_e \) is zero. That is, there was no net change in magma storage in the summit reservoir during this interval.

The long-term ratio of volume changes in edifice and subsurface magma is different from that in short-term deflations accompanying Puu Oo eruptions (table 47.3). This implies that different processes affect the volume changes over longer time intervals. Earlier, the short-term deflations of Kilauea were explained using an elastic model, with inflow and outflow of magma accompanied by compression and decompression of the melt. Because short-term and long-term processes occurred concurrently, variations in the physical properties of the magma cannot explain differences in the volume-change ratios. The differences between the long-term and short-term volume changes must then be due to some superimposed long-term process that affects Kilauea's edifice. That process produces subsidence at the surface without a change in the magma content of the reservoir. One possibility is that horizontal spreading of the volcano's summit removes lateral support of the fluid magma reservoir, thereby decreasing the pressure in the reservoir and causing the summit to subside. Horizontal distance measurements at Kilauea have shown
that the summit region is prone to widening in a south-southeast direction (Dvorak and others, 1983). This widening was particularly notable during the 1975 magnitude 7.2 earthquake along the south flank and for several months afterward (Lipman and others, 1985).

To investigate the possibility of summit extension during 1984–85, horizontal displacements measured by trilateration surveys conducted on January 16, 1984, and July 16, 1985, were analysed. In order to isolate the pattern of regional displacements in the vicinity of Kilauea's summit from the effects of observed deflation, the contribution of changes in the inflation level of the reservoir (approximated by use of a point source of dilatation determined from leveling data) was subtracted from the observed horizontal displacements. Residual line lengths were then used to compute horizontal displacement vectors (fig. 47.5). South-southeastward movement as great as 250 mm of points located on the south side of Kilauea caldera relative to points to the north show that the caldera widened slightly.

A $\Delta g'/\Delta h$ gradient of 0.138 $\mu$Gal/mm was measured by Jachens and Eaton (1980) during the subsidence accompanying the November 29, 1975, earthquake and eruption. A value for the $\Delta V_m/V_e$ ratio of 1.2 was calculated from the gravity gradient and equation 5 using an assumed magma density of 2.75 g/cm$^3$. This ratio indicates a greater component of surface collapse following the 1975 eruption, compared to the 1984–85 deflations (table 47.3). This difference results from 1.8 m of south-southeast crosscaldera extension during the November 1975 event (Lipman and others, 1985), in contrast to no measurable extension during the periods of very brief (1–3 d) deflation associated with eruptions in 1984 and 1985. The increased collapse in November 1975 is consistent with horizontal dilation of the edifice and surface subsidence because of the resultant drop in reservoir pressure.

Horizontal extension of Kilauea's edifice lowers the pressure in the magma reservoir; additional magma can be injected without uplifting the surface. In this way the reservoir volume increases by lateral growth. Growth by horizontal spreading and magma intrusion occurs during rift-zone development (Swanson and others, 1976), and it might also apply to the growth of the summit reservoir. Swanson and others (1976) attributed horizontal dilation of the rift zone to forceful injection of magma into dikes. However, at Kilauea's summit area, observation of subsidence concurrent with extension indicates that magma accumulation there is a passive response to
dilation. Perhaps the extension of Kilauea's summit that increases the capacity of the reservoir is an indirect response to the forceful dilation of the rift zones that bound the caldera on both the east and west sides.

Net gravity and elevation changes for the 16-mo period following the November 1975 deflation imply mass filling of the reservoir concurrent with subsidence. Dzurisin and others (1980) reported a $\Delta g'/\Delta h$ gradient of $-0.294 \ \mu\text{Gal/mm}$ for December 1975 to April 1977, which corresponds to a $\Delta V_g'/\Delta V_e$ ratio of $-2.5$ from equation 5, using a magma density of 2.75 g/cm$^3$. This, combined with observed subsidence of the summit area of as much as 180 mm, indicates that the volume of magma in the reservoir actually increased despite the subsidence. The summit region widened by about 0.5 m in a south-southeast direction (Lipman and others, 1985) during this period; that extension created more reservoir space in which additional magma accumulated.

CONCLUSIONS

Gravity and surface-displacement data demonstrate that volume changes of Kilauea's edifice, as determined by integrating vertical displacements, are dependent, not only on the balance between volume of magma supplied from the mantle and volume of magma lost from the reservoir by eruption but also on the volume of summit collapse owing to horizontal extension of the summit region. During times of elastic inflow or outflow of magma from the reservoir, the ratio of subsurface-magma volume change to edifice volume change is $2.4 \pm 0.21$ (1 standard error). This is consistent with expected changes, calculated using a ratio of shear strength of edifice to bulk modulus of magma of $2.0 \pm 0.24$ (1 standard error). On the basis of a laboratory-determined magma bulk modulus of $K_m = 11.5$ GPa, the shear modulus of the edifice is $\mu_e = 23$ GPa.

Inelastic horizontal dilation of the edifice in a south-southeast direction increases the capacity of the summit reservoir to accept magma without surface uplift. For the 16-mo period following the November 1975 deflation, additional magma was accommodated by this process. When insufficient magma is supplied to fill space created by summit widening, the pressure in the reservoir drops and the surface subsides as it did during and after the November 1975 earthquake and between January 1984 and February 1985.
FIGURE 47.5.—Horizontal displacements in the Kilauea summit region determined from line-length changes (January 16, 1984, to July 16, 1985). To remove effects of summit subsidence, computed horizontal displacements based on inversion of level data obtained on December 28, 1983, and July 8, 1985 (table 47.4), were subtracted from measured changes in line lengths between January 16, 1984, and July 16, 1985. Computed residual displacements indicate south-southeastward widening of Kilauea summit area.
REFERENCES CITED


REFERENCES CITED


